



CRANBROOK  
SCHOOL

## Year 11 Extension 1 Mathematics

### Term 1 Examination

March, 2012

Time Allowed: 1.5 hours *plus* 5 minutes reading time

Total Marks: 48

There are 4 questions, each of equal value.

Start a new booklet for each question.

All necessary working should be shown in every question.

Full marks may not be awarded if work is careless or badly arranged.

Approved calculators may be used.

Question 1—Harder Basic Arithmetic, Algebra and Equations  
Start a new booklet

(12 marks)

- a.  $\left(27 - \frac{y^3}{8}\right)$  factorised is equal to 1
- A.  $\left(3 + \frac{y}{2}\right)\left(9 + 3y + \frac{y^2}{4}\right)$
- B.  $\left(3 + \frac{y}{2}\right)\left(9 - \frac{3y}{2} + \frac{y^2}{4}\right)$
- C.  $\left(3 - \frac{y}{2}\right)\left(9 + \frac{3y}{2} + \frac{y^2}{4}\right)$
- D.  $\left(3 - \frac{y}{2}\right)\left(9 - \frac{3y}{2} + \frac{y^2}{4}\right)$
- b. If  $\sqrt{x+1} = 3$  then  $(x+1)^2$  equals: 1
- A.  $\sqrt{3}$       B. 3      C. 9      D. 27      E. 81
- c. A person's wage increased by  $a\%$  to \$250 per week.  
Find the wage just prior to this increase, in dollars per week, in terms of  $a$ . 1
- d. Find the values for  $x$  and  $y$  if  $15x - y - 7 = 0$  and  $x - y = 0$  2
- e. Solve  $\frac{2}{x+1} \geq 3$  3
- f. Solve  $\left|\frac{x+1}{2}\right| = 3x - 2$  2
- g. Find the coordinates where the hyperbola  $xy = 4$  and the circle  $x^2 + y^2 = 10$  intersect 2

**Question 2 – Harder Surds and Indices**  
Start a new booklet

(12 marks)

- a. A student was asked to rationalise the denominator of  $\frac{1}{2\sqrt{3}}$ . The correct answer would be:

A.  $\frac{2\sqrt{3}}{3}$       B.  $\frac{\sqrt{3}}{4\sqrt{3}}$       C.  $\frac{\sqrt{3}}{6}$       D.  $\frac{2\sqrt{3}}{1}$

- b. If  $2^x = 80$ , what would be the value of  $2^{x+1}$ ?

A. 81      B. 160      C. 40      D. 8.94

- c. Simplify  $\frac{1}{6}\sqrt{3} + \frac{1}{6}\sqrt{12} + \frac{1}{2}\sqrt{27}$ , showing all working. 2

- d. Simplify the following surd by rationalising the denominator. 2

$$\frac{1}{2 + \sqrt{x+1}}$$

- e. Simplify  $\frac{x\sqrt{x}}{x^3}$  2

- f. Simplify  $\frac{9^x \times 3^{x+1}}{27}$  2

- g. Simplify  $\frac{a^{-3} + b^{-3}}{a^{-4} + b^{-4}}$  leaving your answer with positive indices. 2

**Question 3 – Harder Linear Functions**  
Start a new booklet

(12 marks)

- a. The acute angle between the lines  $y = -x + 5$  and  $2x - y + 2 = 0$  can be found using:

A.  $\tan \phi = |-1 - 2|$

B.  $\tan \phi = \left| \frac{-1 - 2}{1 + 2} \right|$

C.  $\tan \phi = -3$

D.  $\tan \phi = \left| \frac{-1 - 2}{1 - 2} \right|$

- b. The points A(3,4) and B(9,10) are divided internally 2:1. The resulting point Q is:

A. (7,8)      B.  $\left(\frac{13}{3}, 6\right)$       C.  $\left(\frac{21}{2}, 12\right)$       D.  $\left(6, \frac{13}{3}\right)$

- c. The points A(0,3), B(-2,-5) and C(8,1) are the vertices of a triangle.

- i. Find the equation of the perpendicular bisector of side BC. 3

- ii. Show that the bisector passes through the vertex A. 1

- iii. What does this tell you about triangle ABC? 1

- d. A is the point (-2, 1) and B is the point (x, y). The point P(13, -9) divides AB externally in the ratio 5 : 3. Find the values of x and y. 3

- e. Find the acute angle between the lines  $y = -x$  and  $\sqrt{3}y = x$ . 2

**Question 4– Harder Functions and Graphs**  
Start a new booklet

(12 marks)

a.  $\lim_{k \rightarrow -4} \frac{x+4}{16-x^2}$  is equal to

1

A.  $-\frac{1}{8}$

B. 0

C.  $\infty$

D.  $\frac{1}{8}$

b. The function  $f(x) = \frac{\sqrt{1-x}}{x}$  has a point of discontinuity at

1

A.  $x=1$

B.  $x < 1$

C.  $x > 1$

D.  $x=0$

c. Sketch  $y = \frac{3}{x-1}$  showing all significant points

3

d. Sketch  $(x-1)^2 + y^2 = 4$

3

e. Sketch  $y = \frac{3x}{x^2 - 2x - 3}$

5

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Q1a)  $(27 - \frac{y^3}{8}) \text{ (C) } \checkmark$  e)  $\frac{2}{x+1} - 3 \geq 0$   $x \neq -1$

$= (3)^3 - (\frac{y}{2})^3$   
 $= (3 - \frac{y}{2})(3^2 + 3 \times \frac{y}{2} + (\frac{y}{2})^2)$   
 $(3 - \frac{y}{2})(9 + \frac{3y}{2} + \frac{y^2}{4})$

$(x+1) \frac{(2-3x-3)}{(x+1)} \geq 0$   $x(x+1)^2$

$(x+1)(-3x-1) \geq 0$   
 $-1 < x \leq -\frac{1}{3}$

1)  $\sqrt{x+1} = 3$  TAKING SQUARE ROOTS  
 $x+1 = 9$   
 $\therefore (x+1)^2 = 81$  **(E)**  $\checkmark$

2) Let  $x =$  wage prior to increase.

$x + a\% \text{ of } x = 250$

$x + (\frac{a}{100} \times x) = 250$

$x(1 + \frac{a}{100}) = 250$

$x = \frac{250}{1 + \frac{a}{100}}$

$= \frac{25000}{100+a}$   $\checkmark$

d)  $15x - y = 7$  ①  
 $x - y = 0$  ②

①-②  $14x = 7$   
 $x = \frac{1}{2}$   $\checkmark$

SUBST into ②

$\frac{1}{2} - y = 0$

$\therefore y = \frac{1}{2}$   $\checkmark$

$\therefore x = \frac{1}{2}, y = \frac{1}{2}$   $\checkmark$

f)  $|\frac{x+1}{2}| = 3x-2$

$|x+1| = 6x-4$

$x+1 = 6x-4$   $x-1 = 6x-4$   
 $x = 1$   $-7x = -3$   
 $x = \frac{3}{7}$

test  $|\frac{1}{2}| = 3-2 = 1$  test  $|\frac{3}{7}| \neq \frac{-5}{7}$   
 $\therefore x = 1$  only

g)  $xy = 4$  ①  $\therefore y = \frac{4}{x}$

$x^2 + y^2 = 10$  ②

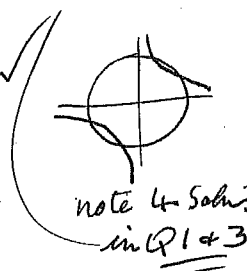
$x^2 + \frac{16}{x^2} = 10$

$x^4 - 10x^2 + 16 = 0$

$(x^2 - 8)(x^2 - 2) = 0$

$x = \pm\sqrt{8}, \pm\sqrt{2}$

$y = \pm\sqrt{2}, \pm\sqrt{8}$



note 4 Solns in Q1+3

$\therefore$  Solutions are

$(2\sqrt{2}, \sqrt{2})$   $(-2\sqrt{2}, -\sqrt{2})$   $\checkmark$

$(\sqrt{2}, 2\sqrt{2})$   $(-\sqrt{2}, -2\sqrt{2})$   $\checkmark$

(always use sketches to help if possible)

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QUESTION 2

a.  $\frac{1}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{2 \times 3} = \frac{\sqrt{3}}{6}$

= **(C)**  $\checkmark$

b.  $2^x = 80$

$2^{x+1} = 2^x \times 2^1$  (INDEX LAWS!)

$= 80 \times 2$

$= 160 =$  **(B)**  $\checkmark$

c.  $\frac{1}{6}\sqrt{3} + (\frac{1}{6} \times 2\sqrt{3}) + (\frac{1}{2} \times 3\sqrt{3})$

$= \frac{1}{6}\sqrt{3} + \frac{2}{6}\sqrt{3} + \frac{3}{2}\sqrt{3}$   $\checkmark$

$= 2\sqrt{3}$   $\checkmark$

- QUITE POORLY DONE... DISAPPOINTING, CONSIDERING THAT A VERY SIMILAR QUESTION WAS IN THE PREVIOUS TEST.

- \*NOTE!!  $\frac{1}{2} \times 3\sqrt{3} \neq 3\frac{1}{2}\sqrt{3}$ !

d.  $\frac{1}{2 + \sqrt{x+1}} \times \frac{2 - \sqrt{x+1}}{2 - \sqrt{x+1}} \checkmark$

$$= \frac{2 - \sqrt{x+1}}{4 - (x+1)} \leftarrow \text{REM. BRACKETS!}$$

$$= \frac{2 - \sqrt{x+1}}{3 - x} \checkmark$$

- COULD HAVE BEEN ANSWERED BETTER
- FORGETTING BRACKETS WAS THE MAIN PROB.

e.  $\frac{x\sqrt{x}}{x^{4/3}} = \frac{x \cdot x^{1/2}}{x^{4/3}}$

$$= \frac{x^{3/2}}{x^{4/3}} \checkmark \quad \frac{9}{6} = \frac{8}{6}$$

$$= x^{1/6} \text{ OR } \sqrt[6]{x} \checkmark$$

- MANY STUDENTS WERE RUSTY WITH THEIR INDEX LAWS!

f.  $\frac{9^x \times 3^{x+1}}{27}$

$$= \frac{(3^2)^x \times 3^{x+1}}{3^3} \checkmark \text{ (ALL SAME BASE)}$$

$$= \frac{3^{2x} \times 3^{x+1}}{3^3}$$

PTO  $\rightarrow$

$$= \frac{3^{3x+1}}{3^3} \text{ (INDEX LAWS)}$$

$$= 3^{3x-2} \checkmark$$

- MANY STUDENTS 'CANCELLED' THE BASE
- MANY STUDENTS THOUGHT  $9^{3x} = 3^{3x}$

g.  $\frac{1}{a^3} + \frac{1}{b^3}$

$$\frac{1}{a^4} + \frac{1}{b^4}$$

MAKE NUMERATOR & DENOMINATOR 1 FRACTION

$$= \frac{b^3 + a^3}{a^3 b^3}$$

$$\frac{a^4 + b^4}{a^4 b^4} \checkmark$$

REM:  $\frac{1}{a} + \frac{1}{b}$  IS NOT THE RECIPROCAL OF  $a+b$ .

$$= \frac{b^3 + a^3}{a^3 b^3} \times \frac{a^4 b^4}{a^4 b^4} = \frac{a^3 + b^3}{1} \times \frac{ab}{a^4 + b^4}$$

$$= \frac{ab(a^3 + b^3)}{a^4 b^4} \text{ OR } \frac{a^4 b + ab^4}{a^4 + b^4} \checkmark$$

- A LOT DID NOT KNOW WHAT TO DO!

YEAR 11 EXT. 1 TERM 1  
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QUESTION 3

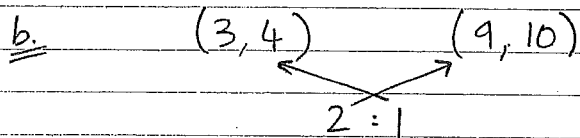
a.  $y = -x + 5 \therefore m_1 = -1$

$y = 2x + 2 \therefore m_2 = 2$

$$\tan \phi = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$\tan \phi = \left| \frac{-1 - 2}{1 - 2} \right| = \boxed{D} \checkmark$$

- ANSWERED QUITE WELL!



$$Q = \left( \frac{3 + 2 \times 9}{3}, \frac{4 + 2 \times 10}{3} \right)$$

$$= \left( \frac{21}{3}, \frac{24}{3} \right)$$

$$= (7, 8) = \boxed{A} \checkmark$$

- NOT ANSWERED AS WELL. MANY MUDDLED THE FORMULA.

c.  $B(-2, -5) \quad C(8, 1)$

$$i. m_{BC} = \frac{1 + 5}{8 + 2} = \frac{6}{10} = \frac{3}{5}$$

$$\therefore m_{\text{perp. bisector}} = -\frac{5}{3} \checkmark$$

PERP. BISECTOR PASSES THRU MIDPT. OF BC. MANY DID NOT FIND THIS!

$$M_{BC} = \left( \frac{-2 + 8}{2}, \frac{-5 + 1}{2} \right)$$

$$= \left( \frac{6}{2}, \frac{-4}{2} \right)$$

$$= (3, -2) \checkmark$$

$\therefore$  EQN OF PERP. BISECTOR...

USE  $(3, -2)$   
NOT  $(0, 3)$

$$\rightarrow y + 2 = -\frac{5}{3}(x - 3)$$

$$3y + 6 = -5x + 15$$

$$5x + 3y - 9 = 0 \checkmark \text{ OR } y = -\frac{5}{3}x + 3$$

- ANSWERED QUITE POORLY.
- MANY DID NOT FIND THE BISECTOR, JUST USED THE THIRD VERTEX  $(0, 3)$ . AT THIS POINT WE CANT ASSUME THE PERP. BISECTOR PASSES THRU THE VERTEX.
- ALTHOUGH THE CORRECT ANSWER MAY BE ACHIEVED, WE ARE TESTING UNDERSTANDING.

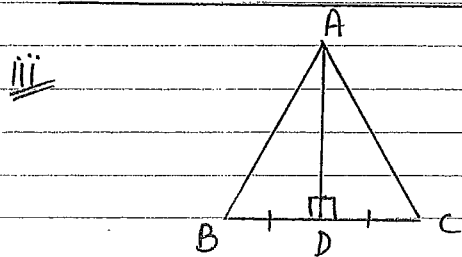
ii) DOES  $5x + 3y - 9 = 0$  PASS THRU  $(0,3)$ ?  
SUB  $(0,3)$  IN!

$$5 \times 0 + 3 \times 3 - 9 = 0$$

$$9 - 9 = 0$$

$$\text{LHS} = \text{RHS} \checkmark$$

- COULD HAVE BEEN ANSWERED MUCH BETTER.
- SOME SIMPLY HAD THE STATEMENT:  
 $0 + 9 - 9 = 0$
- WHILE THIS IS TRUE, IT IS A RANDOM EQUATION NOT DIRECTLY LINKED TO THE Q'N.



BD = DC  
AD COMMON  
 $\angle ADB = \angle ADC$   
 $\therefore \triangle ABD \equiv \triangle ADC \Rightarrow AB = AC$   
 $\therefore$  TRIANGLE IS ISOSCELES  $\checkmark$

- MARKING WAS VERY GENEROUS FOR THIS Q'N
- TO STATE THAT THE TRIANGLE WAS EQUILATERAL YOU HAD TO SHOW THAT ALL SIDES WERE =.
- PLEASE LEARN TO SPELL 'ISOSCELES'  $\ddot{\circ}$

d.  $(-2, 1)$   $\rightarrow$   $(x, y)$   
 $5 : -3$  ..... HAS TO BE -VE:EXT!

$$(13, -9) = \left( \frac{5x + (-3x - 2)}{2}, \frac{5y + (-3 \times 1)}{2} \right) \checkmark$$

$$\therefore 13 = \frac{5x + 6}{2} \quad / \quad -9 = \frac{5y - 3}{2}$$

$$26 = 5x + 6 \quad / \quad -18 = 5y - 3$$

$$20 = 5x \quad / \quad -15 = 5y$$

$$\therefore x = 4 \quad / \quad \therefore y = -3 \checkmark$$

$(4, -3)$

- ANSWERED REASONABLY WELL.
- MANY DIDN'T MAKE ONE OF THE NUMBERS IN THE RATIO -VE.

e.  $y = -x \quad \therefore m_1 = -1$

$$\sqrt{3}y = x$$

$$y = \frac{1}{\sqrt{3}}x \quad \therefore m_2 = \frac{1}{\sqrt{3}}$$

$$\tan \phi = \left| \frac{-1 - \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} \right| \checkmark$$

$$\phi = 75^\circ \checkmark$$

CAN BE TOO CONVOLUTED, JUST PUT IN CALC! (USE MEMORY)

- A LOT OF STUDENTS THOUGHT...

IF YOU FACTORISE PROPERLY...

$$\frac{-1 - \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} = 1 \quad \text{(NO!)}$$

$$\rightarrow \frac{-(1 + \frac{1}{\sqrt{3}})}{1 - \frac{1}{\sqrt{3}}} \neq 1$$

- DON'T ROUND OFF NUMERATOR & DENOMINATOR
- ALSO DON'T WRITE DOWN A 10-DIGIT DECIMAL.
- USE THE CALC'S MEMORY.

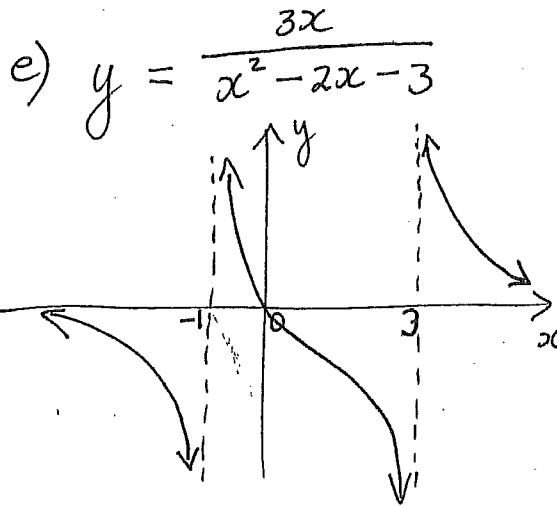
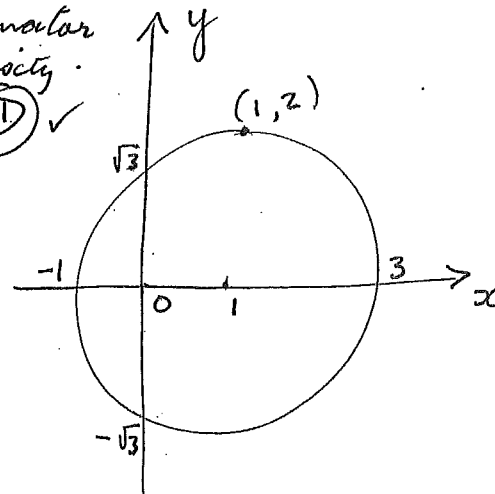
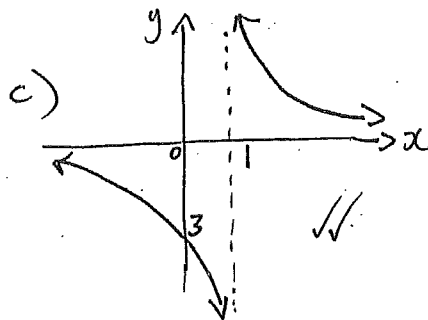
Q4a) 2012 EXT 1 TERM 1 EXAM

$$\lim_{h \rightarrow -4} \frac{(x+4)}{(4+h)(4-x)}$$

MUST FACTORISE FIRST

$$= \frac{1}{8} \quad \text{(D) } \checkmark$$

b) zero in the denominator is undefined.  $\therefore$  pt of discontinuity occurs at  $x=0$  (D)  $\checkmark$



PLEASE READ

Marks were generously awarded to those who understood the equation and associated graphs and made this clear. A variety of methods of arriving at these is possible and acceptable

HOWEVER (D) The majority of students do need to work on detail, labelling, and the actual lines of their graphs. In many cases this needs a serious effort. Please make sure it happens as part of your corrections - even if you did not lose marks THIS TIME!! (D)