SYDNEY GRAMMAR SCHOOL

2010 Half-Yearly Examination

FORM II

MATHEMATICS

Thursday 6th May 2010

General Instructions
- Writing time — 1 hours 30 minutes
- Write using black or blue pen.
- Calculators are not to be used.
- All necessary working should be shown in every question.
- Start each question on a new page.

Structure of the paper
- Total marks — 108
- All nine questions may be attempted.
- All nine questions are of equal value.

Collection
- Write your name, class and master clearly on each page of your answers.
- Staple your answers in a single bundle.
- Write your name and master on this question paper and submit it with your answers.

2A: KWM	2B: SJE	2C: JMR
2D: LYL	2E: TCW	2F: BR
2G: RCF	2H: SO	2I: MW

Checklist
- Writing paper required.
- Candidature — 188 boys

Examiner
BR

QUESTION ONE   (12 marks)   Start a new page.

(a) Evaluate:

  (i) $\dfrac{7}{10} - \dfrac{3}{5}$

  (ii) $0{\cdot}173 + 0{\cdot}21$

  (iii) $234 \div 3$

  (iv) $\dfrac{1}{4}$ of 48

(b) Write as percentages:

  (i) $\dfrac{1}{5}$

  (ii) $0{\cdot}7$

(c) Reduce each fraction to its lowest terms:

  (i) $\dfrac{50}{60}$
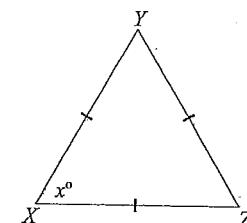
  (ii) $\dfrac{12}{15}$

(d) Simplify:

  (i) $4y - 2y$

  (ii) $-6m \times (-7)$

(e)

Find the value of x giving a clear geometric reason.
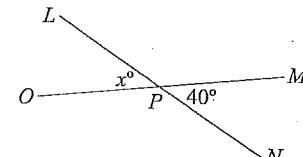
QUESTION TWO (12 marks) Start a new page.

(a) Solve $m + 10 = 7$.

(b) Evaluate $3a^2$ when $a = 2$.

(c) Factorise $7a + 21$.

(d) If three angles in a quadrilateral are $50°$, $160°$ and $30°$, what is the size of the fourth angle?

(e) Evaluate:

  (i) $\dfrac{2}{3} \div \dfrac{3}{4}$

  (ii) $19 - (7 - 11)$

  (iii) $0.9 - 0.35$

(f) Simplify:

  (i) $16 + 3 \times p$

  (ii) $4 \times x \times 3 \times y \times x$

  (iii) $\dfrac{17pq^2}{pq}$

(g) Find 30% of 120.

(h) What percentage is 20 of 50?
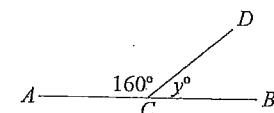
QUESTION THREE (12 marks) Start a new page.

(a) Write $\dfrac{38}{5}$ as a mixed fraction.

(b) Write $\dfrac{384}{300}$ as a percentage.

(c) I double a number x and then add 4. The result is 16.

  (i) Write an equation for this calculation.

  (ii) Solve the equation to find x.

(d) Simplify:

  (i) $7x + 3 - 4x + 9$

  (ii) $8a^2 - 3b + 7b - 5a$

(e) Solve $\dfrac{x+3}{5} = 7$.

(f) (i)

Find the value of x, giving reasons.

  (ii)

Find the value of y, giving reasons.

<u>QUESTION FOUR</u> (12 marks) Start a new page.

(a) Factorise $3x - 108xy$.

(b) Simplify:

   (i) $\dfrac{3x}{2} + \dfrac{x}{5}$

   (ii) $\dfrac{2x}{5} \div \dfrac{3x}{10}$

(c) Evaluate:

   (i) $2\frac{1}{2} - 1\frac{3}{4}$
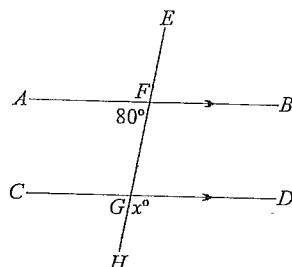
   (ii) $3\frac{4}{5} \times 1\frac{1}{3}$

(d) Evaluate:

   (i) $(0\cdot2)^3$

   (ii) $84 \div 7 - 2 \times 5$

(e)

Find the value of x, giving clear reasons for each step.

<u>QUESTION FIVE</u> (12 marks) Start a new page.

(a) After a 20% reduction in price a mountain bike cost $288. How much did the bike cost before the reduction?

(b) Solve $\dfrac{2x}{7} > 8$.

(c) Evaluate $0\cdot6 \div 0\cdot002$.
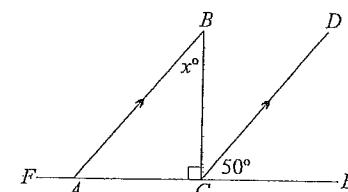
(d) When a number is decreased by three and the result doubled, the answer is equal to five more than three times the original number.

   (i) Let the number be n and write an equation showing the given information.

   (ii) Solve the equation to find the value of n.

(e) At the start of Form I, Juan was 160cm tall. At the end of Form II, he was 172cm tall. What was his percentage increase in height?

(f)

Find the value of x, giving clear reasons for each step.

QUESTION SIX (12 marks) Start a new page.

(a) Express $\frac{6}{7}$ as a recurring decimal.
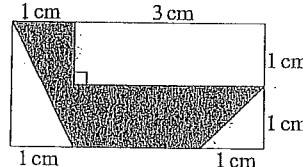
(b) Solve the equations:

    (i) $13a - 27 = 25a - 33$

    (ii) $7x - 3(x - 2) = 62$

(c) A tennis player has won 32 out of 48 matches. His sponsor says that he must win 60% of his total number of matches to qualify for a bonus. If there are 17 matches remaining on the tour, what is the minimum number of games he must win to collect his bonus?

(d)

1 cm   3 cm   1 cm   1 cm   1 cm   1 cm

This is a rectangle which has been divided into 4 sections. What percentage is the shaded area of the total rectangle?

QUESTION SEVEN (12 marks) Start a new page.
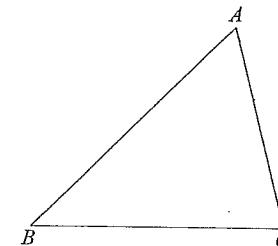
(a) Find the simple interest on $6000 at 15% p.a. for 18 months.

(b) Evaluate $105 \div (((16 + 8) \div 12) + 5)$.

(c) (i) Solve the inequation: $2(2x - 5) > 6x + 3$.

    (ii) Graph the solution on a number line.

(d) Solve the equation $\dfrac{x}{3} - \dfrac{1 - x}{9} = -5$.

(e)

Copy the diagram of $\triangle ABC$, shown above, and then complete the proof below.

AIM: To prove that the angle sum of a triangle is 180°.

CONSTRUCTION:

PROOF:

QUESTION EIGHT (12 marks)   Start a new page.
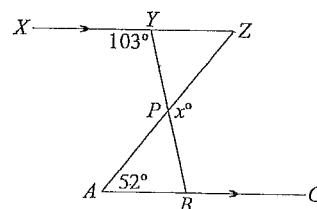
(a) I bought real estate valued at $500 000. In the first year my investment increased by 12%, but in the second year the value dropped by 7%. What is the value of the property at the end of the second year?

(b) Find the value of z when $x = 2\frac{3}{4}$ and $y = 1\frac{1}{2}$ if

$$\frac{1}{2x} - \frac{1}{z} = \frac{3}{y} \; .$$

(c) A triangle is to be drawn with one angle 16° less than another and the third angle is 20° less than the sum of the other two. Find the angles of the triangle, clearly showing your working.

(d)

Find the value of x, giving clear reasons for each step.

QUESTION NINE (12 marks)   Start a new page.

(a) If $\dfrac{1}{y+3} = 5$, find the value of $\dfrac{1}{y+5}$.

(b) If x is 250% of y, then what percentage is $2y$ of x?

(c) I have two twelve hour watches each with a minute hand and an hour hand. One gains a minute per day and the other loses $1\frac{1}{2}$ minutes per day. If I set them both to the correct time, how long before they next together tell the correct time?

(d) A four digit number is a perfect square and it is of the form *aabb* where a and b are digits. Find the number, clearly showing your working.

END OF EXAMINATION

Solution - HY Form II 2010

Question 1

a) i) $\dfrac{1}{10}$ ✓

ii) 0.383 ✓

iii) 78 ✓

iv) 12 ✓

b) i) 20% ✓

ii) 70% ✓

c) i) $\dfrac{5}{6}$ ✓

ii) $\dfrac{4}{5}$ ✓

d) i) $2y$ ✓

ii) $42m$ ✓

e) $x = 60°$ [angle of equilateral $\triangle XYZ$] ✓ ✓

$\boxed{12}$

Question 2

a) $m = -3$ ✓

b) $3a^2 = 12$ ✓

c) $7a + 21 = 7(a + 3)$ ✓

d) $360° - 50° - 160° - 30° = 120°$ ✓

e) i) $\dfrac{2}{3} \times \dfrac{4}{3} = \dfrac{8}{9}$ ✓

ii) $19 - (-4) = 23$ ✓

iii) 0.55 ✓

f) i) $16 + 3p$ ✓

ii) $12x^2y$ ✓

iii) $17q$ ✓

g) 36 ✓

h) $\dfrac{20}{50} \times 100\% = 40\%$ ✓

Question 3

a) $\dfrac{38}{5} = 7\dfrac{3}{5}$ ✓

b) $\dfrac{384}{300} = \dfrac{128}{100}$ ✓

  $= 128\%$ ✓

c) i) $2x + 4 = 16$ ✓

  ii) $2x = 12$ ✓

   $x = 6$

d) i) $3x + 12$ ✓

  ii) $8a^2 + 4b - 5a$ ✓

e) $\dfrac{x+3}{5} = 7$

  $x + 3 = 35$ ✓

  $\therefore x = 32$ ✓

f) i) $x = 40°$ [Vertically opposite angle] ✓

  ii) $y = 20°$ [straight angle ACB] ✓

part (f) must have the reason
for the mark.

$\boxed{12}$

Question 4

a) $3x(1 - 36y)$ ✓

b) i) $\dfrac{17x}{10}$ ✓

  ii) $\dfrac{4}{3}$ ✓ or $\left[1\dfrac{1}{3}\right]$

c) i) $2\dfrac{1}{2} - 1\dfrac{3}{4} = \dfrac{5}{2} - \dfrac{7}{4}$ ✓

   $= \dfrac{10}{4} - \dfrac{7}{4}$

   $= \dfrac{3}{4}$ ✓

  ii) $3\dfrac{4}{5} \times 1\dfrac{1}{3} = \dfrac{19}{5} \times \dfrac{4}{3}$ ✓

   $= \dfrac{76}{15}$ or $5\dfrac{1}{15}$ ✓

d) i) 0.008 ✓

  ii) $84 \div 7 - 2 \times 5 = 12 - 10$ ✓

   $= 2$ ✓

e) $\angle EGC = 100°$ [co-interior angles, AB||CD] ✓

  $\therefore x = 100°$ [Vertically opposite angles] ✓

OR any two steps to $x = 100°$
Must have reasons for mark.

$\boxed{12}$

Question 5

a) 80% of sales price is $288 ✓
   10% of sales price is $36
   100% of sales price is $360
   ∴ The bike cost $360 before the reduction.

b) $\dfrac{2x}{7} > 8$

   $2x > 56$ ✓

   $x > 28$ ✓

c) $0.6 \div 0.002 = 600 \div 2$

   $= 300$ ✓

d) i) $2(n-3) = 3n + 5$ ✓

   ii) $2n - 6 = 3n + 5$

   $-6 = n + 5$ ✓

   $n = -11$

e) % increase $= \dfrac{172 - 160}{160} \times 100\%$ ✓

   $= \dfrac{12}{160} \times 100\%$

   $= \dfrac{15}{2}\% \text{ or } 7\dfrac{1}{2}\%$ ✓

f) $\angle BAC = 50°$ [corresponding angle $AB \parallel CD$] ✓
   $x + 90° + 50° = 180°$ [angle sum of $\triangle ABC$]
   $x = 40°$ ✓

(12)

Question 6

a)
```
      0.8571428
   7 )6.0⁶0⁵0¹0³0²0⁶0⁴0
```
✓ for using divisi
✓ for correct div

   ∴ $\dfrac{6}{7} = 0.\dot{8}5714\dot{2}$ ✓ for repeaters in the right place

b) i) $13a - 27 = 25a - 33$
   $-12a - 27 = -33$ ✓
   $-12a = -6$
   ∴ $a = \dfrac{1}{2}$ ✓

   ii) $7x - 3(x-2) = 62$ ✓
   $7x - 3x + 6 = 62$ ✓
   $4x + 6 = 62$
   $4x = 56$
   $x = 14$ ✓

c) let w be the min. matches he must win to collect his bonus
   $\dfrac{32 + w}{48 + 17} = \dfrac{60}{100}$ ✓
   $\dfrac{32 + w}{65} = \dfrac{3}{5}$
   $160 + 5w = 195$
   $5w = 35$
   $w = 7$
   ∴ He must win a minimum of ✓
   7 more matches
   OR 39 total matches.

d) Area of rectangle $= (4)(2)$ ✓
   $= 8 \, cm^2$ ✓

Area shaded $= 8 - \frac{1}{2}(1)(2) - 3(1) - \frac{1}{2}(1)(1)$

$= \frac{7}{2} \, cm^2 \quad or \quad 3\frac{1}{2} \, cm^2$ ✓

% shaded area $= \frac{\frac{7}{2}}{8} \times 100\%$

$= \frac{\frac{7}{\cancel{2}}}{\cancel{16}4} \times \cancel{100}^{25}\%$

$= \frac{175}{4}\% \quad or \quad 43\frac{3}{4}\%$ ✓

(12)
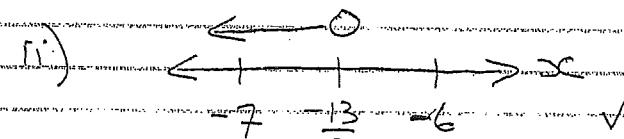
Question 7

a) $I = PRT$
   $= (6000)(0.15)(1.5)$ ✓
   $= 900(1.5)$
   $= \$1350$
   ∴ The simple interest is $\$1350$. ✓
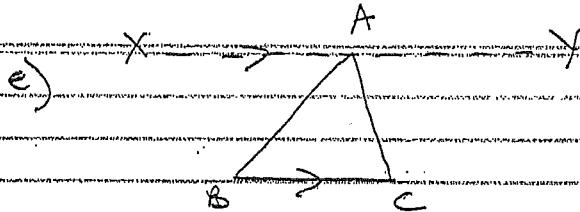
b) $105 \div (((16+8) \div 12) + 5)$
   $= 105 \div ((24 \div 12) + 5)$
   $= 105 \div (2 + 5)$ ✓
   $= 105 \div 7$
   $= 15$ ✓

c) i) $2(2x - 5) > 6x + 3$
   $4x - 10 > 6x + 3$
   $-2x - 10 > 3$ ✓
   $-2x > 13$
   $x < -\frac{13}{2}$ ✓

   ii)

d) $\frac{x}{3} - \frac{1-x}{9} = -5$
   $3x - (1-x) = -45$
   $3x - 1 + x = -45$ ✓
   $4x = -44$
   $x = -11$ ✓

e)

Aim: To prove that the angle sum
   of a triangle is 180°
Construction: Construct XY ∥ BC through A
Proof: ∠XAB = ∠ABC [alternate ∠'s, XY∥BC]
        ∠YAC = ∠ACB [alternate ∠'s, XY∥BC]
∴ ∠XAB + ∠BAC + ∠YAC = 180° [straight ∠XAY]
∴ ∠ABC + ∠BAC + ∠ACB = 180°
∴ The angle sum of a triangle is 180°
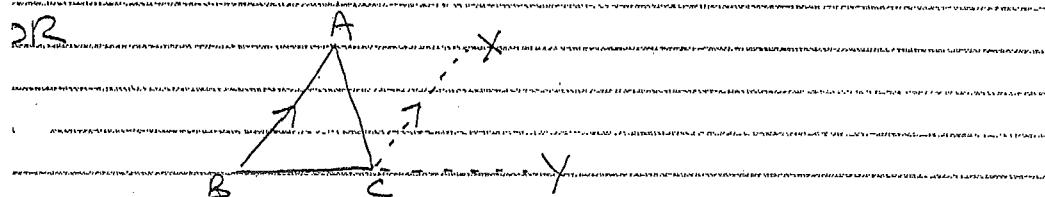
2R

Construction: Construct ray CX ∥ AB
              Produce BC to Y
Proof: ∠XCY = ∠ABC [corresponding ∠'s, AB∥XC]
        ∠XCA = ∠BAC [alternate ∠'s, AB∥XC]
∠XCY + ∠XCA + ∠ACB = 180° [straight ∠BCY]
∠ABC + ∠BAC + ∠ACB = 180°
∴ The angle sum of a triangle is 180°

(12)

Question 8

a) Value @ 1 year = (500 000)(1·12)
                   = $560 000

93 x
56
558
+1,050
52,08

    Value @ 2nd year = (560 000)(0.93)
                     = $520 800

∴ The property is worth
  $520 800 at the end of
  the 2nd year.

b) x = 2¾   y = 1½
      = 11/4   = 3/2

    2(11/4)   1/2 = 3/2
    ──────   ───   ───
                      ½   3/2

    2/11   1/2 = 2
    ────   ───
    11   ½

                    -1/2 = 20
                    ───   ──
                     ½   11

∴ -2 = -11
        ──
        20

c) Let one angle of the triangle be θ
    θ, θ-16°, (θ+θ-16°)-20°
Angles θ, θ-16°, 2θ-36°
    θ + θ-16° + 2θ-36° = 180°
                  4θ = 232°
                   θ = 58°
                θ-16° = 42°
               2θ-36° = 80°
∴ The angles of the triangle
   are 58°, 42° and 80°.

d) $\angle YBC = 103°$ [alternate angles XZ||AC]
$\angle APB = 103° - 52°$ [exterior angle of $\triangle APB$]
$ = 51°$
$\therefore x = 129°$ [straight $\angle APZ$]

[They will probably do the 2nd part in two steps, ie adjacent angles on a straight line and angle sum of $\triangle APB$]

(12)

<div>

Question 9

a) $\dfrac{1}{y+3} = 5$

$1 = 5y + 15$

$5y = -14$

$y = \dfrac{-14}{5}$

$\therefore \dfrac{1}{y+5} = \dfrac{1}{\frac{-14}{5}+5}$

$= \dfrac{1}{\frac{11}{5}}$

$= \dfrac{5}{11}$

b) $\dfrac{x}{y} \times 100\% = 250\%$

$100x = 250y$

$\dfrac{2y}{x} \times 100\% = \dfrac{200y}{x} \%$

$= \dfrac{20000y}{100x} \%$

$= \dfrac{20000y}{1250y} \%$

$= 80\%$

$\therefore 2y$ is 80% of x

</div>

c)   12 hours = 720 mins

1st watch tells correct time after
it has gained 720, 1440.... mins.
As it gains 1 min/day this takes
720, 1440... days

2nd watch is correct after it loses
720, 1440... mins. As it loses
$1\frac{1}{2}$ min/day this takes:

$\frac{720}{1\frac{1}{2}} = 430$ or $\frac{1440}{1\frac{1}{2}} = 960$ or $\frac{2160}{1\frac{1}{2}} = 1440$ days

∴ The first time both watches will
tell the correct time together is
after 1440 days

d) $aabb = 1000a + 100a + 10b + b$
$= 1100a + 11b$
$= 11(100a + b)$

As the number is a perfect square:
$100a + b = 11x^2$ for some integer x
So, $11^2 x^2$ is a 4 digit number
∴ $1000 < 11^2 x^2 < 10000$
$31 < 11x < 100$ [square root both sides]
∴ $11x = 33, 44, 55, 66, 77, 88, 99$
∴ $(11x)^2 = 1089, 1936, 3025, 4356,$
$5929, 7744, 9801$

∴ The number is 7744 ✓

[Give full marks for guess & check]

⑫