CRANBROOK
SCHOOL

Year 11 (2U) Mathematics

Term 1 Examination + Solutions

March, 2012

Time Allowed: 1.5 hours *plus* 5 minutes reading time

Total Marks: 48

There are 4 questions, each of equal value.

Start a new booklet for each question.

All necessary working should be shown in every question.

Full marks may not be awarded if work is careless or badly arranged.

Approved calculators may be used.

Question 1 – Basic Arithmetic, Algebra and Equations **(12 marks)**

Start a new booklet

For parts (a) and (b) select the most appropriate answer from A. B. C. or D.

a) If $x=-3$ and $y=-4$ which of the following statements is true: 1

A. $|x+y|=|x|-|y|$

B. $|x+y|<|x|-|y|$

C. $|x+y|>|x|-|y|$

D. $|x+y|\approx|x|-|y|$

b) Solve $3^{x-3}=27$ 1

A. $x=3$

B. $x=6$

C. $x=0$

D. $x=12$

c) Simplify $\dfrac{4(x-3)+y(x-3)}{x^3-27}$ 2

d) Solve the following:

i) $|7x-1|=34$ 2

ii) $3x^2=11x+4$ 2

iii) $k^2+9k+14\le 0$ 2

iv) $6x^2+5x-2=0$ using the quadratic formula 2

Question 2 – Surds and Indices **(12 marks)**
Start a new booklet

For parts (a) and (b) select the most appropriate answer from A. B. C. or D.

a) Simplify $\sqrt{27}+\sqrt{75}-\sqrt{243}$ 1

A. $\sqrt{-141}$

B. $-\sqrt{141}$

C. $\sqrt{3}$

D. $-\sqrt{3}$

b) What must you multiply $\frac{3}{\sqrt{5}-\sqrt{2}}$ by to rationalise the denominator? 1

A. $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

B. $\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

C. $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}$

D. $\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$

c) Simplify the following:

i) $\left(3\sqrt{5}\right)^2$ 1

ii) $\frac{9-\sqrt{63}}{3}$ 2

d) Expand and simplify $\left(5-\sqrt{3}\right)^2$ 1

e) Solve for a and b if $\frac{\sqrt{5}}{\sqrt{5}-\sqrt{3}}=a+b\sqrt{15}$ 2

f) Simplify $\frac{(5k)^3}{25k^{-2}}$ 2

g) Evaluate $\left(\frac{(k)^{-3}\times(w)^{-4}}{(w)^{-3}\times(k)^{-4}}\right)^{\frac{3}{2}}$ if $k=8$ and $w=2$ 2

Question 3 – Linear Functions **(12 marks)**
Start a new booklet

For parts (a) and (b) select the most appropriate answer from A. B. C. or D.

a) What is the gradient of the line perpendicular to the line $2x+3y-5=0$ 1

A. $-\frac{2}{3}$

B. $\frac{2}{3}$

C. $\frac{3}{2}$

D. $-\frac{1}{2}$

b) The x-intercept of the line $2x+3y-5=0$ is: 1

A. 5

B. $\frac{5}{3}$

C. $\frac{5}{2}$

D. $-\frac{5}{2}$

c) Find the equation of the line that goes through the point A(3,-4) and is parallel to the line $y=\frac{2}{3}x-1$ 2

d) Find the equation of the line that is the perpendicular bisector of the interval joining the points R(-1,5) and S(3,-2). Leave your answer in general form. 3

e) Find the perpendicular distance between the line $y=x+5$ and the point A(4,7). 2
Leave answer in exact form with a rational denominator.

f) Find the equation of the line through Q(2,-5) and the point of intersection of the lines $2x+3y-13=0$ *and* $3x-y-3=0$. 3

Question 4 – Functions and Graphs **(12 marks)**
Start a new booklet

For parts (a) and (b) select the most appropriate answer from A. B. C. or D.

a) If $f(2)=-1$ and $f(-1)=-4$ then $f(x)$ can be 1

A. x^3

B. $x+3$

C. x^2-5

D. $-(x-5)^2$

b) Which of the following are even functions? 1

A. $f(x)=x^3+3$

B. $f(x)=x^4+x^2$

C. $f(x)=\frac{1}{x^3-x^2}$

D. All of the above

c) If $f(x)=\frac{2x+1}{\sqrt{x+1}}$ find:

i) $f(3)$ 1

ii) x if $f(x)=0$ 1

d) Sketch the following functions

i) $f(x)=4x-8$ 2

ii) $f(x)=3x-x^2$ 3

iii) $f(x)=6x^2+x-12$ 3

END OF ASSESSMENT

✓ = 1 MARK

a) ① $|-3+-4| > |-3| - |-4|$

$|-7| > 3-4$

$7 > -1$ TRUE

$\therefore$ (C) ✓

a) Absolute value bars behave like brackets WORK THEM 1ST.

b) ① $3^{6-3} = 3^3$

$= 27$

$\therefore$ (B) ✓

b) Either solve or test the given options

c) ② $\dfrac{4(x-3)+y(x-3)}{x^3-27}$

$= \dfrac{(x-3)(4+y)}{(x-3)(x^2+3x+9)}$ ✓

$= \dfrac{4+y}{x^2+3x+9}$ ✓

c) $(x-3)$ is the common factor

MUST FACTORISE BEFORE Cancelling!

The denominator is a DIFFERENCE OF 2 CUBES LEARN THE RULE

Well done by many — but those who have not bothered to learn it might consider whether this is the course for them!

There are not many new rules yet - learn them as they happen!

d) (i) ② $7x-1=34$ $\quad -7x+1=34$

$7x=35$ $\quad -7x=33$

$x=5$ ✓ $\quad x=-\dfrac{33}{7}$ ✓

d) (i) Most did this well but some showed sloppy algebra - yr 7/8 equation solving! - SET IT OUT carefully and neatly.

(ii) ② $3x^2-11x-4=0$

$(3x+1)(x-4)=0$ ✓

$x=-\frac{1}{3}, 4$ ✓

(ii) Generally well done Some need more practice

(iii) ② $(k+7)(k+2) \leq 0$ ✓

$-7 \leq x \leq -2$ ✓

(iii) Again a new idea that MUST BE understood - it appears again and again later in the course WORK ON these if you did not get it right.

(iv) $x=\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$ ✓

$= \dfrac{-5 \pm \sqrt{73}}{12}$ ✓

(iv) Soooo Simple IF you know the formula To the few who didn't LEARN IT!

Question 2:

(a) $\sqrt{27}+\sqrt{75}-\sqrt{243}$

$= \sqrt{9}\times\sqrt{3} + \sqrt{25}\times\sqrt{3} - \sqrt{81}\times\sqrt{3}$

$= 3\sqrt{3} + 5\sqrt{3} - 9\sqrt{3}$

$= -\sqrt{3}$

$\therefore$ (D) ✓

• Must first express surds in simplest form before addition/subtraction.

(b) $\dfrac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}$ $\therefore$ (C) ✓

(c)

(i) $(3\sqrt{5})^2$

$= 9\times 5$

$= 45$ ✓

• must apply power to 3 & $\sqrt{5}$.
• could also just put it into your calculator.

(ii) $\dfrac{9-\sqrt{63}}{3}$

$= \dfrac{9-\sqrt{9}\times\sqrt{7}}{3}$

$= \dfrac{{}^{3}9-3\sqrt{7}}{\cancel{3}}$ ✓

$= 3-\sqrt{7}$ ✓

• must express surd in simplest form first.
• simplify using fractional laws.

(d) $(5-\sqrt{3})^2$

$= 25-10\sqrt{3}+3$

$= 28-10\sqrt{3}$ ✓

• use normal rule for expanding $(x-y)^2 = x^2-2xy+y^2$

(e) $\dfrac{\sqrt{5}}{\sqrt{5}-\sqrt{3}} = a+b\sqrt{15}$

$= \dfrac{\sqrt{5}(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})}$

$= \dfrac{5+\sqrt{15}}{5-3}$

$= \dfrac{5}{2} + \dfrac{\sqrt{15}}{2}$ ✓

$\therefore a=\frac{5}{2}$, $b=\frac{1}{2}$ ✓

• For all these sorts of questions you must always start by rationalising the denominator.
• break up fraction into a form similar to $a+b\sqrt{15}$.

(f) $\dfrac{(5k)^3}{25k^{-2}}$

$= \dfrac{125k^3\times k^2}{25}$ ✓

$= 5k^5$ ✓

• follow your index laws of power to a power, then remove negative index, and simplify.

(g) $\left(\dfrac{k^{-3}\times w^{-4}}{w^{-2}\times k^{-4}}\right)^{3/2}$ if $k=8$, $w=2$.

$= \left(\dfrac{w^2k^4}{k^3w^4}\right)^{3/2}$

$= \left(\dfrac{k}{w^2}\right)^{3/2}$ ✓

$= \left(\dfrac{8}{2^2}\right)^{3/2}$

$= 4^{3/2}$

$= 8$ ✓

• remove negative index and then simplify.
• from this point sub in values for k & w.
• you could also plug this into your calculator.

Question 3:

(a) $3y = -2x + 5$

$y = \frac{-2x}{3} + \frac{5}{3}$

$\therefore$ Perp. gradient $= \frac{3}{2}$

$\therefore$ (C) ✓

(b) $2x + 3y - 5 = 0$

x-int @ $y = 0$.

$\therefore 2x + 3(0) - 5 = 0$.

$2x = 5$

$x = 5/2$. $\therefore$ (C) ✓

(c) $m = \frac{2}{3}$, Point $A(3, -4)$

$y - (-4) = \frac{2}{3}(x - 3)$ ✓

$y + 4 = \frac{2x}{3} - 2$

$y = \frac{2}{3}x - 6$ ✓

or $2x - 3y - 18 = 0$

(d) Midpoint of $R(-1, 5)$ & $S(3, -2)$

$= \left(\frac{-1+3}{2}, \frac{5-2}{2}\right)$

$\therefore M = (1, 3/2)$ ✓

Gradient $= \frac{5 - (-2)}{-1 - 3}$

$= \frac{7}{-4}$

$\therefore$ Perp gradient $= \frac{4}{7}$ ✓

$\therefore y - \frac{3}{2} = \frac{4}{7}(x - 1)$

$7y - \frac{21}{2} = 4(x - 1)$

$14y - 21 = 8(x - 1)$

* Q4 answer to leave in general form !!

$14y - 21 = 8x - 8$

$\therefore 8x - 14y + 13 = 0$ ✓

(e) Perp. distance $y = x + 5$ & $A(4, 7)$

$\therefore x - y + 5 = 0$ & $A(4, 7)$

$d = \left| \frac{4 - 7 + 5}{\sqrt{1^2 + (-1)^2}} \right|$ ✓

$= \frac{2}{\sqrt{2}}$

$= \frac{2(\sqrt{2})}{\sqrt{2}(\sqrt{2})}$ * Must rationalise

$= \frac{2\sqrt{2}}{2}$

$= \sqrt{2}$ ✓

(f)

→ Method 1: k method

$(2x + 3y - 13) + k(3x - y - 3) = 0$

↳ sub $Q(2, -5)$

$(4 - 15 - 13) + k(6 + 5 - 3) = 0$ ✓

$\therefore 8k = 24$

$\therefore k = 3$ ✓

$\therefore (2x + 3y - 13) + 3(3x - y - 3) = 0$.

$2x + 3y - 13 + 9x - 3y - 9 = 0$.

$\therefore 11x - 22 = 0$.

$\therefore x = 2$. ✓ * Had to be simplified

OR

→ Method 2: simultaneous equations.

$2x + 3y - 13 = 0$ — (1)

$3x - y - 3 = 0$ — (2)

Question 3 - continued...

Manipulate (2)

↳ $y = 3x - 3$ — (3)

sub (3) into (1)

$\therefore 2x + 3(3x - 3) - 13 = 0$.

$2x + 9x - 9 - 13 = 0$.

$11x = 22$

$\therefore x = 2$ ✓

sub $x = 2$ into (3)

$y = 3(2) - 3$

$\therefore y = 6 - 3$

$= 3$

$\therefore$ Equation between $Q(2, -5)$ and intersection point $(2, 3)$ ✓

↳ by inspection, x-values are the same.

$\therefore$ equation of line is $x = 2$ ✓

* Students lost marks if they used second method and decided that

$m = \frac{8}{0} = 0$

Q4 a) $f(2) = 2^2 - 5 = -1$ $\quad f(-1) = (-1)^2 - 5 = 1 - 5 = -4$

$\therefore$ (C) ✓

b) $f(-x) = (-x)^4 + (-x)^2$
$= x^4 + x^2$
$= f(x)$ $\therefore$ (B) ✓

a) b) c) i) Well done
tho' a few needed to simplify
$\sqrt{4} = 2$!

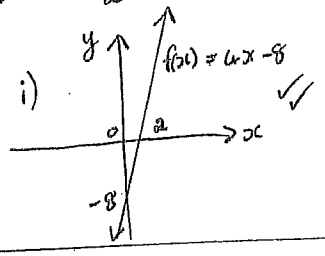
c) i) $f(3) = \frac{2(3)+1}{\sqrt{3+1}}$
$= \frac{7}{\sqrt{4}}$
$= \frac{7}{2}$ ✓

c) ii) Remember XO always gives O

ii) $f(x) = 0$
$\frac{2x+1}{\sqrt{x+1}} = 0$
$\sqrt{x+1} \times \frac{2x+1}{\sqrt{x+1}} = 0 \times \sqrt{x+1}$
$2x+1 = 0$
$x = -\frac{1}{2}$ ✓

d) SKETCHING NEEDS more care and practise
AND LABELLING
LABEL AXES, ORIGIN, INTERCEPTS and the graph.
PUT arrows on axes and graph.

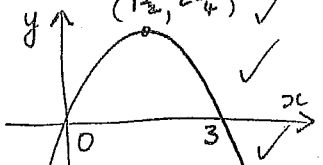
d) i)



✓✓

d(iii) $f(x) = 6x^2 + x - 12$
$(3x-4)(2x+3) = 0$
$x = 1\frac{1}{3}, -1\frac{1}{2}$ ✓
y-intercept $(0, -12)$
axis of symmetry $= -\frac{1}{12}$ ✓
$f(-\frac{1}{12}) = -\frac{289}{24} = -12\frac{1}{24}$

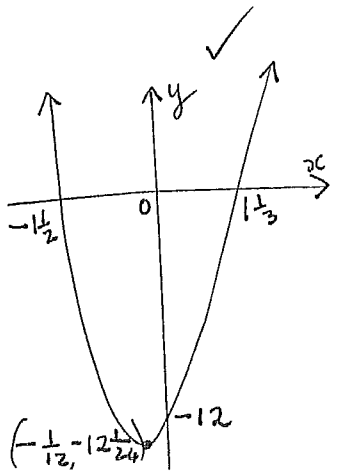
ii) $f(x) = 3x - x^2$



✓ ✓ ✓

$f(x) = x(3-x)$
FOR x intercepts let $y = 0$
$0 = x(3-x)$
$x = 0, 3$
NOTE WE NOW HAVE the y intercept

axis of symmetry
$x = 1\frac{1}{2}$
VERTEX $(1\frac{1}{2}, 2\frac{1}{4})$
$f(x) = 1\frac{1}{2}(3 - 1\frac{1}{2})$
$= 2\frac{1}{4}$
(OR use $x = -\frac{b}{2a}$)



✓