

Name: _____



KAMBALA

Mathematics Extension 1

Preliminary HSC Assessment Task 1

April 2007

Time Allowed: 50 minutes

INSTRUCTIONS

- This task contains 3 questions of 13 marks each. Marks for each question are shown.
- Answer all questions on the paper provided.
- Start each question on a new page.
- Calculators may be used.
- Show all necessary working.
- Marks may be deducted for careless or badly arranged work.

Question 1 Start a new page. 13 Marks

- (a) Divide xy by $\left(\frac{1}{x} + \frac{1}{y}\right)$, expressing your answer as a single fraction in simplest form. 2
- (b) Solve for x , in exact form, the following equation : 3
- $$x^2 + 2x + \frac{3}{x^2 + 2x} = 4$$
- (c) Find the point(s) of intersection, if any, for the curves $y = 25 - x^2$ and $x + y = 13$. 3
- (d) Show that $\frac{2\sqrt{5}+1}{2\sqrt{5}-1} + \frac{2\sqrt{5}-1}{2\sqrt{5}+1}$ is a rational number. 3
- (e) Without using a calculator, find the value of : 2
- $$(\sqrt{3}+1)^4 + (\sqrt{3}-1)^4$$

Question 2 Start a new page. **13 Marks**

- (a) Use the technique of completing the square to show that $7 + 4x - x^2$ never exceeds 11. 2
- (b) (i) Factorise $2^{n+1} + 2^n$ 1
- (ii) Hence, write $\frac{2^{1001} + 2^{1000}}{3}$ as a power of 2. 1
- (c) Solve, for all real values of x : $\frac{3-x^2}{x} \geq 2$ 3
- (d) Let $f(x) = \frac{x^2}{x^2-1}$.
- (i) Show that $f\left(\frac{1}{2}\right) = f\left(-\frac{1}{2}\right)$ 1
- (ii) For what value(s) of x is $f(x)$ undefined? 1
- (iii) Determine whether the function $y = f(x)$ is odd, even or neither. 2
- (e) The centre of a circle has co-ordinates (1, 2). The end-points of a diameter are (1, -3) and (1, 7). Find the equation of the circle. 2

Question 3 Start a new page. **13 Marks**

- (a) Consider the following set of numbers:

$$5^{-\frac{n}{3}}, 5^{\frac{n}{3}}, 5^n, 5^{-n}$$

Given that n is a positive number, state which number is :

- (i) the largest in the set 1
- (ii) the smallest in the set 1
- (b) For an odd function, $f(-x) = -f(x)$ for all values of x . What is the geometrical significance of this fact (i.e. what does this say about the graph of an odd function)? 1
- (c) Sketch $y = [(x-a)(x+b)]$, clearly showing the x and y intercepts, (where a and b are both positive). 2
- (d) (i) Show that $\frac{4x-7}{x-2} = 4 + \frac{1}{x-2}$. 1
- (ii) Hence sketch the function $y = 4 + \frac{1}{x-2}$, showing all essential features. 2
- (e) (i) Solve algebraically $|x+3| = 1-x$. 2
- (ii) On the same diagram, sketch the graphs of $y = |x+3|$ and $y = 1-x$. 2
- (iii) Use these graphs to solve $|x+3| > 1-x$. 1

End of Assessment Task

Kambala

Year 11 Extension 1 Mathematics

Assessment Task 1

April 2007

SOLUTIONS

Question One

a) $\frac{xy}{\frac{1}{x} + \frac{1}{y}}$
 $= \frac{xy}{\frac{x+y}{xy}}$
 $= xy \times \frac{xy}{x+y}$
 $= \frac{x^2 y^2}{x+y}$

b) $x^2 + 2x + \frac{3}{x^2 + 2x} = 4$

let $u = x^2 + 2x$
 $\therefore u + \frac{3}{u} = 4$
 $\therefore u^2 + 3 = 4u$
 $\therefore u^2 - 4u + 3 = 0$
 $(u-3)(u-1) = 0$
 $\therefore u = 1, 3$
 $\therefore x^2 + 2x = 1$ or $x^2 + 2x = 3$
 $x^2 + 2x - 1 = 0$ $x^2 + 2x - 3 = 0$
 $x = \frac{-2 \pm \sqrt{4+4}}{2}$ $(x+3)(x-1) = 0$
 $x = -2 \pm \sqrt{2}$ $x = -3, 1$

$\therefore x = \frac{2(-1 \pm \sqrt{2})}{2}$

$\therefore x = -1 \pm \sqrt{2}$

c) $y = 25 - x^2$
 $x + y = 13$
 $\therefore y = 13 - x$
 $\therefore 13 - x = 25 - x^2$
 $\therefore x^2 - x - 12 = 0$
 $(x-4)(x+3) = 0$
 $\therefore x = 4, -3$
 when $x = 4, y = 9$
 when $x = -3, y = 16$
 \therefore pts of intersection are $(4, 9)$ and $(-3, 16)$

d) $\frac{2\sqrt{5}+1}{2\sqrt{5}-1} + \frac{2\sqrt{5}-1}{2\sqrt{5}+1}$
 $= \frac{(2\sqrt{5}+1)^2 + (2\sqrt{5}-1)^2}{4(5)-1}$
 $= \frac{20 + 4\sqrt{5} + 1 + 20 - 4\sqrt{5} + 1}{19}$
 $= \frac{42}{19}$, which is a rational number

e) $(\sqrt{3}+1)^4 + (\sqrt{3}-1)^4$
 $= (\sqrt{3}+1)^2(\sqrt{3}+1)^2 + (\sqrt{3}-1)^2(\sqrt{3}-1)^2$
 $= (4+2\sqrt{3})(4+2\sqrt{3}) + (4-2\sqrt{3})(4-2\sqrt{3})$
 $= 16 + 16\sqrt{3} + 12 + 16 - 16\sqrt{3} + 12$
 $= 56$

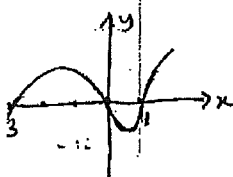
Question Two

a) $7 + 4x - x^2$
 $= -(x^2 - 4x - 7)$
 $= -(x^2 - 4x + 4 - 7 - 4)$

$= -(x-2)^2 + 11$
 but $(x-2)^2 \geq 0$ for all x
 $\therefore -(x-2)^2 \leq 0$ for all x
 $\therefore -(x-2)^2 + 11$ cannot exceed 11
 as 11 is its maximum value

b) i) $2^{n+1} + 2^n$
 $= 2^n(2+1)$
 $= 3 \times 2^n$
 ii) $2^{1001} + 2^{1000}$
 $= 3 \times 2^{1000}$
 $= 3 \times 2^{1000}$

c) $\frac{3-x^2}{x} \geq 2$
 $\therefore (3-x^2)x \geq 2x^2, x \neq 0$
 $3x - x^3 \geq 2x^2$
 $x^3 + 2x^2 - 3x \leq 0$
 $x(x+3)(x-1) \leq 0$



$\therefore x \leq -3, 0 \leq x \leq 1$
 but $x \neq 0$
 $\therefore x \leq -3, 0 < x \leq 1$

d) i) $f(x) = \frac{x^2}{x^2-1}$
 $f(\frac{1}{2}) = \frac{(\frac{1}{2})^2}{(\frac{1}{2})^2-1}$
 $= \frac{\frac{1}{4}}{\frac{1}{4}-1}$
 $= \frac{\frac{1}{4}}{-\frac{3}{4}}$
 $= -\frac{1}{3}$
 $f(-\frac{1}{2}) = \frac{(-\frac{1}{2})^2}{(-\frac{1}{2})^2-1}$
 $= \frac{\frac{1}{4}}{\frac{1}{4}-1}$
 $= -\frac{1}{3}$
 $= f(\frac{1}{2})$
 $\therefore f(\frac{1}{2}) = f(-\frac{1}{2})$ as required

ii) $f(x)$ undefined when $x^2 - 1 = 0$
 $\therefore (x-1)(x+1) = 0$
 $\therefore x = \pm 1$

iii) $f(x) = \frac{x^2}{x^2-1}$
 $f(-x) = \frac{(-x)^2}{(-x)^2-1}$
 $= \frac{x^2}{x^2-1}$
 $= f(x)$

$\therefore f(x) = f(-x)$
 $\therefore f(x)$ is an even function

e) $C(1, 2)$
 $r = 2 - (-3) = 7 - 2 = 5$
 \therefore eqn of circle is $(x-1)^2 + (y-2)^2 = 25$

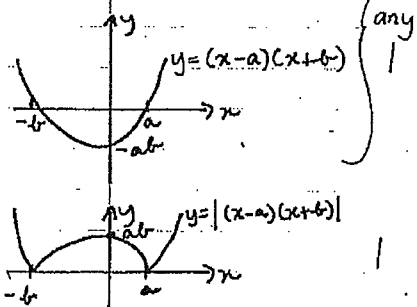
Question Three

- a) i) S^n
ii) S^{-n}

b) The graph of an odd function has point symmetry about the origin.

c) $y = |(x-a)(x+b)|$

x-intercepts at $x=a, -b$
y-intercept at $|ab| = ab$



e) i) $|x+3| = 1-x$

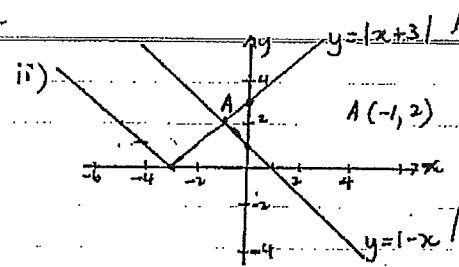
$\therefore x+3 = 1-x$ or $x+3 = x-1$
 $2x = -2$ or $0 = 4$
 $x = -1$ no real solns

check: $|-1+3| = 1-(-1)$

$|2| = 2$

true

$\therefore x = -1$ only



ii) for $|x+3| > 1-x$
By observation, $x > -1$

d) i) RTP: $\frac{4x-7}{x-2} = 4 + \frac{1}{x-2}$

RHS: $4 + \frac{1}{x-2}$

$= \frac{4(x-2)+1}{x-2}$

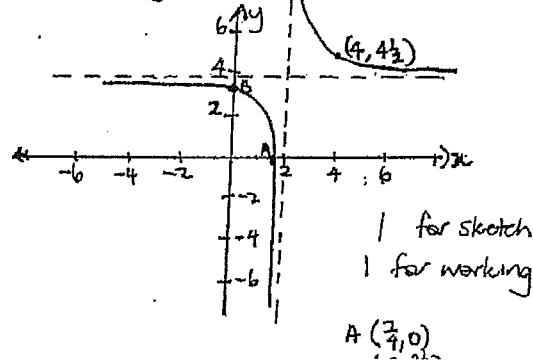
$= \frac{4x-8+1}{x-2}$

$= \frac{4x-7}{x-2} = \text{LHS}$

$\therefore \frac{4x-7}{x-2} = 4 + \frac{1}{x-2}$ as required

ii) $y = 4 + \frac{1}{x-2}$

u/d when $x=2, y=4$ $-4(x-2)=1$
when $x=0, y=3\frac{1}{2}$ $-4x+8=1$
when $y=0, \frac{1}{x-2} = -4$ $x = \frac{7}{4}$



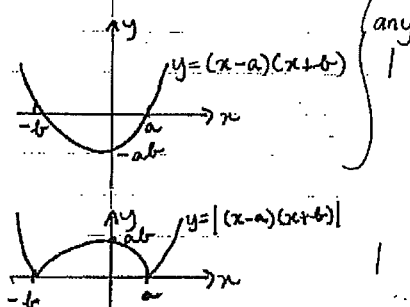
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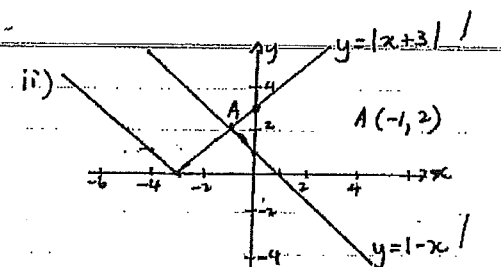
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