Student Number: ____________

Class Teacher (*circle*): KM GP

KAMBALA

Mathematics Extension 1

Preliminary HSC Assessment Task 1

April 2009

Time Allowed: 50 minutes

INSTRUCTIONS

- This task contains 3 questions. Marks for each question are shown.
- Answer all questions on the paper provided.
- **Start each question on a new page.**
- Calculators may be used.
- Show all necessary working.
- Marks may be deducted for careless or badly arranged work.

Question 1 **Start a new page.** **13 Marks**

(a) Expand and simplify $\left(\frac{1}{x}+2\right)^3$. 2

(b) Factorise fully $x^2-2xy+y^2-4$. 2

(c) Simplify $\frac{a}{m^2-n^2}-\frac{b}{m^2+mn}$. 2

(d) If $\frac{m}{mx-3}=7$ express m in terms of x. 2

(e) Express $\frac{a^2-b^2}{b^{-1}+a^{-1}}$ in simplest form without negative indices. 3

(f) Rationalise and write in simplest form $\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}$ where $a \neq b$. 2

Question 2 **Start a new page.** **13 Marks**

(a) Solve the equation $x^2 = \frac{3}{x^2} + 2$ for x. 3

(b) (i) Show that $\frac{3x-1}{x-2} = 3 + \frac{5}{x-2}$. 1

(ii) Hence or otherwise, state the domain and range of $y = \frac{3x-1}{x-2}$. 2

(iii) Sketch the curve $y = \frac{3x-1}{x-2}$, showing all its features. 2

(c) Consider the region given by the intersection of:

$$y < 4 - |2x|, \quad x \geq 0 \quad \text{and} \quad y \geq -2$$

(i) Sketch the region. 3

(ii) Calculate the area of the region. 2

Question 3 **Start a new page.** **13 Marks**

(a) Solve $\frac{3x+7}{x+4} > 2$ for x. 3

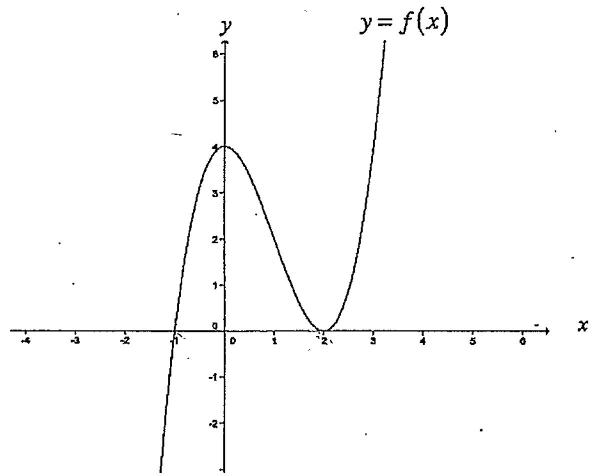
(b) Solve $|2x+3| = 3x+2$ for x. 3

(c) Consider the function defined by 2

$$f(x) = \begin{cases} x^2 - 4x + 4 & \text{for } x < 0 \\ 4x + 4 & \text{for } 0 \leq x < 2 \\ 16 - x^2 & \text{for } x \geq 2 \end{cases}$$

Find the value of $f(-a^2)$.

(d) (i) Find the equation of the curve $y = f(x)$ below. 1



(ii) Consider the line $y = b$. For what value(s) of b will $f(x) = b$ have three solutions? 2

(iii) Sketch the graph of $y = -f(x)$ showing all its features. 2

End of Assessment Task

Qn	Solutions	Marks	Comments+Criteria
1	(a) $\left(\frac{1}{x}+2\right)^3$ 1 3 3 1 $= \left(\frac{1}{x}\right)^3 + 3\left(\frac{1}{x}\right)^2\times 2 + 3\left(\frac{1}{x}\right)\times 2^2 + 2^3$ $= \frac{1}{x^3} + \frac{6}{x^2} + \frac{12}{x} + 8$	1 1	
	(b) $x^2-2xy+y^2-4$ $=(x-y)^2-4$ $=(x-y-4)(x-y+4)$	1 1	
	(c) $\frac{a}{m^2-n^2} - \frac{b}{m^2+mn}$ $= \frac{a}{(m-n)(m+n)} - \frac{b}{m(m+n)}$ $= \frac{a\times m - b(m-n)}{m(m-n)(m+n)}$ $= \frac{am-bm+bn}{m(m-n)(m+n)}$	1 1	
	OR $\frac{a}{m^2-n^2} - \frac{b}{m^2+mn}$ $= \frac{a(m^2+mn)-b(m^2-n^2)}{(m^2-n^2)(m^2+mn)}$ $\left[= \frac{am^2+amn-bm^2+bn^2}{(m^2-n^2)(m^2+mn)}\right]$ $= \frac{am(m+n) - b(m-n)(m+n)}{(m^2-n^2)(m^2+mn)}$	OR } 1	$= \frac{am-bm+bn}{m(m-n)(m+n)}$ 1

$= \frac{\cancel{(m+n)}\left[am - b(m-n)\right]}{m(m-n)(m+n)\cancel{(m+n)}}$ ↗

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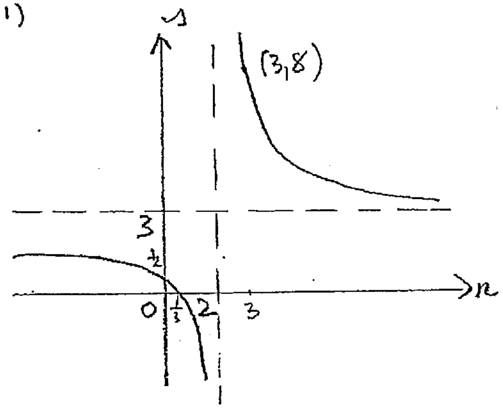
Qn	Solutions	Marks	Comments+Criteria
Q1 ctd	(d) $\frac{m}{mx-3}=7$ $m=7(mx-3)$ $m=7mx-21$ $m-7mx=-21$ $m(1-7x)=-21$ $m=\frac{-21}{1-7x}$	1 1	
	OR $\frac{m}{mx-3}=7$ $m=7(mx-3)$ $m=7mx-21$ $21=7mx-m$ $21=m(7x-1)$ $\therefore m=\frac{21}{7x-1}$	OR 1 1	
	(e) $\frac{a^2-b^2}{b^{-1}+a^{-1}}$ $=\frac{a^2-b^2}{\frac{1}{b}+\frac{1}{a}}$ 1 $=(a^2-b^2)\div\left(\frac{b+a}{ab}\right)$ $=(a-b)\cancel{(a+b)}\times\frac{ab}{\cancel{a+b}}$ $=ab(a-b)$	1	

Qn	Solutions	Marks	Comments+Criteria
Q1 ctd	(f) $\dfrac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} \times \dfrac{\sqrt{a}+\sqrt{b}}{\sqrt{a}+\sqrt{b}}$ $= \dfrac{a+2\sqrt{ab}+b}{a-b}$	1 1	
2	(a) $x^2 = \dfrac{3}{x^2}+2$ Let $M = x^2$ $M = \dfrac{3}{M}+2$ $M^2-2M-3=0$ $(M-3)(M+1)=0$ $\therefore M=3$ or $M=-1$ ie $x^2=3$ or $x^2=-1$ $x=\pm\sqrt{3}$ 1 no solution 1	1	
b	(i) Show $\dfrac{3x-1}{x-2} = 3+\dfrac{5}{x-2}$ LHS $= \dfrac{3x-1}{x-2}$ $= \dfrac{3x-6+5}{x-2}$ $= \dfrac{3(x-2)+5}{x-2}$ $= 3+\dfrac{5}{x-2}$ $=$ RHS	1	correct proof

Qn	Solutions	Marks	Comments+Criteria
Q2 ctd	(b) (i) ctd OR RHS $= 3+\dfrac{5}{x-2}$ $= \dfrac{3(x-2)+5}{x-2}$ $= \dfrac{3x-6+5}{x-2}$ $= \dfrac{3x-1}{x-2}$ $=$ LHS		
	(ii) $y = \dfrac{3x-1}{x-2} = 3+\dfrac{5}{x-2}$ D: all real x except $x=2$ R: all real y except $y=3$	1 1	
	(iii)  if $x=0$, $y = 3+\dfrac{5}{-2} = 3+-2\frac{1}{2} = \frac{1}{2}$ $x=3$, $y = 3+\dfrac{5}{1} = 8$	1 1	curves correct asymptotes (as per (ii)) and intercepts labelled

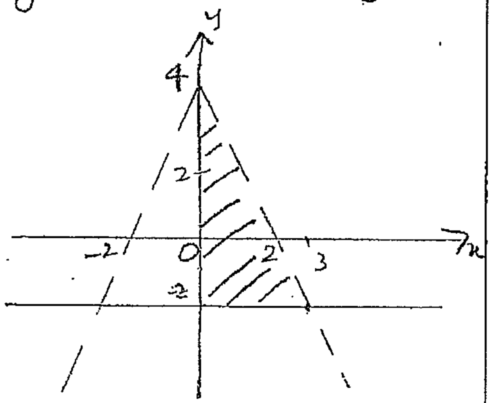
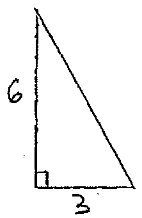
if $y=0$, $0 = 3+\dfrac{5}{x-2}$

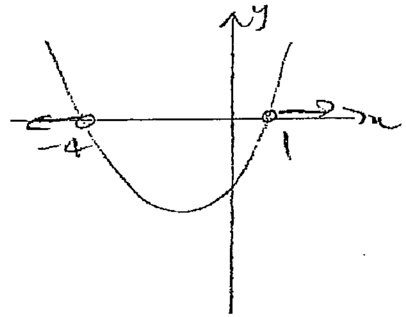
$-3(x-2)=5$

$-3x+6=5$

$-3x=-1$

$x=\frac{1}{3}$

Qn	Solutions	Marks	Comments+Criteria
2 cont'd	(c) $y < 4-\|2x\|,\ x \geq 0\ \ y \geq -2$		
	(i) 	1 1 1	absolute value $x \geq 0$ and $y \geq -2$ region
	(ii)  when $y=-2$, $-2 = 4-2x$ $-6 = -2x$ $x = 3$	1	
	$A = \frac{1}{2}bh$ $= \frac{1}{2}.3.6$ $= 9\ u^2$	1	

Qn	Solutions	Marks	Comments+Criteria
3	(a) $\frac{3x+7}{x+4} > 2 \quad \boxed{x \neq -4}$		
	$(x+4)^2 \cdot \frac{3x+7}{x+4} > 2(x+4)^2$	1	
	$(x+4)(3x+7) > 2(x+4)^2$ $(x+4)(3x+7) - 2(x+4)^2 > 0$ $(x+4)[3x+7 - 2(x+4)] > 0$ $(x+4)(3x+7-2x-8) > 0$		
	$(x+4)(x-1) > 0$	1	
			
	$x < -4$ and $x > 1$.	1	

Qn	Solutions	Marks	Comments+Criteria
3	(b) $\lvert 2x+3 \rvert = 3x+2$ $2x+3 = 3x+2$ Case 1 $-x = -1$ $x = 1$	 1	
	OR $2x+3 = -3x-2$ $5x = -5$ $x = -1$	 1	
	Check: LHS $= \lvert 2+3 \rvert = 5$ RHS $= 3\times1+2 = 5$ $=$ LHS ✓ LHS $= \lvert -2+3 \rvert = 1$ RHS $= 3\times -1+2 = -3+2 = -1 \neq$ LHS $\therefore$ only soln. is $\boxed{x=1}$	 1	testing and 1 soln only
	(c) $f(-a^2) = (-a^2)^2 - 4(-a^2) + 4$ $= a^4 + 4a^2 + 4$ if $a \neq 0$. If $a=0$, $f(-a^2) = 4(-a^2)+4$ $= -4a^2+4$	1 1	

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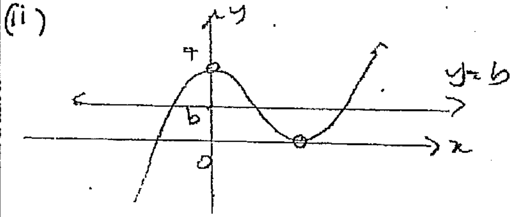
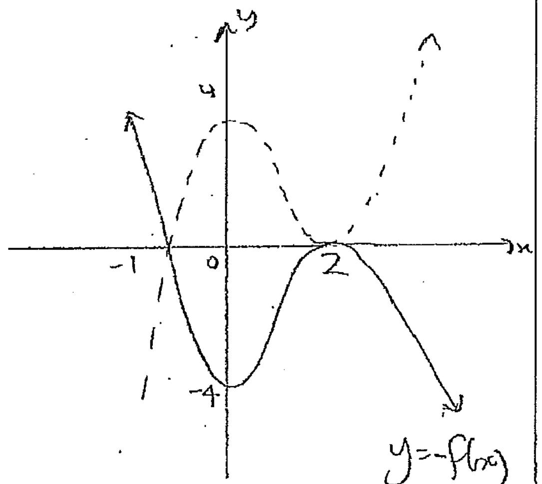
Qn	Solutions	Marks	Comments+Criteria
3	(d)(i) $f(x) = a(x+1)(x-2)^2$ passes through $(0,4)$: $4 = a(1)(-2)^2$ $4 = a.4$ $\therefore a = 1$ $\therefore f(x) = (x+1)(x-2)^2$	 1	
	(ii)  $0 < b < 4$	1 1	$<$ sign correct range
	(iii). $y = -f(x)$  	1 1	shape x and y-intercepts