



2011 Half-Yearly Examination

FORM II MATHEMATICS

Tuesday 17th May 2011

General Instructions

- Writing time — 1 hour 30 minutes
- Write using black or blue pen.
- Calculators are not to be used.
- All necessary working should be shown in every question.
- Start each question on a new page.

Structure of the paper

- Total marks — 105
- All seven questions may be attempted.
- All seven questions are of equal value.

Collection

- Write your name, class and master clearly on each page of your answers.
- Staple your answers in a single bundle.
- Write your name and master on this question paper and submit it with your answers.

2A: BR	2B: SJG	2C: TCW
2D: JMR	2E: BDD	2F: REP
2G: KWM	2H: MW	2I: FMW

Checklist

- Writing paper required.

Examiner

JMR

QUESTION ONE (15 marks) Start a new page.

(a) Simplify:

(i) $5x - 4x$

(ii) $5y \times 2y$

(iii) $24x \div x$

(b) Evaluate:

(i) $-9 - 8$

(ii) $(-3)^2$

(iii) 0.038×100

(c) Simplify:

(i) $2a - 11a$

(ii) $(2x)^2$

(iii) $\frac{2x}{3} - \frac{x}{3}$

(d) Evaluate:

(i) $200.6 - 13.9$

(ii) $1\frac{3}{4} + 3\frac{1}{2}$

(iii) $\frac{2}{7} \div \frac{1}{14}$

(iv) $12.06 \div 6$

(v) 5^3

(e) Find the supplement of 40° .

QUESTION TWO (15 marks) Start a new page.

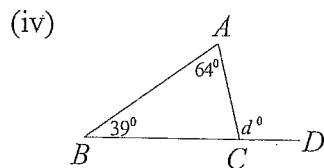
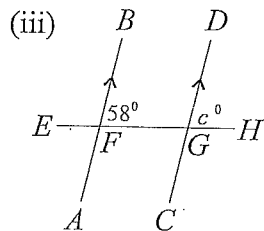
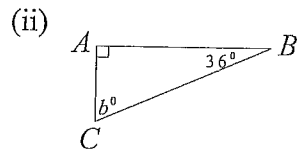
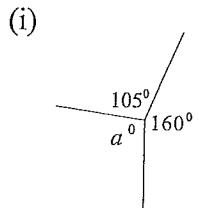
(a) If $x = \frac{2}{3}$, evaluate $3x - 2$.

(b) Evaluate:

(i) $\frac{3}{16} \times \frac{4}{15}$

(ii) 15% of \$5.00

(c) Find the value of the pronumeral in each of the diagrams below, giving reasons for your answers.



Question Two continues overleaf.

Exam continues overleaf ...

QUESTION TWO (Continued)

(d) Change 20% to a fraction in lowest terms.

(e) Write $1\frac{1}{4}$ as a percentage.

(f) Write 0.78 as a percentage.

(g) Simplify:

(i) $\frac{5m}{14} - \frac{3m}{7}$

(ii) $\frac{2x}{5} \times \frac{10}{x}$

(iii) $\frac{45x^2y}{5x}$

(h) Expand $2x(x^2 - y)$.

(i) Decrease \$50 by 50% .

Exam continues next page ...

QUESTION THREE (15 marks) Start a new page.

(a) Solve:

(i) $2x + 1 = 5$

(ii) $\frac{2x}{3} = 12$

(b) Factorise by taking out the highest common factor.

(i) $6x - 24$

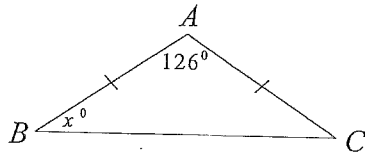
(ii) $7xy^3 + 21x^2y$

(c) Expand and simplify $3(2x - 5) + 4(3x + 4)$.

(d) Given that $A = \frac{h(a+b)}{2}$, find the value of A when $h = 6$, $a = 3$ and $b = 7$.

(e) Evaluate $1\frac{4}{5} - \frac{3}{7}$.

(f)

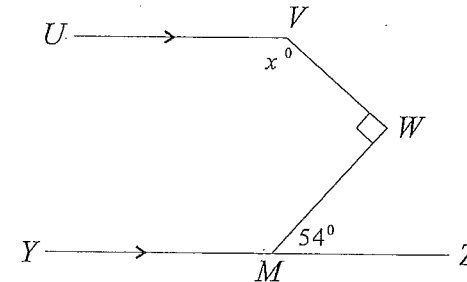


In the diagram above, find the value of x , giving all reasons. $\triangle ABC$ is isosceles.

(g) Solve the inequation $-2x \leq 14$.

QUESTION FOUR (15 marks) Start a new page.

(a)



In the diagram above, UV is parallel to YZ , $\angle VWM$ is 90° and $\angle WMZ$ is 54° . Copy the diagram onto your page.

Find the value of x , showing any constructions and giving all reasons.

(b) I invested \$750 with a bank which offered me a simple interest rate of 5.25% per annum. How much interest would I earn from the investment over four years?

(c) Divide 780 by 0.013.

(d) Given the formula $v = u + at$, find a if $v = 53$, $u = 13$ and $t = 5$.

(e) Simplify $2\frac{1}{13} \div \frac{9}{26} \times 1\frac{1}{2}$.

(f) Solve the inequation $4(x - 2) < 18$ and graph your solution on a number line.

(g) On average, Sydneysiders each consumed 2.05 kg of bacon in 2010. If the population of Sydney in 2010 was 4.15 million, what was the total amount of bacon consumed by Sydneysiders in 2010?

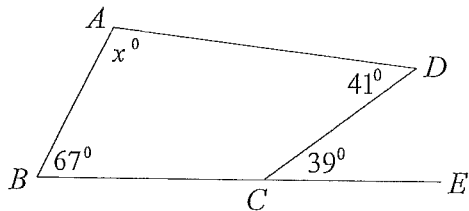
QUESTION FIVE (15 marks) Start a new page.

- (a) Expand and simplify $3x^2 - 9x(x - 1)$.
- (b) Solve $\frac{2x + 1}{7} - 3 = \frac{5x - 9}{6}$.
- (c) In a 500 gram packet of Brittos breakfast cereal, the weight (in grams) of fat, carbohydrate, dietary fibre, protein and other constituents is shown in the table below.

Contents	Weight(g)
Fat	105
Carbohydrate	190
Dietary Fibre	75
Protein	80
Others	50

- (i) Find the percentage of fat in a 500 g packet.
- (ii) Find the percentage of dietary fibre in a 500 g packet.
- (iii) If the amount of protein was reduced by 25% and replaced by the same weight of fat, what percentage of a packet would now be fat?
- (d) Increase 12.5 metres by 140%.

(e)



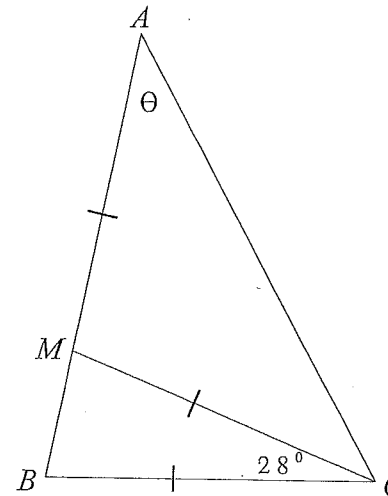
Find the value of x in the diagram above. Give all reasons.

- (f) James is training hard to become a better swimmer. He spends 6 hours per week doing butterfly, 7 hours per week doing freestyle and 4 hours per week in the gym.
- (i) What fraction of his training time is spent out of the pool?
- (ii) What percentage of a whole week is spent doing freestyle?

QUESTION SIX (15 marks) Start a new page.

- (a) Write the following fractions in order from largest to smallest.
 $\frac{3}{4}$, $\frac{4}{7}$, $\frac{2}{3}$, $\frac{7}{10}$.

(b)



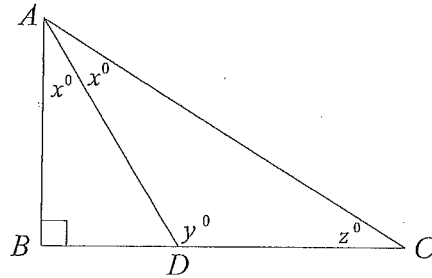
In the diagram above, find the value of θ , giving all reasons.

- (c) Two rugby league teams, the Bears and the Wolves play a match. The Bears Club has 10 000 members and the Wolves Club has 12 000 members. 15% of the Bears Club members and 20% of the Wolves Club members attend the match. What percentage of the combined membership of the clubs attend the match?
- (d) Find the value of the expression $\frac{2 - x}{x^2 - 5}$ when $x = -4$.
- (e) Simplify $(2a^3b^2)^4 \times 5a^9b^7$.
- (f) The sum of four and a certain number is doubled and the result is subtracted from 1. If the answer is equal to the number itself, form an equation and solve it to find the number.
- (g) A shopkeeper marks the price of a microwave oven at \$600. After allowing 10% discount on the oven, the shopkeeper still makes a 20% profit on his cost price. What price did the shopkeeper pay for the oven?

QUESTION SEVEN (15 marks) Start a new page.

(a) Find the last digit of 3^{2011} .

(b)



In the right angled triangle $\triangle ABC$ above, $\angle ABC = 90^\circ$. The bisector of $\angle BAC$ meets BC at D . Let $\angle BAD = x^\circ$, $\angle ADC = y^\circ$, and $\angle ACD = z^\circ$. Show that $y^\circ = 135^\circ - \frac{1}{2}z^\circ$.

(c) An amount of money $\$X$ is increased by $y\%$ and then the resulting amount is decreased by 4% . If this produces a net increase of 8% in X , find the value of y .

(d) Adam, Barry and Christina's mother left a bowl of licorice allsorts on their kitchen table. Adam took one third of the lollies, but then threw back four because he didn't like the green ones. Barry then ate six lollies and took one quarter of those still remaining. Christina then took one third of the remaining lollies and then picked out the last four orange ones from the bowl, because she really liked them. If after all this there were twelve licorice allsorts left, how many were in the bowl to begin with?

(e) If a , b and c are positive integers such that $\frac{a}{3} = \frac{b}{4} = \frac{c}{5}$ and $abc = 1620$, find b .

END OF EXAMINATION

QUESTION ONE (15 marks)

- (a) (i) $5x - 4x = x$ ✓
 (ii) $5y \times 2y = 10y^2$ ✓
 (iii) $24x \div x = 24$ ✓
 (b) (i) $-9 - 8 = -17$ ✓
 (ii) $(-3)^2 = 9$ ✓
 (iii) $0.038 \times 100 = 3.8$ ✓
 (c) (i) $2a - 11a = -9a$ ✓
 (ii) $(2x)^2 = 4x^2$ ✓
 (iii) $\frac{2x}{3} - \frac{x}{3} = \frac{x}{3}$ ✓
 (d) (i) $200 \cdot 6 - 13 \cdot 9 = 186.7$ ✓
 (ii) $1\frac{3}{4} + 3\frac{1}{2} = 5\frac{1}{4}$ ✓
 (iii) $\frac{2}{7} \div \frac{1}{14} = \frac{2}{7} \times \frac{14}{1} = 4$ ✓
 (iv) $12.06 \div 6 = 2.01$ ✓
 (v) $5^3 = 125$ ✓
 (e) 140° ✓

QUESTION TWO (15 marks)

- (a) $3x - 2 = 3 \times \frac{2}{3} - 2$
 $= 2 - 2$
 $= 0$ ✓
 (b) (i) $\frac{1}{16} \times \frac{1}{15} = \frac{1}{240}$ ✓
 (ii) $15\% \text{ of } \$5.00 = \frac{15}{100} \times \frac{500}{1}$
 $= 75 \text{ cents}$ ✓
 or $\$0.75$ ✓
 (c) (i) $a^\circ = 95^\circ$ (Revolution)
 (ii) $b^\circ = 54^\circ$ (Angle sum $\triangle ABC$)
 (iii) $c^\circ = 58^\circ$ (Corresponding angles, $AB \parallel CD$)
 (iv) $d^\circ = 103^\circ$ (Exterior angle $\triangle ABC$)

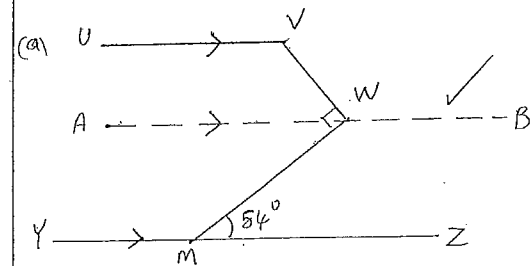
(* Lose 1 mark for 1 or 2 incorrect angles or reasons)

- (d) $20\% = \frac{20}{100} = \frac{1}{5}$ ✓
 (e) $1\frac{1}{4} \times 100 = \frac{5}{4} \times \frac{100}{1} = 125\%$ ✓
 (f) $0.78 = 78\%$ ✓
 (g) (i) $\frac{5m}{14} - \frac{3m}{7} = \frac{5m - 6m}{14} = -\frac{m}{14}$ ✓
 (ii) $\frac{2x^2}{81} \times \frac{10^2}{x} = 4$ ✓
 (iii) $\frac{948x^2y}{18x} = 9xy$ ✓
 (h) $2x(x^2 - y) = 2x^3 - 2xy$ ✓
 (i) $\$50 \times \frac{50}{100} = \25 ✓

QUESTION THREE (15 marks)

- (a) (i) $2x + 1 = 5$
 $2x = 4$
 $x = 2$ ✓
 (ii) $\frac{2x}{3} = 12$
 $2x = 36$
 $x = 18$ ✓
 (b) (i) $6x - 24 = 6(x - 4)$ ✓
 (ii) $7xy^3 + 21x^2y = 7xy(y^2 + 3x)$ ✓
 (c) $3(2x - 5) + 4(3x + 4) = 6x - 15 + 12x + 16 = 18x + 1$ ✓
 (d) $A = \frac{h(a+b)}{2} = \frac{6(3+7)}{2} = \frac{60}{2} = 30$ ✓
 (e) $\frac{1\frac{4}{5} - \frac{3}{7}}{1} = \frac{9}{5} - \frac{3}{7} = \frac{63 - 15}{35} = \frac{48}{35} = 1\frac{13}{35}$ ✓
 (f) $2x^\circ = 54^\circ$ (Angle sum of isosceles \triangle)
 $x = 27$ ✓
 (g) $-2x \leq 14$
 $-x \leq 7$ ✓
 $x \geq -7$ ✓

QUESTION FOUR (15 marks)



- (a) Construct $AB \parallel UV$ through W .
 $\angle AWM = 54^\circ$ (Alternate angles, $AB \parallel UV$)
 $\angle VWA = 36^\circ$ (Adjacent complementary angles)
 $\angle UZW = 144^\circ$ (Co-interior angles, $AB \parallel UV$)
 $x = 144$ ✓
 (b) Interest = $\frac{5.25}{100} \times 750 \times 4 = \frac{21 \times 15}{2} = \157.50 ✓
 (c) $780 \div 0.013 = 780000 \div 13 = 60000$ ✓

- (d) $v = u + at$
 $53 = 13 + 5a$
 $5a = 40$
 $a = 8$ ✓
 (e) $2\frac{1}{3} \div \frac{9}{26} \times 1\frac{1}{2} = \frac{27}{13} \times \frac{26}{9} \times \frac{3}{2} = 9$ ✓

- (f) $4(x - 2) < 18$
 $4x - 8 < 18$
 $4x < 26$
 $x < 6\frac{1}{2}$ ✓

 (g) $2.05 \text{ kg} \times 4.15 \text{ million} = 8.5075 \text{ million kg}$

4.15
2.05
83000
8.5075

QUESTION FIVE (15 marks)

(a) $3x^2 - 9x(x-1) = 3x^2 - 9x^2 + 9x$
 $= -6x^2 + 9x$

(b) $\frac{2x+1}{7} - 3 = \frac{5x-9}{6}$

$6(2x+1) - 126 = 7(5x-9)$

$12x + 6 - 126 = 35x - 63$

$-57 = 23x$

$x = -\frac{57}{23}$

(c) (i) $\frac{105}{500} \times \frac{100}{1} = 21\%$

(ii) $\frac{75}{500} \times \frac{100}{1} = 15\%$

(iii) 25% of $80g = 20g$
 Total fat now = $105 + 20 = 125g$

$\frac{125}{500} \times \frac{100}{1} = 25\%$

25% of a packet would be fat.

(d) Increase by 140% , find 240% of 12.5

$12.5 \times \frac{240}{100} = 30$ metres

(e) $\angle BCD = 141^\circ$ (straight angle)

$x^\circ = 360^\circ - (67^\circ + 41^\circ + 141^\circ)$
 (Angle sum quadrilateral)

$x^\circ = 111^\circ$

(f) (i) 17 hours training
 $\frac{4}{17}$ out of the pool.

(ii) 7 hours of 7×24 hours spent doing freestyle.

$\frac{7}{7 \times 24} \times \frac{100}{1} = \frac{100}{24}$
 $= 4\frac{1}{6}\%$

QUESTION SIX (15 marks)

(a) $\frac{3}{4} = \frac{315}{420}$, $\frac{4}{7} = \frac{240}{420}$, $\frac{2}{3} = \frac{280}{420}$
 $\frac{7}{10} = \frac{294}{420}$

$\frac{3}{4} > \frac{7}{10} > \frac{2}{3} > \frac{4}{7}$

(b) $\angle CMB = \frac{1}{2}(180^\circ - 28^\circ)$
 $= 76^\circ$ (Base angles isosceles $\triangle BMC$)

$\angle CMB = 2\theta$ (Exterior angle isosceles $\triangle AMC$)

$\theta = 38^\circ$

(c) 15% of 10000 Bears + 20% of 12000 Wolves

$= 1500 + 2400$
 $= 3900$ members attending

$\frac{3900}{22000} \times \frac{100}{1} = \frac{390}{22}$
 $= 17\frac{8}{11}\%$

(d) $\frac{2-x}{x^2-8} = \frac{2-(-4)}{(-4)^2-8}$
 $= \frac{6}{11}$

(e) $(2a^3b^2)^4 \times 5a^9b^7$
 $= 16a^{12}b^8 \times 5a^9b^7$
 $= 80a^{21}b^{15}$

(f) Let the number be x

$1 - 2(4+x) = x$

$1 - 8 - 2x = x$

$-7 = 3x$

$x = -\frac{7}{3}$

The number is $-\frac{7}{3}$

(g) Allowing 10% discount, price is \$540
 \$540 is 120% of the cost price.

Cost price = $\frac{540}{120} \times \frac{100}{1}$
 $= \$450$

The shopkeeper paid \$450 for the oven.

QUESTION SEVEN (15 marks)

(a) $3^1 = 3$

$3^2 = 9$

$3^3 = 27$

$3^4 = 81$

$3^5 = 243$

$3^6 = 729$

$3^7 = 2187$

$3^8 = 6561$

Sequence of last digit

3, 9, 7, 1, 3, 9, 7, 1, ...

So 3^{2011} will end with a 7.

(b) $y^\circ = 90^\circ + x^\circ$ (Exterior angle $\triangle ABD$)

So, $x^\circ = y^\circ - 90^\circ$

$x^\circ + y^\circ + z^\circ = 180^\circ$ (Angle sum $\triangle ACD$)

hence $y^\circ - 90^\circ + y^\circ + z^\circ = 180^\circ$

$2y^\circ + z^\circ = 270^\circ$

$2y^\circ = 270^\circ - z^\circ$

$y^\circ = 135^\circ - \frac{1}{2}z^\circ$

(c) $X \times \frac{100+y}{100} \times \frac{96}{100} = \frac{108}{100} \times X$

$\frac{96(100+y)}{100 \times 100} = \frac{108}{100}$

$96(100+y) = 10800$

$9600 + 96y = 10800$

$96y = 1200$

$y = 12\frac{1}{2}$

(d) Let the number of licorice allsorts to begin with = x .

After Adam, remaining lollies = $\frac{2}{3}x + 4$

After Barry, remaining lollies

$= \left[\left(\frac{2}{3}x + 4 \right) - 6 \right] \times \frac{3}{4}$

After Christina, remaining lollies

$= \frac{2}{3} \left\{ \left[\left(\frac{2}{3}x + 4 \right) - 6 \right] \times \frac{3}{4} \right\} - 4$

So, $\frac{2}{3} \left\{ \left[\left(\frac{2}{3}x + 4 \right) - 6 \right] \times \frac{3}{4} \right\} - 4 = 12$

$\frac{2}{3} \left\{ \left[\frac{2x}{3} - 2 \right] \times \frac{3}{4} \right\} = 16$

$\left[\frac{2x}{3} - 2 \right] \times \frac{3}{4} = 24$

$\frac{2x}{3} - 2 = 32$

$\frac{2x}{3} = 34$

$x = 51$

There were 51 licorice allsorts to begin with. (Any suitable method)

(e) $\frac{a}{3} = \frac{b}{4}$ $\frac{b}{4} = \frac{c}{5}$

$4a = 3b$ $4c = 5b$

$a = \frac{3b}{4}$ $c = \frac{5b}{4}$

So, $\frac{3b}{4} \times b \times \frac{5b}{4} = 1620$

$b^3 = \frac{1620 \times 16}{15}$

$= 108 \times 16$

$= 27 \times 4 \times 16$

$= 3^3 \times 4^3$

So, $b = 3 \times 4 = 12$