Randwick Girls High School

2012
HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics Extension 1

General Instructions

- Reading Time - 5 minutes
- Working Time - 2 hours
- Write using a blue or black pen
- Board approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Show all necessary working in Questions 11 - 14
- Begin each question on a new sheet of paper.

Total marks (70)

Section I

10 marks

- Attempt Questions 1-10
- Answer on the Multiple Choice answer sheet provided.
- Allow about 15 minutes for this section

Section II

60 marks

- Attempt questions 11 – 14
- Answer on the blank paper provided, unless otherwise instructed. Start a new page for each question.
- Allow about 1 hour 45 minutes for this section

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\frac{x}{a}, \quad a \neq 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}} dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

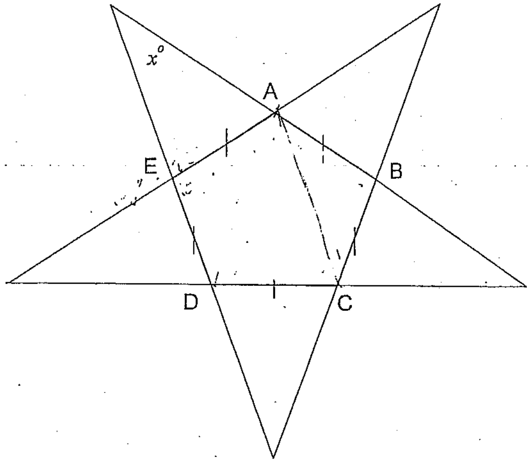
$$\int \frac{1}{\sqrt{x^2+a^2}} dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

1. Calculate to 3 significant figures $\sqrt[5]{\dfrac{18.7+3.65}{\sqrt{(4.25)^3}}} =$

(A) 1.20 (B) 1.206 (C) 1.21 (D) 12.6

2. In the diagram, ABCDE is a regular pentagon. The value of x is:



(A) 90° (B) 36° (C) 108° (D) 72°

3. Find $\lim\limits_{x\to 0}\dfrac{\sin 7x}{5x}$

(A) $\dfrac{7}{5}$

(B) 1

(C) $\dfrac{5}{7}$

(D) 0

4. $\dfrac{d}{dx}[\cos(\ln x)] =$

(A) $-\sin(\ln x)$

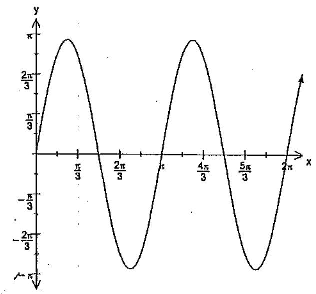
(B) $\dfrac{\cos(\ln x)}{x}$

(C) $\sin(\ln x)$

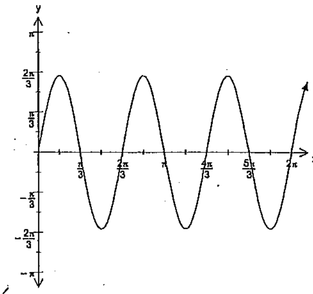
(D) $\dfrac{-\sin(\ln x)}{x}$

5. Which graph represents the curve $y = 3\sin 2x$

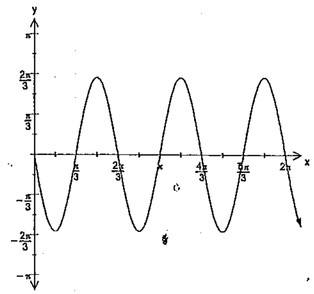
(A)



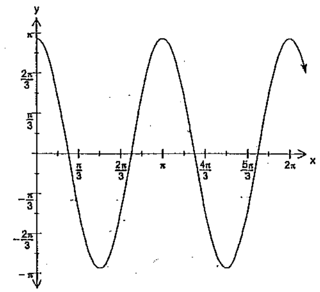
(B)



(C)



(D)



6. Find an approximation of the root of $y = e^x - 3x^2$ by using Newton's Method once and substituting with an approximation of $x = 3.8$ (answer correct to 2 decimal places)

(A) 3.74
(B) 4.22
(C) -12.06
(D) 3.7

7. $4 + \frac{3}{x^2} - \frac{2}{x^3} + \frac{7}{x^5}$ can also be written as:

(A) $\frac{4x+3x-2x+7}{4x}$
(B) $\frac{4x^5+3x^3-2x^2+7}{x^5}$
(C) $\frac{4x^2+3x^2-2x^3-7x^4}{x^2}$
(D) $4+\frac{3x^5-2x^3+7x^2}{x^{10}}$

8. Two dice are rolled and the sum of the numbers is written down. Find the probability of rolling a total less than 6.

(A) $\frac{1}{4}$
(B) $\frac{5}{36}$
(C) $\frac{5}{12}$
(D) $\frac{5}{18}$

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9. The correct factorisation of $8x^3 - 27y^3$ is:

(A) $(x-y)(x^2+xy+y^2)$
(B) $(2x-3y)(4x^2+6xy+9y^2)$
(C) $(4x-9y)(x^2+xy+y^2)$
(D) $(2x-3y)(4x^2-6xy+9y^2)$

10. We can express $\sin x$ and $\cos x$ in terms of $\tan\frac{x}{2}$, for all values of x except……

(A) $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$……..
(B) $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$……..
(C) $x = \pi, 3\pi, 5\pi$……..
(D) $x = 2\pi, 6\pi, 8\pi$……..

End of Section 1

Section II

Total marks (60)
Attempt Questions 11 - 14
Allow about 1 hour 45 minutes for this section.

Answer all questions, starting each question on a new sheet of paper with your name and question number at the top of the page. Do not write on the back of sheets.

Question 11 (15 Marks) Use a Separate Sheet of paper **Marks**

a) Find the obtuse angle between the lines (to nearest degree). **2**

$2x+3y=8$ and $x-2y=-5$.

b) Given that $x=5\sin\theta$ and $y=5\cos\theta+1$.

i) Show the equation relating x and y by eliminating θ is $y^2+x^2-2y-24=0$ **2**

ii) Graph this equation stating major features. **2**

c) A test consists of 10 multiple choice questions.
Each question has 4 possible answers. A student guesses the answers to all the questions.

What is the probability that the student:

i) Guesses all questions correctly? **1**

ii) Only guesses 2 right answers? **1**

iii) Gets over 75% on the test? **2**

d) Find $\int 3x\sqrt{4-x}\,dx$ using the substitution $u=4-x$. **3**

e) Solve $\left(3+\frac{1}{x}\right)^2+4\left(3+\frac{1}{x}\right)-21=0$. **2**

End of Question 11

Question 12 (14 Marks) Use a Separate Sheet of paper **Marks**

a) Prove by induction, that **3**

$4^n>1+3n$ for $n>1$, where n is an integer..

b) Evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\cos^2 x\,dx$ **2**

c) Evaluate $\int_0^{\frac{\pi}{4}}\sin x\cos^2 x\,dx$ using the substitution $u=\cos x$.

d) ABCDE are points on a circle radius 4 cm and $\angle DBC=\angle DAE$ **2**

i) Draw a diagram to represent this information.

ii) Prove that the triangle formed by the points CDE is isosceles. **1**

e) Sketch the graph of $y=\cos^{-1}(x+3)$. **2**

2

f) Find the 6^{th} term in the expansion of $\left(3x-\frac{4}{5x^2}\right)^9$.

2

End of Question 12

Question 13 (16 Marks) Use a Separate Sheet of paper **Marks**

a) Evaluate $\int_0^1 \frac{1}{\sqrt{4-2x^2}}\,dx$ in exact form. **2**

b) Storm is making a toffee dessert. The rate at which the toffee cools is proportional to the difference between the temperature of the toffee (T) and room temperature (R).

$$\text{ie. } \frac{dT}{dt} = -k(T-R)$$

i) Show that $T = R + Ce^{-kt}$, where C is a constant, is a solution of this differential equation. **1**

ii) Storm notices that a 2L pot of toffee initially cools from 540°C to 100°C in 50 minutes in a room whose temperature is 20°C. Storm can not put the toffee into the dessert until it reaches 40°C. How much longer does Storm need to wait to be able to add the toffee and finish her dessert (to the nearest minute)? **3**

iii) Explain or show by calculations, if it would take more or less time to create this dessert if the room temperature was 25°C. Assuming k and C remain the same. **2**

c) Calculate the exact volume generated by the solid formed when $y = \ln x - 1$ is rotated about the y-axis between $y = 0$ and $y = 1$. **2**

d) $P(x) = x^4 - 2x^3 + 5x^2 - 16x + 12$

i) Show that $(x-1)(x-2)$ is a factor of $P(x)$. **1**

ii) Hence find the remaining factor of $P(x)$. **2**

e) i) Show that the area of an equilateral triangle of side length (x-2) is given by: **1**

$$A = \frac{\sqrt{3}(x-2)^2}{4}$$

ii) The sides of the equilateral triangle are increasing at the rate of 5mm/s. At what rate is the area increasing at the instant when the sides are 10cm long? **2**

End of Question 13

Question 14 (16 Marks) Use a Separate Sheet of paper **Marks**

a) Zanthie bought a 'Splat Blaster' that fires paint balls at a velocity of 20ms^{-1}.
A target has been placed on a tree, with its centre 2.5m from the ground. The base of the tree is 25m horizontally away from Zanthie.
Zanthie holds the Splat Blaster at a height of 1.5m and wants to hit the centre of the target with a paint ball.

i) The equation of horizontal motion is given by $x = 20t\cos\theta$. Derive the equation of vertical motion. **2**

ii) To avoid overhead power lines; Zanthie must fire at an angle less than 45° At what angle should she fire the paint ball to hit the target on the tree? (assume $g = -9.8ms^{-2}$ and give your answer to the nearest degree). **3**

b) Find the greatest coefficient in the expansion of $(2+3x)^{10}$. **3**

c) The chord of contact of the tangents to the parabola $x^2 = 4ay$ from an external point $A(x_1, y_1)$ passes through the point $B(0,2a)$. Find the equation of the locus of the midpoint of AB. **2**

d) A particle moves in a straight line and its position at time (t) is given by:

$$x = 4 + \frac{\sin 4t}{\sqrt{3}} - \cos 4t$$

i) Express $\frac{\sin 4t}{\sqrt{3}} - \cos 4t$ in the form $R\sin(4t - \alpha)$, where α is in radians. **2**

ii) The particle is undergoing Simple Harmonic Motion, show the equation for acceleration is: **2**

$$\ddot{x} = -16(x-4)$$

iii) When does the particle first reach its maximum speed? **2**

End of Examination

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Multiple Choice Answer Sheet

Name ______________

Completely fill the response oval representing the most correct answer.

	A	B	C	D
1.	A ○	B ○	C ●	D ○
2.	A ○	B ●	C ○	D ○
3.	A ●	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ●
5.	A ●	B ○	C ○	D ○
6.	A ●	B ○	C ○	D ○
7.	A ○	B ●	C ○	D ○
8.	A ○	B ○	C ○	D ●
9.	A ○	B ●	C ○	D ○
10.	A ○	B ○	C ●	D ○

$\frac{10}{10}$

Question 11

$\tan\theta = \left|\frac{m_1 - m_2}{1+m_1m_2}\right|$

a) $\tan\theta = \left|\frac{m_1 - m_2}{1-m_1m_2}\right|$

$2x+3y=8$

$3y = 8-2x$

$3y = -2x+8$

$y = -\frac{2}{3}x + \frac{8}{3}$

$\therefore m_1 = -\frac{2}{3}$

$x - 2y = -5$

$2y = x+5$

$y = \frac{x}{2} + \frac{5}{2}$

$m_2 = \frac{1}{2}$

$\tan\theta = \left|\frac{-\frac{2}{3} - \frac{1}{2}}{1-\left(-\frac{2}{3}\right)\left(\frac{1}{2}\right)}\right|$

$= \left|\frac{-\frac{7}{6}}{\frac{4}{3}}\right|$

$= \left|-\frac{7}{8}\right|$

$= \frac{7}{8} \times \frac{7}{4}$

$\frac{1}{2}$

$\theta = 41°11'$

$\therefore$ obtuse $\angle = 180 - 41°11'$

$= 138°49'$ $120°$

$= 139°$ (to nearest degree)

bi) $x = 5\sin\theta$ $y = 5\cos\theta + 1$

$\sin\theta = \frac{x}{5}$ $y-1 = 5\cos\theta$

$\sin^2\theta = \frac{x^2}{25}$ $\cos\theta = \frac{y-1}{5}$

$\cos^2\theta = \frac{(y-1)^2}{25}$

$\frac{0}{2}$

Using $\sin^2\theta + \cos^2\theta = 1$

$\frac{x^2}{25} + \frac{(y-1)^2}{25} = 1$

$x^2 + y^2 - 2y + 1 = 25$

$x^2 + y^2 - 2y - 24 = 0$

(i) ✗

b i) $y^2+x^2-2y-24=0$

$y^2-2y+x^2-24=0$

$(y-2)^2+(x-12)^2 = 4+144$

$(y-2)^2+(x-12)^2 = 148$ ✗

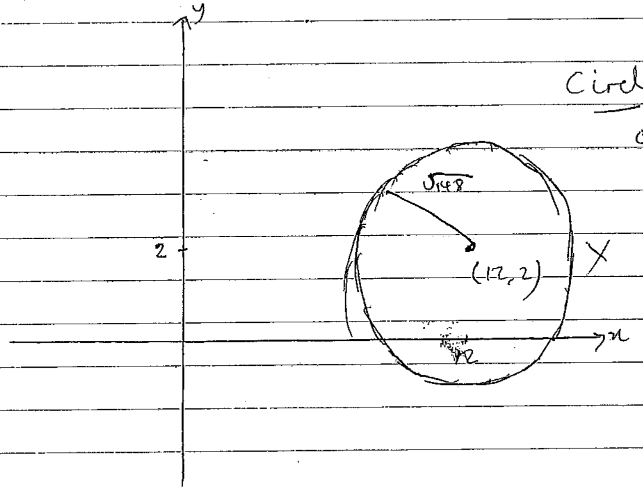
$y^2-2y+x^2=24$

$y^2-2y+1-1+x^2=24$

$(y-1)^2+x^2=25$

$\therefore r=5$

$c=(0,1)$



Circle

centre (0,1)

rad 5

✗

$\frac{1}{2}$ / 2

e i) $P(\text{success}) = P(\text{correct}) = \frac{1}{4}$

$P(\text{fail}) = P(\text{incorrect}) = \frac{3}{4}$

1/1

$P(10\text{ successes}) = {}^{10}C_{10}\left(\frac{3}{4}\right)^0\left(\frac{1}{4}\right)^{10}$

$= \frac{1}{1048576}$ ✓

$\frac{1}{2}$ / 1

ii) $P(2\text{ successes}) = {}^{10}C_2\left(\frac{3}{4}\right)^8\left(\frac{1}{4}\right)^2$ ✓

$= 0.282$

iii) $\frac{75}{100}\times 10 = \frac{15}{2} = 7.5$

0/2

P(

✗

P(

P(8 correct) or P(9 correct) or P(10 correct)

$= {}^{10}C_8\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)^8 + {}^{10}C_9\left(\frac{3}{4}\right)^1\left(\frac{1}{4}\right)^9 + \frac{1}{1048576}$

$= 4.15\times10^{-4}$

d) $\int 3x\sqrt{4-x}\,dx$ $\quad u = 4-x$, $\frac{du}{dx} = -1$, $du = -dx$

$= -3\int (-x)\sqrt{4-x}\,(dx)$

$= -3\int \sqrt{u}\,du$

$= -3\left[\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right] + C$

$= -2\sqrt{u^3} + C$

$= -2\sqrt{(4-x)^3} + C$ ✗

1/3

e) $\left(3+\frac{1}{x}\right)^2 + 4\left(3+\frac{1}{x}\right) - 21 = 0 \quad x \neq 0$

let $u = 3+\frac{1}{x}$

$u^2 + 4u - 21 = 0$

$(u+7)(u-3) = 0$

$u = -7$ or $u = 3$

$3+\frac{1}{x} = -7$ $\quad$ $\cancel{3+\frac{1}{x}=3}$

$\frac{1}{x} = -10$ or $\cancel{x=0}$, $x \neq 0$

$x = -\frac{1}{10}$

$\therefore x = -\frac{1}{10}$ ✓

2/2

1?/15

Question 12

a) Step 1: prove true for $n=2$ $\quad 4^n > 1+3n$, $4^n - 1 - 3n > 0$

LHS $= 4^2 - 1 - 3n$
$= 9$
> 0
$\therefore$ true for $n=2$ ✓

Step 2: Assume true for $n=k$
$4^k > 1+3k$
$4^k - 1 - 3k > 0$ ← Ⓐ $4^k > 1+3k$

Step 3: Prove true for $n=k+1$
$4^{k+1} - 1 - 3(k+1) > 0$ → Ⓟ $4^{k+1} - 3k - 4 > 0$

LHS $= 4^{k+1}$
$= 4^1 . 4^k$
$> 4(1+3k)$
$= 4+12k$
$> 4+3k$
$=$ RHS
LHS > RHS
$\therefore$ true for $n=k+1$

LHS $= 4.4^k - 1 - 3(k+1)$
$= 4.4^k - 1 - 3k - 3$
$= 4.4^k - 4 - 3k$ ✓
$= 4(4^k - 1 - 3k) + 9k$ which must be >0 since $4^k - 3k - 1 > 0$

$\therefore$ true for $n=k+1$

$\therefore$ By MI true for all n integers needs to be better!

2/3

b) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^2 x\,dx$

$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{2}(1+\cos 2x)\,dx$

$= \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$= \frac{1}{2}\left[\frac{\pi}{2} + \cancel{\frac{\sin\pi}{2}} - \left(\frac{\pi}{4} + \frac{\sin\frac{\pi}{2}}{2}\right)\right]$

$= \frac{1}{2}\left[\frac{\pi}{2} - \frac{\pi}{4} - \frac{1}{2}\right]$

$= \frac{1}{2}\left[\frac{\pi}{4} - \frac{1}{2}\right]$

$= \frac{\pi}{8} - \frac{1}{4}$ ✓

2/2

c) $\int_0^{\frac{\pi}{4}} \sin x \cos^2 x \, dx$

$u = \cos x$ $\qquad u^2 = \cos^2 x$

$\frac{du}{dx} = -\sin x$

$du = -\sin x \, dx$

when $x = \frac{\pi}{4}$, $u = \frac{1}{\sqrt{2}}$

when $x = 0$, $u = 1$

$= -\int_0^{\frac{\pi}{4}} -\sin x \cos^2 x \, dx$

$= -\int_1^{\frac{1}{\sqrt{2}}} u^2 \, du$

$= \int_{\frac{1}{\sqrt{2}}}^{1} u^2 \, du$

$= \left[\frac{u^3}{3}\right]_{\frac{1}{\sqrt{2}}}^{1}$

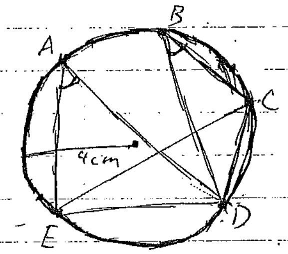
$= \frac{1}{3} - \frac{\left(\frac{1}{\sqrt{2}}\right)^3}{3}$

$= \frac{1}{3} - \frac{1}{6\sqrt{2}}$

2/2 $= \frac{1}{3} - \frac{\sqrt{2}}{12}$

$= \frac{4-\sqrt{2}}{12}$ ✓

d i)



1/1 ✓

ii) ✗

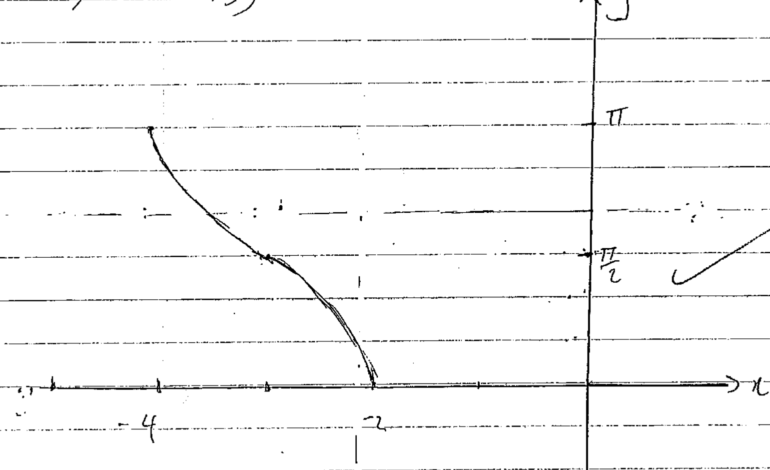
$\angle EAD = \angle DBC$ (given)

$ED = CD$ (equal $\angle$s at the circumference stand on equal chords)

$\therefore \triangle CDE$ is isosceles (2 sides equal)

0/2

e) $y = \cos^{-1}(x+3)$



2/2 ✓

f) $\left(3x - \frac{4}{5x^2}\right)^9$

$T_6 = {}^9C_5 (3x)^4 \left(\frac{-4}{5x^2}\right)^5$

$= 126 \times 81x^4 \times \frac{1024}{3125x^{10}}$

$= \frac{10450944x}{3125x^{10}}$

1/2 $= \frac{10450944}{3125x^{6}}$ ×

10/14

-8-

Question 13

a) $\int_0^1 \frac{1}{\sqrt{4-2x^2}}\,dx$

$= \int_0^1 \frac{1}{\sqrt{2(2-x^2)}}\,dx$

$= \int_0^1 \frac{1}{\sqrt{2}\sqrt{(\sqrt{2})^2 - x^2}}\,dx$

$= \frac{1}{\sqrt{2}}\left[\sin^{-1}\frac{x}{\sqrt{2}}\right]_0^1$

$= \frac{1}{\sqrt{2}}\left[\frac{\pi}{4} - 0\right]$

2/2 $= \frac{\pi}{4\sqrt{2}}$

$= \frac{\pi\sqrt{2}}{8}$ ✓

bi) $T = R + Ce^{-kt} \Rightarrow Ce^{-kt} = T - R$

$\frac{dT}{dt} = Ce^{-kt} \cdot -k$

$= -k \cdot Ce^{-kt}$

1/1 $= -k(T-R)$ ✓

ii)

$t = 0 \quad T = 540 \quad R = 20$

$540 = 20 + Ce^0$

$C = 520$

$t = 50$

$T = 100°C$

$R = 20°C$

$C = 520$ ✓

$100 = 20 + 520e^{-k50}$

$520e^{-k50} = 80$

$e^{-k50} = \frac{2}{13}$

$-50k = \ln\frac{2}{13}$ ✓

$\ln\frac{2}{13}$ ...

$t = ?$

$T = 40$

$40 = 20 + 520e^{-(\ldots)t}$

$520e^{-kt} = 20$

$e^{-kt} = \frac{1}{26}$

$-kt = \ln\frac{1}{26}$

$t = 87\text{ min}$ ✓

$\therefore 87 - 50 = 37$ minutes ✓

3/3

-9-

iii) $40° = 25 + 520e^{-MRt}$ ✓

$520e^{-MRt} = 15$

$e^{-MRt} = \frac{3}{104}$

$-MRt = \ln\frac{3}{104}$

$t = \frac{\ln\left(\frac{3}{104}\right)}{-MR}$

$= \cancel{98}$ min (to nearest min) ✓

95 mins

∴ longer

1 / 1

e) $y = \ln x - 1$

$\ln x = y + 1$

$\log_e x = y + 1$

$e^{(y+1)} = x$

$x^2 = e^{2y+2}$

$V = \pi \int_c^d x^2 \, dy$

$= \pi \int_0^1 e^{2y+2} \, dy$

$= \frac{1}{2}\pi \int_0^1 2 \cdot e^{2y+2} \, dy$

$= \frac{1}{2}\pi \left[e^{2y+2}\right]_0^1$

2/2 $= \frac{\pi}{2}\left[e^4 - e^2\right]$ ✓

d) $P(x) = x^4 - 2x^3 + 5x^2 - 16x + 12$

i) $P(1) = 1^4 - 2 + 5 - 16 + 12$
$= 0$ ✓

∴ $x - 1$ is a factor

$P(2) = 2^4 - 2(2)^3 + 5(2)^2 - 16(2) + 12$
$= 0$ ✓

∴ $x - 2$ is a factor

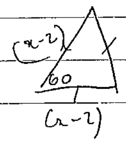
1

ii) $(x-1)(x-2)$
$= x^2 - 2x - x + 2$
$= x^2 - 3x + 2$

$$
\begin{array}{r}
x^2 + x + 6 \\
x^2 - 3x + 2 \,\big)\, x^4 - 2x^3 + 5x^2 - 16x + 12 \\
-\; x^4 - 3x^3 + 2x^2 \\
\hline
x^3 + 3x^2 - 16x \\
-\; x^3 - 3x^2 + 2x \\
\hline
6x^2 - 18x + 12 \\
-\; 6x^2 - 18x + 12 \\
\hline
0
\end{array}
$$

2/2 ∴ $x^2 + x + 6$ ✓

e i)



$A = \frac{1}{2}(x-2)(x-2)\sin 60$

$= \frac{(x-2)^2}{2} \times \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}(x-2)^2}{4}$ ✓

1/1

ii) $\frac{d(x-2)}{dt} = 5\text{mm}$

$\frac{dA}{d(x-2)} = \frac{\sqrt{3}}{4} \times 2(x-2)$

$\frac{dA}{dt} = ?$

$= \frac{\sqrt{3}(x-2)}{2}$

$(x-2) = 10\text{cm}$
$= 100$

$\frac{dA}{d(x-2)} = \frac{dA}{dt} \times \frac{dt}{d(x-2)}$

$\frac{\sqrt{3}(x-2)}{2} = \frac{dA}{dt} \times \frac{1}{5}$

$5 \times \frac{\sqrt{3}(100)}{2} = \frac{dA}{dt}$

2/2

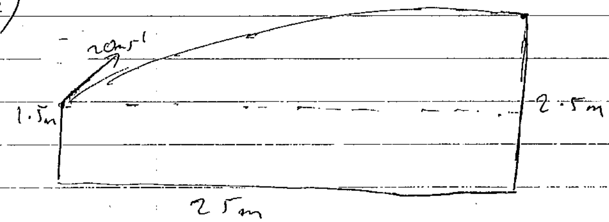
$\frac{dA}{dt} = 250\sqrt{3}\ \text{mm}^2/\text{s}$ ✓ $= 433\ \text{mm}^2\text{s}^{-1}$

$= 4.33\ \text{cm}^2\text{s}^{-1}$

15/16

Question 14

a)



i) $\ddot{y} = -g$

[I.C. $t=0,\ x=0,\ y=1.5,\ \frac{dx}{dt} = v\cos\theta,\ \frac{dy}{dt} = v\sin\theta$
$= 20\sin\theta$]

Int wrt t

$\frac{dy}{dt} = -gt + C_1$

$20\sin\theta = 0 + C_1$

$C_1 = 20\sin\theta$

$\frac{dy}{dt} = -gt + 20\sin\theta$ — (4)

~~Int wrt t~~

$y = \frac{-gt^2}{2} + 20\sin\theta\, t + C_2$

$1.5 = 0 + 0 + C_2$

$C_2 = 1.5$

2/2 $y = \frac{-gt^2}{2} + 20\sin\theta t + 1.5$ ✓

ii

$x = 20t\cos\theta$

$t = \dfrac{x}{20\cos\theta}$

X

$y = \dfrac{-g}{2}\,\dfrac{x^2}{20^2\cos^2\theta} + 20\sin\theta \cdot \dfrac{x}{20\cos\theta} + 1.5$

$\theta = ?$

0/3

$y = -\dfrac{g}{800}(1+\tan^2\theta)x^2 + \tan\theta\, x + 1.5$

$(25, 2.5)$ sat this

$2.5 = \dfrac{-9.8}{800}(1+\tan^2\theta)625 + \tan\theta \cdot 25 + 1.5$

$2000 = -6125(1+\tan^2\theta) + 20000\tan\theta + 1200$

$2000 = -6125 - 6125\tan^2\theta + \ldots$

$6125\tan^2\theta - 20000\tan\theta + 6925 = 0$

$\tan\theta = \dfrac{20000 \pm \sqrt{20000^2 - 4\times 6125\times 6925}}{2(6125)}$

$\tan\theta = 2.87$ or $\tan\theta = 0.39$

$\theta = 70°48'$ $\qquad \theta = 21°29'$

$\theta \geq 45°$

b) $\dfrac{T_{k+1}}{T_k} \geq 1$

$T_{k+1} = {}^{10}C_k\, 2^{10-k}(3x)^k$

$T_k = {}^{10}C_{k-1}(2)^{11-k}(3x)^{k-1}$

$\dfrac{10-k+1}{k} \times \dfrac{3}{2} \geq 1$

$\dfrac{33-3k}{2k} \geq 1$

$33 - 3k \geq 2k$

$33 \geq 5k$

$k \leq 6.6$

$\therefore k = 6$ ✓

3/3

coeff $T_{6+1} = {}^{10}C_6\, 2^4 (3)^6$

$= 210 \times 16 \times 729$

$= 2449440$ ✓

c)

tangent at P?

$x^2 = 4ay$

$y = \dfrac{x^2}{4a}$

$\dfrac{dy}{dx} = \dfrac{x}{2a}$

slope of tangent at P

$\dfrac{2ap}{2a} = p$

equation of tangent at P

$y - y_1 = m(x - x_1)$

$y - ap^2 = p(x - 2ap)$

$y - ap^2 = px - 2ap^2$

$px - y - ap^2 = 0$ — (1)

tangent at Q

$qx - y - aq^2 = 0$ — (2)

(1) − (2)

$px - qx - ap^2 + ap^2 = 0$

$(p-q)x = ap^2 - ap^2$

$(p-q)x = a(p+q)(p-q)$

$x = a(p+q)$

$p(a(p+q)) - y - ap^2 = 0$

$ap^2 + apq - y - ap^2 = 0$

$y = apq$

$m: x = \dfrac{x_1 + x_2}{2}$ $\qquad y = \dfrac{y_1 + y_2}{2}$

$= \dfrac{0 + a(p+q)}{2}$ $\qquad = \dfrac{2a + apq}{2}$

$= \dfrac{a}{2}(p+q)$

(next page)

$= \dfrac{2a + a(-2)}{2}$

$= \dfrac{2a - 2a}{2}$

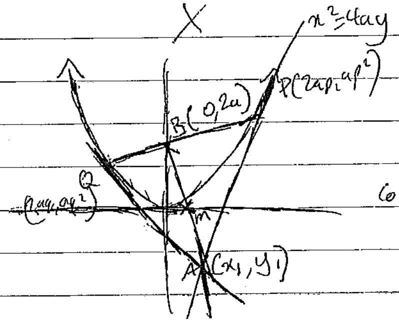
$= 0$

O R

c) $xx_0 = 2a(y + y_0)$

0/2

X



chord of contact: $xx_1 = 2a(y + y_1)$

$(0, 2a)$ sat. this

$0x_1 = 2a(2a + y_1)$

$0 = 2a(2a + y_1)$

$2a + y_1 = 0$

$y_1 = -2a$

Coords of m

$x = \dfrac{x_1 + x_2}{2}$ $\qquad y = \dfrac{y_1 + y_2}{2}$

$= 2a + \dfrac{-2a}{2}$

$= 0$

locus

d i) $x = 4 + \frac{\sin 4t}{\sqrt{3}} - \cos 4t$

i) $\frac{\sin 4t}{\sqrt{3}} - \cos 4t$ $\qquad R(\sin 4t \cos\alpha - \cos 4t \sin\alpha)$

$= \sqrt{\frac{1}{\sqrt{3}}^2 + 1^2}\left(\frac{\frac{1}{\sqrt{3}}}{\frac{\sqrt{4}}{\sqrt{3}}}\sin 4t - \frac{1}{\frac{\sqrt{4}}{\sqrt{3}}}\cos 4t\right)$

$= \frac{2}{\sqrt{3}}\left(\frac{1}{2}\sin 4t - \frac{\sqrt{3}}{2}\cos 4t\right)$ $\qquad R = \frac{2}{\sqrt{3}}$

$\alpha = \frac{\pi}{3}$ $\begin{cases}\cos\alpha = \frac{1}{2}\\ \sin\alpha = \frac{\sqrt{3}}{2}\end{cases}$

$= \frac{2}{\sqrt{3}}\left(\sin 4t \cdot \frac{1}{2} - \cos 4t \cdot \frac{\sqrt{3}}{2}\right) = \frac{2}{\sqrt{3}}\sin\left(4t - \frac{\pi}{3}\right)$

1/2

$= R(\sin 4t \cos\alpha - \cos 4t \cdot \sin\alpha)$

ii) $\dot{x} = \frac{1}{\sqrt{3}}\cos 4t \cdot 4 + \sin 4t \cdot 4$

$= \frac{4}{\sqrt{3}}\cos 4t + 4\sin 4t$

$x - 4 = \frac{\sin 4t}{\sqrt{3}} - \cos 4t$

$\ddot{x} = -\frac{4}{\sqrt{3}}\sin 4t \cdot 4 + 4\cos 4t \cdot 4$

$= -16\left(\frac{1}{\sqrt{3}}\sin 4t - \cos 4t\right)$

2/2

$= -16(x - 4)$

iii) v_{max} at $\ddot{x} = 0$ $\qquad 4 + \frac{\sin 4t}{\sqrt{3}} - \cos 4t = 4$

$-16(x-4) = 0$ $\qquad \frac{\sin 4t}{\sqrt{3}} - \cos 4t = 0$

$x - 4 = 0$

$x = 4$

2/2

$LHS = \frac{2}{\sqrt{3}}\left(\sin 4t \cdot \frac{1}{2} - \cos 4t \cdot \frac{\sqrt{3}}{2}\right)$

$= R(\sin 4t \cos\alpha - \cos 4t \sin\alpha)$

$R = \frac{2}{\sqrt{3}}$ $\qquad \frac{2}{\sqrt{3}}\sin\left(4t - \frac{\pi}{3}\right) = 0$

$\cos\alpha = \frac{1}{2}$, $\alpha = \frac{\pi}{3}$

$4t - \frac{\pi}{3} = 0$

$4t = \frac{\pi}{3}$

$t = \frac{\pi}{12}$ s

/16

chord PQ

slope $= \frac{ap^2 - aq^2}{2ap - 2aq}$

$= \frac{a(p+q)(p-q)}{2a(p-q)}$

$= \frac{p+q}{2}$

$y - y_1 = m(x - x_1)$

$y - ap^2 = \frac{p+q}{2}(x - 2ap)$

$(0, 2a)$ sat this

$2a - ap^2 = \frac{p+q}{2}(-2ap)$

$4a - 2ap^2 = (p+q)(-2ap)$

$4a - 2ap^2 = -2ap^2 - 2apq$

$4a = -2apq$

$\frac{4a}{-2a} = pq$

$pq = -2$