



2009 Half-Yearly Examination

FORM II MATHEMATICS

Monday 18th May 2009

General Instructions

- Writing time — 1 hours 30 minutes
- Write using black or blue pen.
- Calculators are not to be used.
- All necessary working should be shown in every question.
- Start each question on a new page.

Structure of the paper

- Total marks — 112
- All eight questions may be attempted.
- All eight questions are of equal value.

Collection

- Write your name, class and master clearly on each page of your answers.
- Staple your answers in a single bundle.
- The question papers will be collected separately.

2A: MW
2E: DNW
2I: SJE

2B: SO
2F: MLS
2J: TCW

2C: JMR
2G: BDD

2D: BR
2H: KWM

Checklist

- Writing paper required.
- Candidature — 192 boys

Examiner
SJE

QUESTION ONE (14 marks) Start a new page.

(a) Evaluate:

- (i) $1006 - 19$
- (ii) 0.56×1000
- (iii) -8^2
- (iv) $5.107 - 4.68$

(b) Simplify:

- (i) $9a - 5a + a$
- (ii) $3pq \times 2q$
- (iii) $\frac{3x}{7} + \frac{2x}{7}$

(c) Express the following as percentages:

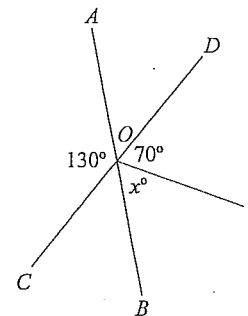
- (i) 0.012
- (ii) $\frac{3}{20}$

(d) Expand $2(x - y)$.

(e) Factorise fully:

- (i) $3x - 6$
- (ii) $ab^2 + 2b$

(f)



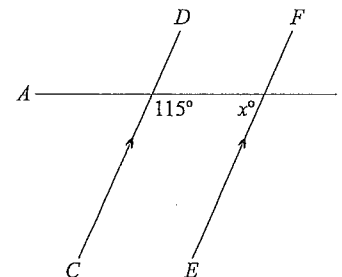
In the diagram above, AB and CD are straight lines. Find x , giving a reason.

QUESTION TWO (14 marks) Start a new page.

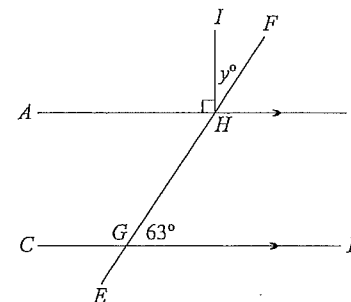
- (a) Evaluate $2x^2 - 3$, when $x = 4$.
- (b) Calculate:
- (i) $-3 - (-17)$
 - (ii) $1.6 \div 0.04$
 - (iii) 15% of \$300
 - (iv) $\frac{2}{3} \times \frac{2}{5}$
- (c) Express the following as fractions in simplest form:
- (i) 35%
 - (ii) $1\frac{1}{4}\%$
 - (iii) 1.88
- (d) Simplify:
- (i) $\frac{2t}{5} - \frac{t}{10}$
 - (ii) $x^2 - 2x^2 + 4x$
- (e) Express 45 g as a percentage of 1.5 kg.
- (f) Solve:
- (i) $3x - 11 = -5$
 - (ii) $\frac{a}{4} = \frac{1}{2}$

QUESTION THREE (14 marks) Start a new page.

- (a) Decrease \$120 by 30%.
- (b) Find the value of the pronumerals in the following diagrams. You must give reasons.
- (i)



(ii)



- (c) Find the simple interest on \$1200 invested at 3% per annum for 4 years.
- (d) Evaluate:
- (i) $\frac{2}{3} \div \frac{7}{9}$
 - (ii) $7\frac{3}{5} - 6\frac{1}{4}$
- (e) Fully factorise $-6y^2 - 9xy$.

QUESTION FOUR (14 marks) Start a new page.

- (a) (i) Solve $3x + 11 > -1$.
 (ii) Graph your solution on a number line.

(b) Simplify:

(i) $\frac{8y}{x} \times \left(-\frac{3xy}{2}\right)$

(ii) $\frac{3a}{b} \div \frac{a^2}{6b}$

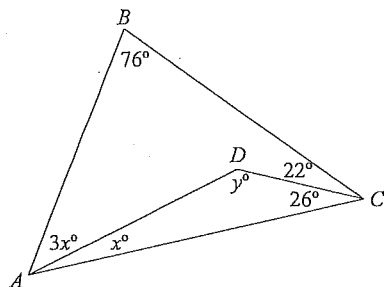
(c) Solve:

(i) $3(k + 6) = 12$

(ii) $3p + 5 = 3 - 2p$

(iii) $2 + \frac{y}{4} = 7$

(d)



Find the values of x and y , giving reasons.

QUESTION FIVE (14 marks) Start a new page.

(a) Consider the formula $y = mx + b$.

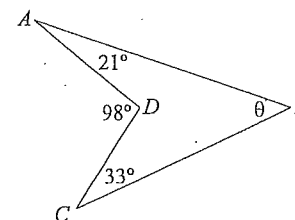
(i) Find y when $m = \frac{3}{2}$, $x = -2$ and $b = 3$.

(ii) Find m when $y = 1$, $x = 3$ and $b = 5$.

(b) Expand and simplify $6(m - 1) + 3(m + n) - 2(3 - n)$.

(c) Evaluate $15 - \frac{x}{2} - \frac{y}{4}$, given $x = -1$ and $y = \frac{1}{2}$.

(d)



Find the value of θ in the quadrilateral above, giving reasons.

(e) A family of four (two adults and two children) are considering travelling to Broome. The two airlines that fly there, Phoenix and Sunshine, both quote a standard return adult fare and children pay half that standard adult fare. However, both airlines are currently offering different deals.

Phoenix offers a Family Saver package in which you pay the standard fare for two adults and a child and the next child travels free. The standard return adult fare to Broome is \$550.

As members of the Sunshine Frequent Flyer Club the family are entitled to a 40% discount on the first fare purchased and a 25% discount on the second fare purchased. All other travellers must pay the applicable fare. The standard return adult fare to Broome is \$600.

Determine which is the cheaper option for the family.

QUESTION SIX (14 marks) Start a new page.

(a) Evaluate:

(i) $\sqrt{0.81}$

(ii) $\left(\frac{1}{2}\right)^3 + \left(-\frac{3}{4}\right)^2$

(b) In a cricket match, Michael scored twice as many runs as David and together they scored one-third of the team's runs. Let x be the the number of runs that David scored.

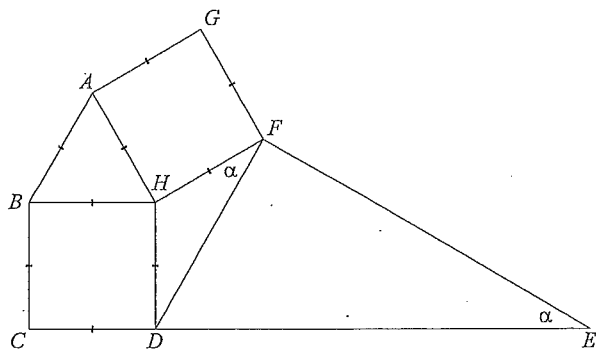
(i) Write an expression for the number of runs Michael and David scored together.

(ii) If the team scored 126 runs, form an equation and solve it for x .

(iii) How many runs did Michael score?

(c) A truck with its full load of 36 identical crates passes through a weigh station and weighs 6.0 tonnes. After making a delivery of 24 crates it then passes through another weigh station and it now weighs 4.2 tonnes. Find the unladen weight of the truck.

(d)



In the diagram above, the equilateral triangle ABH lies between the squares $BHDC$ and $AHFG$ and CD is produced to E such that $\angle HFD = \angle FED$.

Let $\alpha = \angle HFD = \angle FED$.

Show that $\angle DFE$ is a right angle. You must set out your solution giving reasons.

(e) Find the smallest positive whole number that gives a number entirely comprised of 5's, when multiplied by 7.

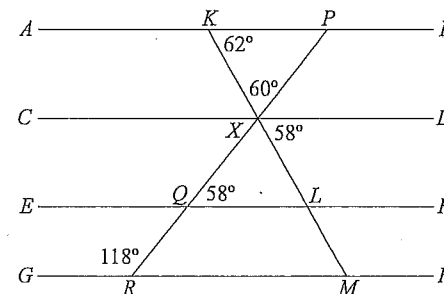
QUESTION SEVEN (14 marks) Start a new page.

(a) Calculate:

(i) $1.385 - 3.705$

(ii) $\frac{3}{4} - 2\frac{1}{8} \times \frac{2}{3} + \frac{1}{6}$

(b)



State a pair of parallel lines from the diagram above, giving reasons.

(c) Given $X = \sqrt{\frac{a-b}{a+b}}$

(i) Evaluate X if $a = 5$ and $b = 3$.

(ii) Find another pair of integers for a and b that give the same value for X .

(d) Three containers of different juices are mixed together. The volumes and concentrations are given below:

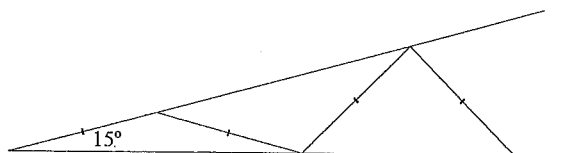
Apple juice	1.3 litres (20% pure juice)
Orange juice	2.0 litres (18% pure juice)
Pineapple juice	1.5 litres (12% pure juice)

Find the percentage of pure fruit juice in the mixture once they have all been mixed together.

QUESTION EIGHT (14 marks) Start a new page.

(a) Simplify $\frac{a-b}{b-a} - \frac{c+d}{d+c}$.

(b)

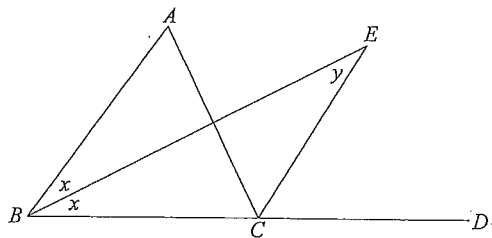


Isosceles triangles are formed between two lines radiating from a point. The angle between the lines is 15° . Find the angles of the last possible isosceles triangle that can be formed.

Hint: Copy the diagram and sketch in the triangles. No formal proof is required.

(c) During the recent global financial crisis, the share price of Torus Bank fluctuated significantly. It first fell 40%, then rose 25%, before falling another 60%. Confidence came back into the market and the price then doubled. What percentage must the share price rise, in order to return to the price it was before the financial crisis started?

(d)



Refer to the diagram above.

- (i) State why $\angle ECD = x + y$.
- (ii) If $\angle ECD = \angle ECA$, find an expression for $\angle BAC$ in terms of y . Show your reasoning.

- (e) A solid cube with side lengths 10 cm is painted on all six of its faces. The cube is then cut into 1000 smaller cubes each with side lengths 1 cm.
 - (i) How many of the small cubes have respectively exactly 3, 2, 1 or no faces painted?
 - (ii) The small cubes are arranged to form solid cubes of side length 4 cm with all outside surfaces painted. Only unpainted small cubes are used within the interior of these cubes. How many cubes can be formed with the blocks available?
 - (iii) The remaining unpainted cubes after the construction in part (ii) are themselves used to build larger solid cubes with side length 4 cm. How many such cubes can be constructed with the available blocks?
 - (iv) How many of each type of small cube, with 3, 2, 1 or no faces painted, remain after these constructions in parts (ii) and (iii)?

END OF EXAMINATION

Question 1

(a) (i) $1006 - 19 = 987$ ✓
 (ii) $0.56 \times 1000 = 560$ ✓
 (iii) $-8^2 = -64$ ✓
 (iv) $5.107 - 4.68 = 0.427$ ✓

(b) (i) $9a - 5a + a = 5a$ ✓
 (ii) $3pq \times 2q = 6pq^2$ ✓
 (iii) $\frac{3x}{7} + \frac{2x}{7} = \frac{5x}{7}$ ✓

(c) (i) $0.012 = 1.2\%$ ✓
 (ii) $\frac{3}{20} = 15\%$ ✓

(d) $2(x-y) = 2x - 2y$ ✓

(e) (i) $3x - 6 = 3(x-2)$ ✓
 (ii) $ab^2 + 2b = b(ab+2)$ ✓

(f) $x + 70^\circ = 130^\circ$ (vertically opposite) ✓
 $x = 60^\circ$ ✓

} $\frac{1}{2}$ correct answer,
 incorrect/no
 reason

(14)

Question 2

(a) $2x^2 - 3 = 2(4)^2 - 3$
 $= 29$ ✓

(b) (i) $-3 - (-17) = 14$ ✓

(ii) $1.6 \div 0.04 = 40$ ✓

(iii) $15\% \times \$300 = \45 ✓

(iv) $\frac{2}{3} \times \frac{2}{5} = \frac{4}{15}$ ✓

(c) (i) $35\% = \frac{7}{20}$ ✓
 (ii) $14\% = \frac{1}{80}$ ✓
 (iii) $1.88 = 1\frac{22}{25}$ ✓

} must be simplified.

(d) (i) $\frac{2t}{5} - \frac{t}{10} = \frac{3t}{10}$ ✓

(ii) $x^2 - 2x^2 + 4x = -x^2 + 4x$ ✓

(e) $\frac{45g}{1500g} \times 100\% = \frac{45}{15}\%$ ✓
 $= 3\%$ ✓

(f) (i) $3x - 11 = -5$
 $3x = 6$
 $x = 2$ ✓

(ii) $\frac{a}{4} = \frac{1}{2}$
 $a = 2$ ✓

(14)

Question 3

(a) New Amount = $\frac{7}{10} \times \$120$ ✓
 $= \$84$ ✓

} $\frac{1}{2}$ for finding 30

(b) (i) $x + 115^\circ = 180^\circ$ (co-interior angles, $CD \parallel EF$) ✓
 $x = 65^\circ$ ✓

(ii) $\angle FHB = 63^\circ$ (corresponding angles, $AB \parallel CD$) ✓
 $90^\circ + y^\circ + 63^\circ = 180^\circ$ (straight angle) ✓
 $y = 27^\circ$ ✓

(only penalise once if parallel lines are not named)

Question 3 (cont.)

$$(c) \text{ Interest} = \$1200 \times 0.03 \times 4 \quad \checkmark$$

$$= \$144 \quad \checkmark$$

$$(d) \text{ (i)} \quad \frac{2}{3} \div \frac{7}{9} = \frac{2}{3} \times \frac{9}{7} \quad \checkmark$$

$$= \frac{6}{7} \quad \checkmark$$

$$\text{(ii)} \quad 7\frac{3}{5} - 6\frac{1}{4} = 1 + \frac{3}{5} - \frac{1}{4} \quad \checkmark$$

$$= 1 + \frac{12}{20} - \frac{5}{20} \quad \checkmark$$

$$= 1\frac{7}{20} \quad \checkmark$$

$$(e) \quad -6y^2 - 9xy = -3y(2y + 3x) \quad \checkmark \quad (14)$$

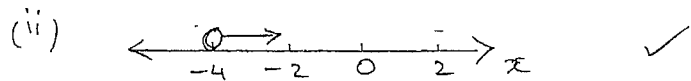
(accept $3y(-2y - 3x)$)

Question 4

$$(a) \text{ (i)} \quad 3x + 11 > -1$$

$$3x > -12$$

$$x > -4 \quad \checkmark$$



$$(b) \text{ (i)} \quad \frac{8y}{x} \times \left(-\frac{3xy}{2}\right) = -12y^2 \quad \checkmark \checkmark$$

$$\text{(ii)} \quad \frac{3a}{b} \div \frac{a^2}{6b} = \frac{3a}{b} \times \frac{6b}{a^2} \quad \checkmark$$

$$= \frac{18}{a} \quad \checkmark$$

$$(c) \text{ (i)} \quad 3(k+b) = 12$$

$$k+b = 4$$

$$k = -2 \quad \checkmark$$

$$\text{(ii)} \quad 3p + 5 = 3 - 2p \quad \checkmark$$

$$5p = -2 \quad \checkmark$$

$$p = -\frac{2}{5} \quad \checkmark$$

$$\text{(iii)} \quad 2 + \frac{y}{4} = 7$$

$$\frac{y}{4} = 5 \quad \checkmark$$

$$y = 20 \quad \checkmark$$

$$(d) \quad (3x+x) + (22+26^\circ) + 76^\circ = 180^\circ \text{ (angle sum of tri)} \quad \checkmark$$

$$4x + 124^\circ = 180^\circ$$

$$4x = 56^\circ$$

$$x = 14^\circ \quad \checkmark$$

$$x^\circ + y^\circ + 26^\circ = 180^\circ \text{ (angle sum of a triangle)}$$

$$14^\circ + y^\circ + 26^\circ = 180^\circ$$

$$y = 140^\circ \quad \checkmark \quad (14)$$

Question 5

$$(a) \text{ (i)} \quad y = \frac{3}{2}(-2) + 3 \quad \checkmark$$

$$= 0$$

$$\text{(ii)} \quad 1 = m(3) + 5 \quad \checkmark$$

$$3m = -4$$

$$m = -\frac{4}{3} \quad \checkmark$$

$$(b) \quad 6(m-1) + 3(m+n) - 2(3-n) = 6m - 6 + 3m + 3n - 6 + 2n$$

$$= 9m + 5n - 12 \quad \checkmark$$

Question 5 (cont.)

$$(c) \quad 15 - \frac{-1}{2} - \frac{\frac{1}{2}}{4} = 15 + \frac{1}{2} - \frac{1}{8} \\ = 15\frac{3}{8}$$

$$(d) \quad \text{Reflex } \angle D = 360^\circ - 98^\circ \text{ (revolution)} \\ = 262^\circ$$

$$\theta = 360^\circ - 21^\circ - 33^\circ - 262^\circ \text{ (angle sum of quadrilateral)} \\ = 44^\circ$$

$$(e) \quad \text{Total Cost (Phoenix)} = 2 \times \$550 + \$275 \\ = \$1375$$

$$\text{Total Cost (Sunshine)} = 0.6 \times \$600 + 0.75 \times \$600 \\ + 2 \times \$300$$

$$= \$1410 \text{ (with correct conclusion)}$$

\therefore Phoenix is cheaper

(14)

Question 6

$$(a) \quad (i) \quad \sqrt{0.81} = 0.9$$

$$(ii) \quad \left(\frac{1}{2}\right)^3 + \left(-\frac{3}{4}\right)^2 = \frac{1}{8} + \frac{9}{16} \\ = \frac{11}{16}$$

(b) Let x be the number of runs David scored

$$(i) \quad 2x + x$$

$$(ii) \quad 2x + x = \frac{1}{3}(126) \\ 3x = 42$$

$$x = 14$$

(iii) Michael scored 28 runs

$$(c) \quad 24 \text{ crates weigh } 1.8 \text{ tonnes}$$

$$36 \text{ crates weigh } 2.7 \text{ tonnes}$$

$$\therefore \text{Weight of truck is } 6.0 - 2.7 = 3.3 \text{ tonnes}$$

(d)

$$\angle HDF = \alpha \text{ (angles opposite equal sides of triangle)}$$

$$\angle FDE = 180^\circ - 90^\circ - \alpha \text{ (straight angle)} \\ = 90^\circ - \alpha$$

$$\therefore \angle DFE = 180^\circ - (90^\circ - \alpha) - \alpha \text{ (angle sum of triangle)} \\ = 90^\circ \text{ (as required)}$$

(e)

$$\begin{array}{r} 79365 \\ 7 \overline{) 555555} \end{array}$$

attempt at division.

correct answer

(14)

Question 7

$$(a) \quad (i) \quad 1.385 - 3.705 = -(3.705 - 1.385) \\ = -2.32$$

$$(ii) \quad \frac{3}{4} - 2\frac{1}{8} \times \frac{2}{3} + \frac{1}{6} = \frac{3}{4} - \frac{17}{8} \times \frac{2}{3} + \frac{1}{6} \\ = \frac{3}{4} - \frac{17}{12} + \frac{1}{6} \\ = \frac{9 - 17 + 2}{12} \\ = -\frac{6}{12} \\ = -\frac{1}{2}$$

Question 7 (cont.)

(b) Either: $\angle XPB = 62^\circ + 60^\circ$ (exterior angle of $\triangle KPX$) ✓
 $= 122^\circ$

$\therefore AB \parallel EF$ (supplementary co-interior angles, $\angle XQL$ and $\angle XPB$) ✓

or, $\angle QXL = 60^\circ$ (vertically opposite angles) ✓

$\angle DXQ = 60^\circ + 58^\circ$ (adjacent angles) ✓
 $= 118^\circ$

$\therefore CD \parallel GH$ (equal alternate angles, $\angle GRQ = \angle DXQ$) ✓

(c) (i) $X = \sqrt{\frac{5-3}{5+3}}$
 $= \sqrt{\frac{1}{4}}$ ✓ ($\sqrt{\frac{2}{8}}$ is o.k.)
 $= \frac{1}{2}$ ✓

(ii) Any multiple e.g. $a = 10$

$b = 6$

or $a = -5$

$b = -3$

(d) After mixing total volume is 4800 ml ✓

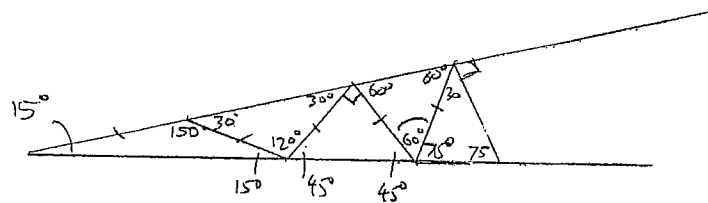
Volume of pure juice = $(260 + 360 + 180)$ ml ✓
 $= 800$ ml ✓

\therefore Percentage of pure juice = $\frac{800}{4800} \times 100\%$
 $= \frac{1}{6} \times 100\%$
 $= 16\frac{2}{3}\%$ ✓ (14)

Question 8

(a) $\frac{a-b}{b-a} - \frac{c+d}{d+c} = \frac{-(b-a)}{b-a} - 1$
 $= -2$ ✓

(b)



\therefore Angles are $75^\circ, 75^\circ, 30^\circ$ ✓

(c) Assume the share price was \$100.

After fall of 40% Price is \$60

After rise of 25% Price is \$75 ✓

After fall of 60% Price is \$30

After doubling Price is \$60

\therefore It must rise \$40 ✓

\therefore Percentage rise required = $\frac{40}{60} \times 100\%$
 $= 66\frac{2}{3}\%$ ✓

(d) (i) Exterior angle of a triangle is equal to the sum of the opposite interior angles ✓

(ii) $\angle ECA = x+y$ (given)

$\angle ACB = 180^\circ - (x+y) + (x+y)$ (straight angle)
 $= 180^\circ - 2x - 2y$

$\angle BAC = 180^\circ - (180 - 2x - 2y) - 2x$ (angle sum triangle)
 $= 2y$

Question 8 (cont.)

(b) (i)

3 faces painted	=	8	(each corner)	
2 faces painted	=	8×12	(edges or sides)	
	=	96		
1 face painted	=	64×6	(faces)	✓
	=	384		
0 faces painted	=	$8 \times 8 \times 8$		
	=	512		

(ii) Fully painted cube uses:

8	(3 faces)
$2 \times 12 = 24$	(2 faces)
$4 \times 6 = 24$	(1 face)
8	(0 faces)

\therefore 1 cube can be formed, since all 8 painted corner cubes are used. ✓

(iii) Remaining cubes with 0 faces painted

$$= 512 - 8$$
$$= 504$$

64 cubes are required \therefore

$$504 \div 64 = 7\frac{7}{8}$$

\therefore 7 solid unpainted cubes can be constructed ✓

(iv) Remaining Cubes:

3 faces painted :	0
2 faces painted :	$96 - 24 = 72$
1 face painted :	$384 - 24 = 360$ ✓
0 faces painted :	$504 - (7 \times 64) = 56$

14