J.M.J.Ch.

MARCELLIN COLLEGE RANDWICK

YEAR 11

ACCELERATED MATHEMATICS

PRELIMINARY ASSESSMENT TASK # 2

2012

Weighting: 70% of Preliminary Assessment Mark.

STUDENT NAME: **MARK:** / 45

Time Allowed: 60 minutes.

Directions: *Answer the multiple choice section on the multiple choice answer sheet.

*Answer all other questions on separate lined paper.

- Show all necessary working.
- Marks may not be awarded for careless or badly arranged work.
- Begin your answers to each new question on a new answer page.

OUTCOMES TO BE ASSESSED:

- P1 – Demonstrates confidence in using mathematics to obtain realistic solutions to problems
- P2 – Provides reasoning to support conclusions which are appropriate to the context
- P3 – performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions and trig identities.
- P4 – chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques
- P5 – Understands the concept of a function and the relationship between a function and its graph
- P7 – determines the derivative of a function through routine application of the rules of differentiation
- P8 – understands and uses the language and notation of calculus

Question 1 – Multiple Choice Questions

1. Find the equation of the parabola with vertex (0,0) and the equation of the directrix $x = -3$.

a. $x^2 = 12y$ b. $x^2 = -12y$ c. $y^2 = 12x$ d. $y^2 = -12x$

2. If $\sin\vartheta = -\frac{3}{5}$ and $\tan\vartheta > 0$, $\cos\vartheta$ is equivalent to:

a. $\frac{3}{5}$ b. $\frac{4}{5}$ c. $-\frac{3}{5}$ d. $-\frac{4}{5}$

3. Find $\lim_{x\to 3} \frac{x^2-5x-84}{x-7}$

a. 0 b. -9 c. 15 d. $\frac{45}{2}$

4. Differentiate $x\sqrt[3]{x}$.

a. $\frac{1}{3}x^{-\frac{2}{3}}$ b. $\frac{4}{3}x^{\frac{1}{3}}$ c. $\frac{2}{3}x^{-\frac{1}{3}}$ d. $\frac{3}{5}x^{\frac{5}{3}}$

5. Given the equation $3x^2 + 4x - 3 = 0$, evaluate $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$.

a. $-\frac{16}{9}$ b. $-\frac{16}{3}$ c. $-\frac{34}{9}$ d. $\frac{34}{9}$

Question 2

a. For the parabola $x^2 - 8y + 24 = 0$, find the: 3

i. coordinates of the vertex.

ii. focal length.

iii. equation of the directrix.

b. Find the equation of the locus of the point $P(x, y)$ that moves so that PM is twice the distance of PN given $M(3,0)$ and $N(0,3)$. 3

c. Find the value/s of k for which the quadratic equation $x^2 - kx + 9 = 0$ has no real roots. 2

d. The roots of a quadratic equation are $3 - 2\sqrt{3}$ and $3 + 2\sqrt{3}$. Find the quadratic equation with these roots in the form $ax^2 + bx + c = 0$. 2

Question 3

a. A helicopter flies 54km from A to B, on a bearing of $055°T$. The helicopter then proceeds a distance of 68km on a bearing of $105°T$. 5

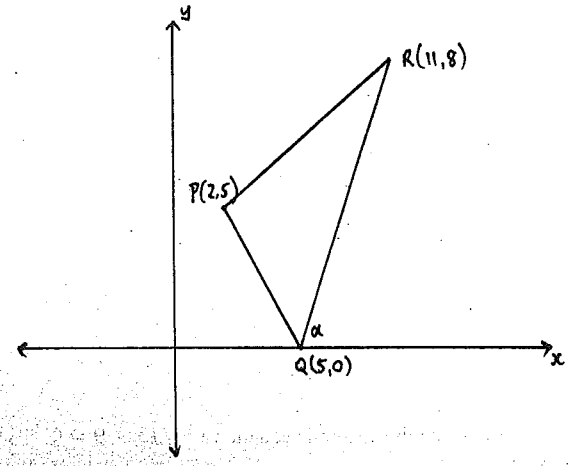
Draw a diagram which represents the above information.

i. Find the size of angle ABC.
ii. Find the distance from C to A.
iii. Find the bearing of C from A to the nearest degree.

b. Solve $sin^2x + cosx = 1$ for $0° \leq x \leq 360°$. 3

c. Prove that $\frac{sin^3\vartheta}{cos\vartheta} + sin\vartheta cos\vartheta = tan\vartheta$. 2

Question 4



a. Find the distance RQ. 1
b. Find the gradient of RQ. 1
c. Find the size of angle α correct to the nearest degree. 1
d. Show that the equation of the line RQ is $4x - 3y - 20 = 0$. 1
e. Find the perpendicular distance of point P from the line RQ. 2
f. Find the area of triangle PQR. 2
g. Point P is the midpoint of the interval RT, where T is a point not shown on the diagram. Find the coordinates of the point T. 2

Question 5

a. Differentiate $\frac{1}{6x^3}$. 1

b. Differentiate the following leaving your answers in simplest factored form:

i. $\frac{x}{\sqrt{x+1}}$ 3

ii. $x^4(2x - 5)^4$ 3

c. Find the equation of the normal to the curve $y = (2x - 3)^4$ at the point $x = 1$. 3

Leave your answer in general form.

Year 11 Accelerated Mathematics
Half Yearly Examination
2012

NAME: _____

Question 1 - Multiple Choice Answer Sheet

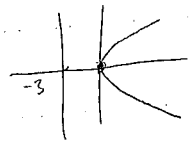
5

Select the alternative A, B, C, or D that best answers the question. Fill in the response oval completely.

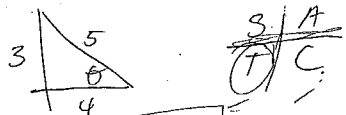
1.	(A) ○	(B) ○	(C) ● ✓	(D) ○
2.	(A) ○	(B) ○	(C) ○	(D) ● ✓
3.	(A) ○	(B) ○	(C) ○	(D) ● ✓
4.	(A) ○	(B) ● ✓	(C) ○	(D) ○
5.	(A) ○	(B) ○	(C) ● ✓	(D) ○

p1

Question 1 - Ignore.

1. 

$y^2 = 4ax$
$y^2 = 12x$.

2. 

$\cos\theta = \frac{-4}{5}$

3. $\frac{9-15-84}{3-7} = \frac{-90}{-4} = \frac{9-15-84}{-4}$ $\frac{10 \quad -9 \quad -90}{(x+7)(x-12)}$ $\frac{7}{(x-7)}$

4. $x \cdot x^{1+\frac{1}{3}}$ $x^{\frac{4}{3}}$ $\frac{4}{3}x^{\frac{1}{3}}$ $\frac{4\sqrt[3]{x}}{3}$.

5. $3x^2 + 4x - 3 = 0$.

$\alpha + \beta = \frac{-b}{a} = \frac{-4}{3}$

$\alpha\beta = \frac{c}{a} = \frac{-3}{3} = -1$.

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$

$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \frac{\left(\frac{-4}{3}\right)^2 - 2(-1)}{(-1)}$

$\frac{\frac{16}{9} + 2}{-1}$ $\frac{1\frac{7}{9} + 2}{-1} = -\frac{25}{3} = -3\frac{7}{3}$

$= -\frac{22}{3}$

(8)

Q2

a(i) $x^2 - 8y + 24 = 0$

$x^2 = 8y - 24$

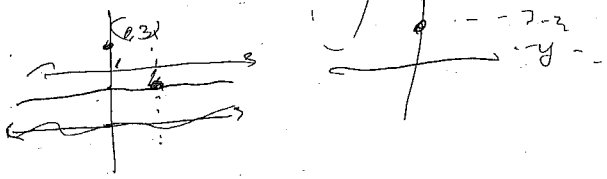
$x^2 = 8(y-3)$ ✓1

vertex $(0, 3)$

(ii) focal lenght $4a = 8$

$a = 2$ ✓1

(iii)



$a = -2$

$y = 1$ ✓1

b) $2PN = PM$ ✓1

$P(x,y)$ $M(3,0)$ $N(0,3)$

$PN = \sqrt{(x-0)^2 + (y-3)^2}$

$= \sqrt{x^2 + y^2 - 6y + 9}$

$PM = \sqrt{(x-3)^2 + (y-0)^2}$

$= \sqrt{x^2 - 6x + 9 + y^2}$

$2\sqrt{x^2 + y^2 - 6y + 9} = \sqrt{x^2 - 6x + 9 + y^2}$ ✓1

$4(x^2 + y^2 - 6y + 9) = x^2 - 6x + 9 + y^2$

$4x^2 + 4y^2 - 24y + 36 = x^2 - 6x + 9 + y^2$

$3x^2 + 3y^2 - 24y + 27 - 6x = 0$

$3x^2 - 6x + 3y^2 - 24y + 27 = 0$

c) $x^2 - kx + 9$.

$\Delta < 0$

$k^2 - 4(9) < 0$ ✓1

$k^2 - 36 < 0$

$(k-6)(k+6) < 0$

$-6 < k < 6$

Ans to 2b)

$x^2 - 2x + y^2 - 8y$

2b) $3x^2 - 6x + 3y^2 - 24y + 27 = 0$ OR

2b) $x^2 - 2x + y^2 - 8y + 9 = 0$

d) $\alpha = 3 - 2\sqrt{3}$

$\beta = 3 + 2\sqrt{3}$

$x^2 - (\alpha+\beta)x + \alpha\beta$.

$\alpha + \beta = 3 - 2\sqrt{3} + 3 + 2\sqrt{3}$

$= 6$.

$\alpha\beta = (3 - 2\sqrt{3})(3 + 2\sqrt{3})$ ✓

$= 9 - 4(3)$

$= 9 - 12$

$= -3$ 2

$x^2 - 6x +$

$x^2 - 6x - 3 = 0$.

Question 3

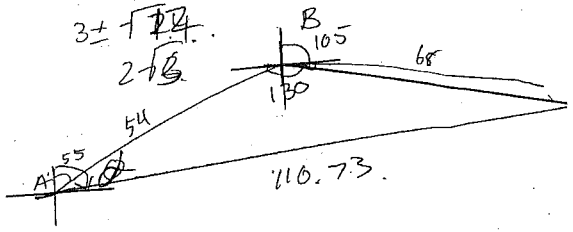
$\frac{6 \pm \sqrt{36 - 4(-3)}}{2}$

$\frac{6 \pm \sqrt{48}}{2}$

$\frac{6 \pm 2\sqrt{12}}{2}$

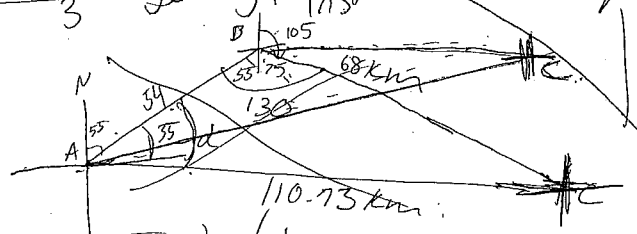
$3 \pm \sqrt{12}$.

$2\sqrt{3}$



Jeremy read the instructions !!

Diagram 3a).



a)(i) $55 + 75 = 130^0$ ✓1

(ii) $c^2 = 54^2 + 68^2 - 2(68)(54)\cos 130$

$c = 110.73\text{km}$ (2dec) ✓1

(iii) $\frac{\sin\alpha}{68} = \frac{\sin 130}{110.73}$ ✓

28° ✓ quad 1

~~+60 28 quad 2~~

~~+152~~

$55 + 28 =$ 083° ✓ C from A

3 S|A T|C.

10

Start each Question on a new pg

b). $\sin^2 x + \cos x - 1 = 0$

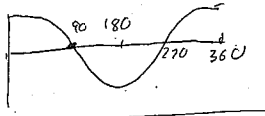
$1 - \cos^2 x + \cos x - 1 = 0$

$-\cos^2 x + \cos x = 0$ ✓

$\cos^2 x - \cos x = 0$

$\cos x(\cos x - 1) = 0$ 3

$\cos x = 0 \qquad \cos x = 1$ ✓



$x = 90^\circ, 270^\circ, 0^\circ, 360^\circ$ ✓ → In order. $0^\circ, 90^\circ, 270^\circ, 360^\circ$

c). $\frac{\sin^3\theta}{\cos\theta} + \sin\theta\cos\theta = \tan\theta$

LHS $\frac{\sin^3\theta + \sin\theta\cos^2\theta}{\cos\theta} =$ ✓

$= \frac{\sin\theta(\sin^2\theta + \cos^2\theta)}{\cos\theta}$ → since $\sin^2\theta + \cos^2\theta = 1$ 2

then $\frac{\sin\theta}{\cos\theta}$

$= \tan\theta$ ✓

$= RHS$

$\therefore \frac{\sin^3\theta}{\cos\theta} + \sin\theta\cos\theta = \tan\theta$

Question 4. (10)

a) $R(11,8)\ Q(5,0)$

$\sqrt{(11-5)^2 + (8-0)^2}$

$\sqrt{36 + 64}$

$\sqrt{100}$

$= 10$ units ✓ 1

b). $\frac{8-0}{11-5} = \frac{8}{6} = \frac{4}{3}$

$m_{RQ} = \frac{4}{3}$ ✓ 1

c). $\tan\alpha = \frac{4}{3}$

$\alpha = 53^\circ$ ✓ 1

d). $y - 0 = \frac{4}{3}(x-5)$

$3y = 4x - 20$

$0 = 4x - 3y - 20 = 0$ ✓ 1

e). $\frac{|4x - 3y - 20|}{\sqrt{4^2+3^2}}$ ✓ $P(2,5)$

$\frac{|8 - 15 - 20|}{5} = \frac{+27}{5}$ units ✓ 2

f). $A = \frac{1}{2}Bh = \frac{1}{2}(10)(\frac{27}{5})$

$= 27$ units^2 ✓ 2

g). $M_P = (2,5)$ $T(x,y)$ $R = (11,8)$ ✓

$(2,5) = \frac{11+x}{2}, \frac{8+y}{2}$ $\frac{11+x}{2} = 2$ $\frac{8+y}{2} = 5$

$11 + x = 4$ $8 + y = 10$

$x = -7$ $y = 2$

$T(-7,2)$

Q5 (10)

a) $\frac{1}{6x^3} = \frac{1}{6}x^{-3} = \frac{-3}{6}x^{-4}$

$= \frac{-1}{2}x^{-4}$

$y' = \frac{-1}{2x^4}$ ✓1

b)(i) $\frac{x}{(x+1)^{\frac{1}{2}}}$ =

$\frac{1(x+1)^{\frac{1}{2}} - x\frac{1}{2}\cdot(x+1)^{-\frac{1}{2}}}{(x+1)}$

~~$(x+1)^{\frac{1}{2}}\left(x+1-\frac{x}{2}\right)$~~

$\frac{(x+1)^{-\frac{1}{2}}\left((x+1)' - x\frac{1}{2}\right)}{x+1}$ ✓

$= \frac{(x+1)^{-\frac{1}{2}}\left(x+1-\frac{x}{2}\right)}{x+1}$

$= \frac{x+1-\frac{x}{2}}{(x+1)(x+1)^{\frac{1}{2}}}$ 3

$= \frac{\frac{x}{2}+1}{(x+1)^{\frac{3}{2}}}$ ✓

$= \frac{x+2}{2(x+1)^3}$ ✓

(ii) $x^4(2x-5)^4$

$4x^3(2x-5)^4 + x^4(2x-5)^3(2)$

$4x^3(2x-5)^4 + 8x^4(2x-5)^3$

~~$4x^3(2x$~~

$(2x-5)^3(4x^3(2x-5)+8x^4)$ 3 ✓

$x^3(2x-5)^3(4(2x-5)+8x)$

$4x^3(2x-5)^3(2x-5+2x)$

$4x^3(2x-5)^3(4x-5)$ ✓

c) $y = (2x-3)^4$

$y' = 4(2x-3)^3(2)$

$= 8(2x-3)^3$

when $x = 1$

$y' = 8(2-3)^3$

$m_1 = -8$

$\therefore m_2 = \frac{1}{8}$ ✓ 3

$y = (2-3)^4$

$y = 1$

$(1,1)$

$y - 1 = \frac{1}{8}(x-1)$ ✓

$8y - 8 = x - 1$

$0 = x - 8y + 7$