Student Number

St. Catherine's School Waverley

August 2012

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading Time – 5 minutes
- Working Time – 3 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Show all necessary working in Questions 11-16
- Task weighting – 40%

Total Marks – 100

Section I Pages 3-5

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section
- Answer on the multiple choice answer sheet provided.

Section II Pages 6-15

90 marks

- Attempt Questions 11-16
- Allow about 2 hours and 45 minutes for this section
- Answer each question in the booklet provided.

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a \neq 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Section I

Total marks - 10
Attempt Questions 1-10
All questions are of equal value.

Answer either *A,B,C* or *D* on the multiple choice answer sheet provided.

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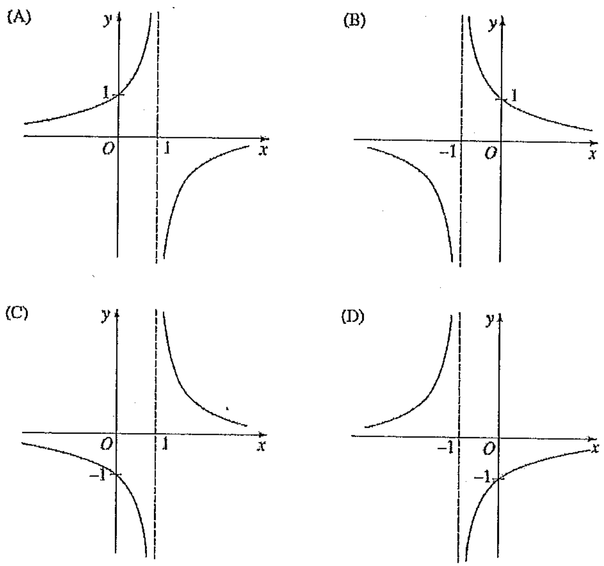
1. Which of the following graphs represents $y = \dfrac{1}{1+x}$?



2. A parabola has its focus at $(0,3)$. The equation of its directrix is $x = 4$. Which of the following is the equation of the parabola?

(A) $(x-2)^2 = 8(y-3)$ (B) $(x-2)^2 = -8(y-3)$

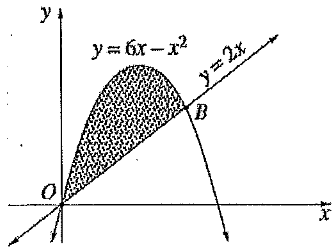
(C) $(y-3)^2 = 8(x-2)$ (D) $(y-3)^2 = -8(x-2)$

3. If $f''(x) < 0$ and $f'(x) > 0$ for all x over a given domain, which of the following describes the graph of $y = f(x)$?

(A) Increasing and concave up (B) Increasing and concave down

(C) Decreasing and concave up (D) Decreasing and concave down

4.



NOT TO SCALE

The graphs of $y = 2x$ and $y = 6x - x^2$ intersect at the origin and point B. An expression for the area bounded by $y = 2x$ and $y = 6x - x^2$ is ?

(A) $\int_0^4 (4x - x^2)\,dx$ (B) $\int_0^4 (x^2 + 4x)\,dx$

(C) $\int_0^6 (4x - x^2)\,dx$ (D) $\int_0^6 (4x + x^2)\,dx$

5. If $\log_a 2 = n$ and $\log_a 5 = m$, which expression gives $\log_a 50$?

(A) $m^2 + n$ (B) $m^2 n$ (C) $2m + n$ (D) $2mn$

6. If $y = 2x^3 + 3x - \dfrac{4}{x}$, then $\dfrac{dy}{dx} = ?$

(A) $6x^2 + 3 + \dfrac{4}{x^2}$ (B) $\dfrac{1}{2}x^4 + \dfrac{3x^2}{2} - 4\ln x + c$

(C) $6x^2 + 3 - \dfrac{4}{x^2}$ (D) $6x^2 - 1$

7. If $f(x) = 3x^2$, then the value of $f(1+h) - f(1)$ is ?

(A) $6x$ (B) $3h^2$ (C) $6h + 3h^2$ (D) $6 + h^2$

8. If the quadratic equation $(n-1)x^2 + nx - 5 = 0$ has real roots, then which of the following statements regarding the discriminant is true?

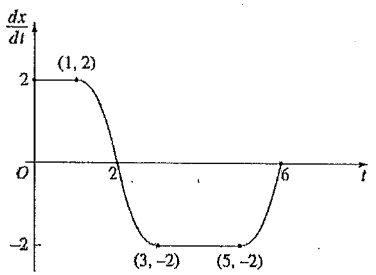
(A) $n^2 + 20n - 20 \geq 0$ (B) $n^2 + 20n - 20 > 0$

(C) $n^2 - 20n - 20 \geq 0$ (D) $n^2 - 20n - 20 > 0$

9. Let $f(x) = 1 + e^x$. What is the range of $f(x)$?

(A) $y > 0$ (B) $y \geq 0$ (C) $y > 1$ (D) $y \geq 1$

10.



The graph above shows the velocity, $\frac{dx}{dt}$, of a particle as a function of time. Initially the particle is at the origin. For what value of t does the particle return to the origin?

(A) $t = 2$ (B) $t = 3$ (C) $t = 6$ (D) $t = 4$

End of Section I

Section II

Total marks - 90
Attempt Questions 11-16
All questions are of equal value

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

Question 11 (15 marks) Use the Question 11 Writing Booklet — Marks

(a) Solve $x^2 - 11x + 10 > 0$ — **2**

(b) Simplify $\dfrac{10m - 5}{40m^3 - 5}$ — **2**

(c) Simplify $3\sqrt{18} - \sqrt{50}$ — **2**

(d) Find $\sum_{n=0}^{10} 2^{-n}$ — **2**

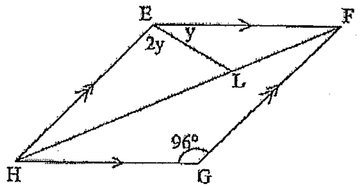
(e) Solve $|4 - 3x| \leq 2$ and sketch your solution on a number line. — **3**

(f) The first four terms of an arithmetic series are 3, x, y, 177.
Find the values of x and y. — **2**

(g) Find the gradient of the normal to the curve $y = 2\sqrt{x+7}$ at the point where $x = 9$. — **2**

Question 12 (15 marks) Use the Question 12 Writing Booklet.

(a)



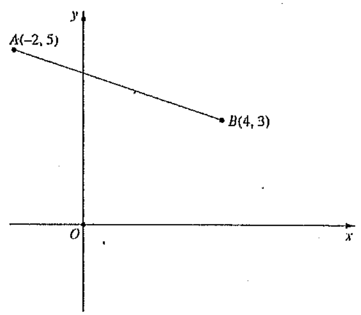
NOT TO SCALE

The diagram shows a rhombus *EFGH*. A line *EL* is drawn through E so that

$\angle LEH = 2 \times \angle FEL$. $\angle FGH = 96°$.

Copy the diagram into your writing booklet and find the size of $\angle ELF$ giving reasons. **2**

(b)



NOT TO SCALE

The diagram shows $A(-2,5)$, $B(4,3)$ and $O(0,0)$.

(i) Show that the equation of *AB* is $x + 3y - 13 = 0$. **2**

(ii) Calculate the perpendicular distance from *O* to the line *AB*. **1**

(iii) If the length of *AB* is $2\sqrt{10}$ units, find the area of *OAB*. **1**

Question 12 continues on page 8

(c) A function $y = f(x)$ has $\frac{d^2y}{dx^2} = 6x - 2$ and a stationary point at (1,2).

(i) Show that $\frac{dy}{dx} = 3x^2 - 2x - 1$. **2**

(ii) Find the equation of the function $y = f(x)$. **2**

(iii) Find the co-ordinates of the second stationary point and determine its nature. **3**

(iv) Find the co-ordinates of the point of inflexion, showing that the concavity changes at this point. **2**

Question 13 (15 marks) Use the Question 13 Writing Booklet.

(a)

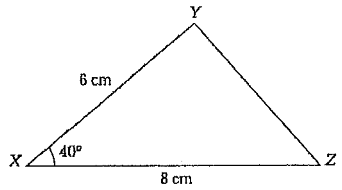
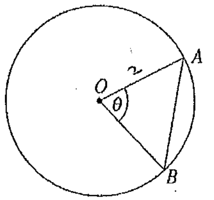


Diagram not to scale

In ΔXYZ above, $XY = 6cm$, $XZ = 8cm$ and $\angle YXZ = 40^\circ$.

Calculate the length of side YZ. (Answer correct to three decimal places) **2**

(b)



NOT TO SCALE

The diagram shows a circle with centre O and radius 2 cm. The points A and B lie on the circumference of the circle and $\angle AOB = \theta$. The area of $\Delta AOB = 1 \ unit^2$.

(i) Find the value of θ if θ is acute. (Answer in radians) **2**

(ii) Hence calculate the area of the minor segment cut off by the chord AB. **2**

(c) Differentiate:

(i) $x \sin 3x$ **1**

(ii) $\log_e \left(\dfrac{2x+1}{x-7} \right)$ (Give your answer as a single fraction) **2**

Question 13 continues on page 10

Question 13 continued

(d) Evaluate $\displaystyle\int_1^2 \frac{1}{(2x+3)^2} dx$ **2**

(e) (i) Use the identity $\sin^2 2x + \cos^2 2x = 1$ to obtain an expression for $\tan^2 2x$ in terms of $\sec^2 2x$. **1**

(ii) Hence find the exact value of $\displaystyle\int_0^{\frac{\pi}{6}} \tan^2 2x \, dx$. **3**

Question 14 (15 marks) Use the Question 14 Writing Booklet.

(a) The following table shows the values of a function $y = f(x)$ for five values of x.

x	0	4	8	12	16
$f(x)$	0.4	0.8	1.5	1.3	0.3

Use Simpson's rule with these five values to estimate $\int_0^{16} f(x)\,dx$. **2**

(b) A rare species of marsupial live on a remote island. A mathematical model predicts that the marsupial population, P, is given by $P = 500e^{-0.05t}$, where t is the number of years after observations began.

(i) According to the model, how many marsupials were there when observations began? **1**

(ii) According to the model, what will be the rate of change in the marsupial population ten years after observations began? **2**

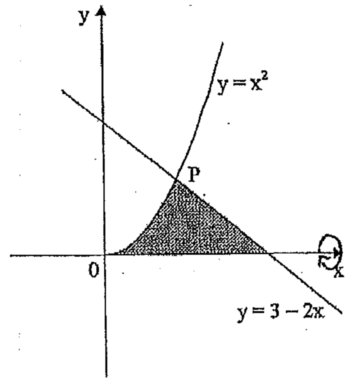
(iii) The species will become eligible for inclusion in the endangered species list when the population falls below 200. When does the model predict this will occur? (Answer in years and months) **2**

Question 14 continues on page 12

(c) Show that:

$$\int_0^{\frac{\pi}{2}} \frac{\cos x}{2 + \sin x}\,dx = \ln \frac{3}{2}.$$ **3**

(d)



NOT TO SCALE

The diagram above shows the parabola $y = x^2$ and the line $y = 3 - 2x$ intersecting at the point P in the first quadrant.

(i) Show that the co-ordinates of P are (1, 1). **2**

(ii) The shaded region is rotated about the x-axis. Find the volume of the solid formed. **3**

Question 15 (15 marks) Use the Question 15 Writing Booklet.

(a) Solve for θ if $\tan\theta - 5 = 0$ and $-\pi \le \theta \le \pi$.

(Answer in radians correct to 2 decimal places) **2**

(b) Solve $3(5^{2x}) - 4(5^{x}) + 1 = 0$. **3**

(c) Emily joined a superannuation fund on her 23rd birthday by depositing \$700 into the fund. This investment is repeated every 6 months until retirement on her 55th birthday. The last payment is made 6 months before her retirement. If the interest rate of 6% p.a is compounded every six months, calculate the amount she will receive on retirement. **3**

(d) The acceleration of a particle moving in a straight line is given by:

$$\ddot{x} = 12\cos 2t,$$

where t is in seconds and x is in metres. Initially, the particle is at rest and $x = 6$.

(i) Find an expression for the velocity, $\dot{x}$. **2**

(ii) Hence show that the displacement, x, is given by:

$$x = 9 - 3\cos 2t$$ **2**

(iii) Neatly sketch the graph of the displacement for $0 \le t \le 4\pi$. **2**

(iv) Hence, state the number of times that the particle changes direction in the first 10 seconds. **1**

Question 16 (15 marks) Use the Question 16 Writing Booklet.

(a) Let α and β be the solutions of $x^2 - 5x + 1 = 0$.

(i) Find $\alpha\beta$. **1**

(ii) Hence find $\alpha + \frac{1}{\alpha}$. **1**

(b) Jennifer borrows \$550 000 to buy a home unit. Interest is calculated at the rate of 7.2% per annum reducible, calculated monthly. She repays the loan with equal monthly instalments of \$$M$ at the end of each month for 25 years. Let A_n be the amount owing after n months.

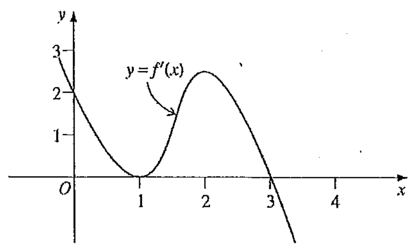
(i) Find an expression for A_1. **1**

(ii) Show that:

$$A_3 = 550000(1.006)^3 - M(1.006)^2 - M(1.006) - M$$ **2**

(iii) Find the amount of each monthly instalment (to the nearest 5 cents) **3**

(c) The graph below shows the gradient function of the curve $y = f(x)$.

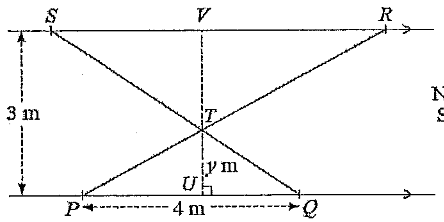


In your writing booklet draw a sketch of $y = f(x)$ given that $f(0) = 1$. **2**

Question 16 continues on page 15

Question 16 continued

(d)



NOT TO SCALE

In the diagram, PQ and SR are parallel railings which are 3 metres apart.

The points P and Q are fixed 4 metres apart on the lower railing. Two crossbars PR and QS intersect at T as shown in the diagram. The line through T perpendicular to PQ intersects PQ at U and SR at V. The length of UT is y metres.

ΔRST is similar to ΔPQT.

(i) Briefly explain why $\dfrac{SR}{PQ} = \dfrac{VT}{UT}$. **1**

(ii) Hence show that $SR = \dfrac{12}{y} - 4$. **1**

(iii) The total area, A, of ΔRST and ΔPQT is given by:

$$A = 4y - 12 + \frac{18}{y}.$$

Find the value of y that minimises A. Justify your answer. **3**

End of Question 16

End of examination

Student Number

YEAR 12 TRIAL HSC MATHEMATICS 2012
MULTIPLE CHOICE ANSWER SHEET

(9)

Section I - Mark your answer in the appropriate box with an X.

	A	B	C	D
1		X		
2				X
3		X		
4	X			
5			X	
6	X			
7			X	
8	X			
9			X	
10		~~X~~		X

Begin Section II using your booklets

St Catherine's School
Waverley

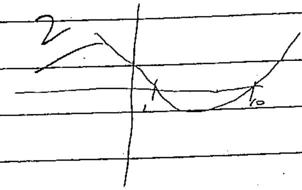
Student number
Course name
Question 11

8 page writing booklet

11 a) $x^2 - 11x + 10 > 0$

$(x-10)(x-1) > 0$

$x < 1$ or $x > 10$



b) $\frac{10m-5}{40m^3-5}$

$= \frac{5(2m-1)}{5(8m^3-1)}$

$= \frac{2m-1}{(2m-1)(4m^2+2m+1)}$

$= \frac{1}{4m^2+2m+1}$

c) $3\sqrt{18} - \sqrt{50}$

$= 9\sqrt{2} - 5\sqrt{2}$

$= 4\sqrt{2}$

d) $\sum_{n=0}^{10} 2^{-n} = 1 + \frac{1}{2} + \frac{1}{4} + \ldots$

~~$= \frac{a(r^n-1)}{r-1}$~~

~~$= \frac{1(0.5^n-1)}{}$~~

~~$= \frac{a(1-r^n)}{1-r}$~~

~~$= \frac{1(1-0.5^n)}{1-0.5}$~~

~~$= 1\frac{511}{512}$~~

$= \frac{a(1-r^n)}{1-r}$

$= \frac{1(1-0.5^{11})}{1-0.5}$

≈ 1.999 (to 3dp)

11e) $|4-3x| \le 2$

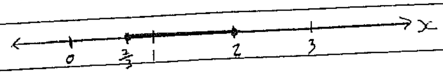
~~$-2 \le 4-3x \le 2$~~

$4-3x \le 2$ or $3x-4 \le 2$

$3x \geq 2$ $\quad$ $3x \le 6$

$x \geq \frac{2}{3}$ $\quad$ $x \le 2$

$\frac{2}{3} \le x \le 2$



3

f) $x-3 = y-x$

$2x-y=3$ — ①

$177-y = x-3$

$x+y=180$ — ②

$3x=183$

$x=61$

$y=119$

2

g) $y=2(x+7)^{\frac{1}{2}}$

$y' = (x+7)^{-\frac{1}{2}}$

when $x=9$

$y' = \frac{1}{4}$

$\therefore$ gradient normal $= -4$

2

St Catherine's School

Waverley

Student number

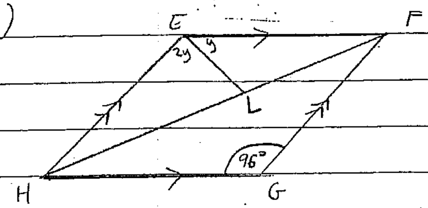
Course name

Question 12

15/15

8 page writing booklet

12a)



$\angle HEF = \angle FGL = 96°$ (opp. ∠'s rhombus equal)

$\therefore \angle HEL = \frac{96}{3} \times 2$

$= 64°$

△FGH is isosceles since $HG = FG$ (~~properties~~ all sides of rhombus equal)

$\therefore \angle HFG = \frac{180-96}{2}$ (base ∠'s isos. △ equal)

$= 42°$

$\angle EHF = \angle HFG = 42°$ (alt ∠'s equal $EH \parallel FG$)

$\therefore \angle ELF = \angle HEL + \angle EHF$ (ext. ∠ equal to sum of 2 opp. interior ∠'s)

$= 64 + 42$

$= 106°$

2/2

DO NOT TEAR ANY PAGES FROM THIS BOOKLET

12.b) i $m_{AB} = \frac{3-5}{4--2}$

$= -\frac{2}{6}$

$= -\frac{1}{3}$

$y - 3 = -\frac{1}{3}(x-4)$

$3y - 9 = -x + 4$

$x + 3y - 13 = 0$ ✓ 2/2

ii $d = \frac{|ax+by+c|}{\sqrt{a^2+b^2}}$

$= \frac{|0+0-13|}{\sqrt{1+9}}$

$= \frac{13}{\sqrt{10}}$ units ✓ 1/1

iii Area $= \frac{1}{2} \times 2\sqrt{10} \times \frac{13}{\sqrt{10}}$

$= 13$ units^2 ✓ 1/1

c) i $\frac{d^2y}{dx^2} = 6x - 2$

$\frac{dy}{dx} = \frac{6x^2}{2} - 2x + C$

$= 3x^2 - 2x + C$

when $x = 1$ $\frac{dy}{dx} = 0$

$0 = 3 - 2 + C$

$C = -1$

$\frac{dy}{dx} = 3x^2 - 2x - 1$ ✓ 2/2

12c) ii $\frac{dy}{dx} = 3x^2 - 2x - 1$

$f(x) = x^3 - x^2 - x + C$

when $x = 1$ $f(x) = 2$

$2 = 1 - 1 - 1 + C$

$C = 3$

$f(x) = x^3 - x^2 - x + 3$ ✓ 2/2

iii $\frac{dy}{dx} = 3x^2 - 2x - 1 = 0$

$(x-1)(3x+1) = 0$

$x = 1, -\frac{1}{3}$

$\frac{d^2y}{dx^2} = 6x - 2$

at $(-\frac{1}{3}, 2\frac{14}{27})$ $\frac{d^2y}{dx^2} < 0$

$\therefore$ max turning pt. ✓ 3/3

iv $f''(x) = 6x - 2 = 0$

$6x = 2$

$x = \frac{1}{3}$

for $x = \frac{1}{3} - \varepsilon$ $f''(x) < 0$

$x = \frac{1}{3} + \varepsilon$ $f''(x) > 0$

$\therefore$ change in concavity

inflexion pt. at $(\frac{1}{3}, 2\frac{16}{27})$ ✓ 2/2

St Catherine's School
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Student number: /
Course name: Mathematics
Question: 13

15/15

8 page writing booklet

13a) $YZ^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \times \cos 40$
$= 26.459...$
$YZ = 5.144$ cm (to 3dp) ✓ 2/2

b) i Area $\triangle AOB = \frac{1}{2} r^2 \sin\theta$
$= \frac{1}{2} \cdot 4 \sin\theta$
$= 2\sin\theta = 1$
$\sin\theta = \frac{1}{2}$
$\theta = \frac{\pi}{6}$ ✓ 2/2

ii Area $= \frac{1}{2} r^2 (\theta - \sin\theta)$
$= \frac{1}{2} \cdot 4 \left(\frac{\pi}{6} - \sin\frac{\pi}{6}\right)$
$= 2\left(\frac{\pi}{6} - \frac{1}{2}\right)$
$= \left(\frac{\pi}{3} - 1\right) u^2$ ✓ 2/2

c) i $\frac{d}{dx} x \sin 3x$
$= \sin 3x + x \cdot 3\cos 3x$
$= \sin 3x + 3x\cos 3x$ ✓ 1/1

DO NOT TEAR ANY PAGES FROM THIS BOOKLET

13 c) ii $\frac{d}{dx} \ln\left(\frac{2x+1}{x-7}\right)$
$= \frac{d}{dx} \ln(2x+1) - \ln(x-7)$
$= \frac{2}{2x+1} - \frac{1}{x-7}$
$= \frac{2(x-7) - 2x - 1}{(2x+1)(x-7)}$
$= \frac{2x - 14 - 2x - 1}{(2x+1)(x-7)}$
~~$= \frac{-15}{(2x}$~~
$=$ ~~$\frac{15}{(2x+1)(x-7)}$~~ $\frac{-15}{(2x+1)(x-7)}$ ✓ 2/2

d) $\int_1^2 \frac{1}{(2x+3)^2}\, dx$
$= \int_1^2 (2x+3)^{-2}\, dx$
$= \left[\frac{(2x+3)^{-1}}{-1 \cdot 2}\right]_1^2$
$= \left[\frac{-1}{2(2x+3)}\right]_1^2$
$= -\frac{1}{2}\left[\frac{1}{2x+3}\right]_1^2$
$= -\frac{1}{2}\left[\frac{1}{7} - \frac{1}{5}\right]$
$= \frac{1}{35}$ ✓ 2/2

e) i $\sin^2 2x + \cos^2 2x = 1$
$\sin^2 2x = 1 - \cos^2 2x$
$\frac{\sin^2 2x}{\cos^2 2x} = \frac{1}{\cos^2 2x} - \frac{\cos^2 2x}{\cos^2 2x}$
$\tan^2 2x = \sec^2 2x - 1$ ✓ 1/1

ii $\int_0^{\frac{\pi}{6}} \tan^2 2x\, dx$
$= \int_0^{\frac{\pi}{6}} \sec^2 2x - 1\, dx$
$= \left[\frac{1}{2}\tan 2x - x\right]_0^{\frac{\pi}{6}}$
$= \frac{\sqrt{3}}{2} - \frac{\pi}{6}$ ✓ 3/3

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Student number: /
Course name: Mathematics
Question: 14

8 page writing booklet

15/15 Excellent

14a) $\int_0^{16} f(x)\,dx$

$\doteq \frac{4}{3}(1st + last + 4(even) + 2(odd))$

$= \frac{4}{3}(0.4 + 0.3 + 4(0.8 + 1.3) + 2(1.5))$ ✓

$= 16\frac{2}{15}$ ✓ 2/2

b) i $P = 500e^{-0.05t}$

when $t = 0$

$P = 500$ ✓ 1/1

ii when $t = 10$ $P = 500e^{-0.05 \times 10}$

$= 303.27\ldots$ ✓

$\frac{dP}{dt} = -0.05.500e^{-0.05t}$

$= -0.05P$ ✓

$= -15.16$ (to 2dp) marsupials/year 2/2

iii $200 = 500e^{-0.05t}$

$e^{-0.05t} = \frac{2}{5}$

$-0.05t = \ln\frac{2}{5}$

$t = -\frac{1}{0.05}\ln\frac{2}{5}$ ✓

$= 18.326$ years

$= 18$ years 4 months ✓ (to nearest month) 2/2

14c) $\int_0^{\frac{\pi}{2}} \frac{\cos x}{2+\sin x}\,dx$ ✓

$= [\ln(2+\sin x)]_0^{\frac{\pi}{2}}$ ✓

$= \ln(2+\sin\frac{\pi}{2}) - \ln 2$ ✓

$= \ln(3) - \ln(2)$ ✓

$= \ln(\frac{3}{2})$ 3/3

d) i $y = x^2$ — ①

$y = 3 - 2x$ — ②

let ① = ②

$x^2 = 3 - 2x$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$ ✓

$x = -3, 1$

since $x > 0$ from diagram

$P(1,1)$ $y = (1)^2$

$= 1$ ✓ 2/2

~~ii $V = \pi\int_0^1 x^4\,dx + \pi\int_1^{\frac{3}{2}} 3 - 2x\,dx$~~

~~$= \pi[\frac{x^5}{5}]_0^1 + \pi[3x - x^2]_1^{\frac{3}{2}}$~~

~~$= \frac{\pi}{5} + \pi[\frac{9}{2} - \frac{9}{4} - 3 + 1]$~~

~~$= \frac{\pi}{5} + \frac{\pi}{4}$~~

~~$= \frac{9\pi}{20}u^3$~~

ii $V = \pi\int_0^1 x^4\,dx + \pi\int_1^{\frac{3}{2}} 9 - 12x + 4x^2\,dx$ ✓

$= \pi[\frac{x^5}{5}]_0^1 + \pi[9x - 6x^2 + \frac{4x^3}{3}]_1^{\frac{3}{2}}$ ✓

$= \frac{\pi}{5} + \pi[13\frac{1}{2} - 13\frac{1}{2} + 4\frac{1}{2} - 9 + 6 - \frac{4}{3}]$

$= \frac{\pi}{5} + \frac{\pi}{6}$

$= \frac{11\pi}{30}u^3$ ✓ 3/3

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15/15 Excellent!

15 a) $\tan\theta - 5 = 0$

$\tan\theta = 5$

~~$\theta = 1.37, 4.51$ (to 2dp)~~

$\theta = 1.37, -1.77$ (to 2dp) ✓✓ 2/2

b) $3(5^{2x}) - 4(5^x) + 1 = 0$

let $k = 5^x$

~~$3k^2 = 4k$~~

$3k^2 - 4k + 1 = 0$ ✓

$(k-1)(3k-1) = 0$

$k = 1, \frac{1}{3}$

$5^x = 1$ or $5^x = \frac{1}{3}$

$x = 0$ ✓ $x \ln 5 = \ln \frac{1}{3}$

$x = -0.68$ (to 2dp) ✓ 3/3

c) $A_1 = 700$ ~~(1.03)~~ $(1.03) + 700$ ✓

$A_2 = 700(1.03)^2 + 700(1.03) + 700$

$A_3 =$ ~~$(700 \times 1.03) + 700)(1.03) + 700$~~ $(700 \times (1.03)^2 + 700 (1.03) + 700)1.03 + 700$

$= 700(1.03)^3 + 700(1.03)^2 + 700(1.03) + 700$ ✓

$A_{64} = 700(1.03 + 1.03^2 + \ldots + 1.03^{64})$ ✓

$= 700\left(\frac{1.03(1.03^{64} - 1)}{1.03 - 1}\right)$ 3/3

$= \$135\,332.93$ (to nearest cent) ✓

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15 d) i $\ddot{x} = 12\cos 2t$ ✓

$\dot{x} = 6\sin 2t + c$ ✓

when $t = 0$ $\dot{x} = 0$ $c = 0$

$\dot{x} = 6\sin 2t$ ✓ 2/2

ii $x = -3\cos 2t + c$

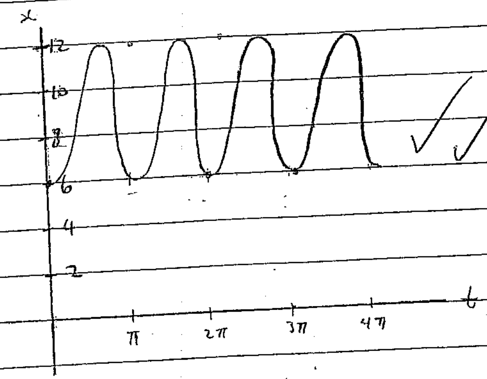
when $t = 0$ $x = 6$ ✓

$6 = -3 + c$

$c = 9$

$x = 9 - 3\cos 2t$ ✓ 2/2

iii



✓✓ 2/2

iv 6 times ✓ 1/1

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(15) 15/15

16a) i $\alpha\beta = 1$ ✓ 1/1

ii ~~$\alpha + \frac{1}{\alpha} = \frac{\alpha+1}{\alpha^2}$~~

$\alpha + \beta = 5$

$\beta = \frac{1}{\alpha}$

$\alpha + \frac{1}{\alpha} = 5$ ✓ 1/1

b) i $A_1 = 550\,000(1.006) - M$ ✓ 1/1

ii $A_2 = (550000(1.006) - M)1.006 - M$

$= 550\,000(1.006)^2 - M(1.006) - M$ ✓

$A_3 = [550\,000(1.006)^2 - M(1.006) - M](1.006) - M$

$= 550\,000(1.006)^3 - M(1.006)^2 - M(1.006) - M$ ✓ 2/2

iii $A_{300} = 550\,000(1.006)^{300} - M(1 + 1.006 + \ldots + 1.006^{299})$

$= 550\,000(1.006)^{300} - M\left(\frac{1.006^{300} - 1}{0.006}\right) = 0$ ✓

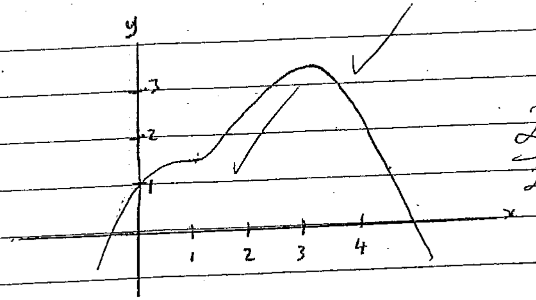
$M\left(\frac{(1.006)^{300} - 1}{0.006}\right) = 550\,000(1.006)^{300}$

$M = \$3957.75$ ✓

(to nearest 5 cents)

3/3

16c)



2/2

d) i corresponding sides and heights in similar triangles are all in the same ratio ✓ 1/1

ii $\frac{SR}{PQ} = \frac{YT}{UT}$

$\frac{SR}{4} = \frac{3-y}{y}$

$= \frac{3}{y} - \frac{y}{y}$

$= \frac{3}{y} - 1$

$SR = \frac{12}{y} - 4$ ✓ 1/1

iii $A = 4y - 12 + \frac{18}{y}$

~~$A' = 4 - 12$~~

$A' = 4 - \frac{18}{y^2} = 0$ ✓

$\frac{18}{y^2} = 4$

$y^2 = \frac{18}{4}$

$= \frac{9}{2}$

$y = \pm\frac{3}{\sqrt{2}}$ since $y > 0$ as it is a length $y = \frac{3}{\sqrt{2}}$ ✓

for $y = \frac{3}{\sqrt{2}} + \varepsilon$ $A' > 0$

$y = \frac{3}{\sqrt{2}} - \varepsilon$ $A' < 0$

_/

$\therefore$ Min turning pt. at $y = \frac{3}{\sqrt{2}}$ ✓ 3/3

$\therefore$ Area is a minimum at $y = \frac{3}{\sqrt{2}}$