Topic 21: Exercises on Resisted Motion
Level 3, Part 2

1. A particle is moving vertically downward under gravity in a medium which exerts a resistance to the motion which is proportional to the speed of the particle. It is released from rest at O, and its terminal velocity is V. Find the distance it has fallen below O and the time taken when its velocity is one half of its terminal velocity.

$$-\frac{V^2}{2g}+\frac{V^2}{g}\ln 2,\ \frac{V}{g}\ln 2$$

2. A parachutist of mass m falls freely until his parachute opens. When it is open he experiences an upward resistance mkv, where v is his speed and k is a positive constant. The parachutist falls freely for a time $\frac{1}{2k}$ and then opens his parachute. Find the total distance he has fallen when his speed is $\frac{3g}{4k}$.

$\frac{g}{8k^2}(8\ln 2 - 1)$

3. A particle of mass m falls from rest under gravity and the resistance to its motion is mkv^2, where v is its speed and k is a positive constant. As the distance it has fallen increases from d_1 to $2d_2$, the speed increases from v_1 to $\frac{5}{4}v_1$. Express the greatest possible speed of the particle in terms of v_1.

$\frac{4v_1}{\sqrt{7}}$

4. A projectile is fired vertically upward from the surface of the earth with speed V. The acceleration due to gravity is $\frac{gR^2}{x^2}$ where R is the radius of the earth and x is the distance from the centre of the earth.

(i) Neglecting air resistance, show that if $V = \sqrt{(gR)}$, then the speed v of the projectile at distance x from the centre of the earth is given by $v = \sqrt{gR}\sqrt{\frac{2R-x}{x}}$

(ii) Hence show that the projectile reaches a height R above the surface of the earth, and find the time taken to reach this height.

$$\left(\frac{\pi}{2}+1\right)\cdot\sqrt{\frac{R}{g}}$$

5. A particle of mass m is projected vertically upward under gravity with speed nV in a medium in which the resistance to motion is mk times the square of the speed of the particle, where k and n are positive constants and V is the terminal velocity of the particle in this medium.

(a) Find the time taken by the particle to return to its starting point.

$$\frac{V}{g}\left\{\tan^{-1} n + \ln\left(n + \sqrt{n^2 + 1}\right)\right\}$$

(b) Find the speed with which the particle returns to its starting point.

$$V \cdot \sqrt{\frac{n^2}{n^2+1}}$$