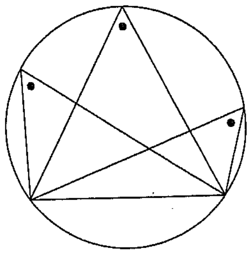
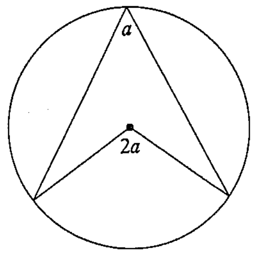
GEOMETRICAL FACTS + CIRCLE GEOMETRY QUESTIONS:

Angle properties of a circle

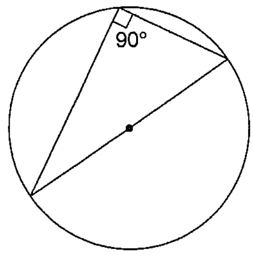
You need to know the the following facts about angles in circles.
(In each diagram, the angles with dots in them are equal.)



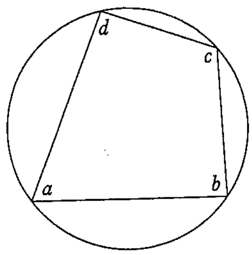
Angles in the same segment are equal.



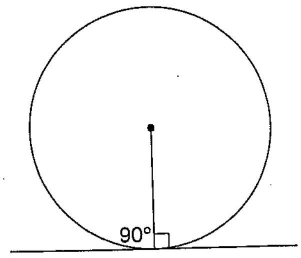
The angle at the centre of a circle is twice that at the circumference.



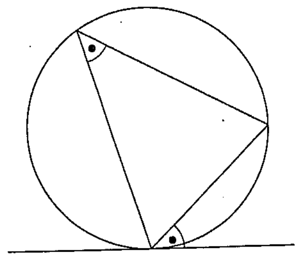
The angle in a semicircle is 90°.



A **cyclic quadrilateral** has all its vertices on a circle. Its opposite angles add up to 180°. ($a + c = 180°, b + d = 180°$)

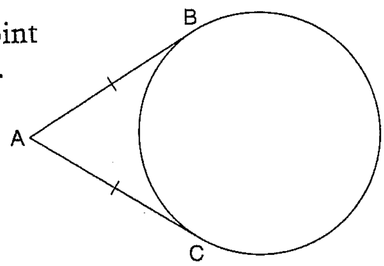


The angle between a tangent and the radius where it touches the circle is 90°.



The angle between a tangent and any chord where it touches the circle is equal to the angle in the alternate segment.

Tangents from a point are equal in length. (AB =AC)



You may be asked to use any of the above properties to prove results. You may need other facts about angles as well.

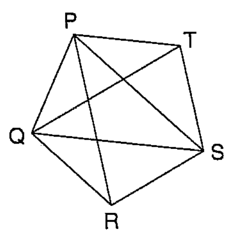
It is a good idea to mark the angles you can work out on a copy of the diagram.

When you are asked to give reasons in a proof, you should give a reason like 'Tangents from a point' or 'Angle in the alternate segment'.

1 The diagram shows a regular pentagon.
Some of the vertices are joined up.

(a) Calculate the sizes of angles (i) PQR (ii) PRQ.

(b) Name two congruent triangles in the diagram.

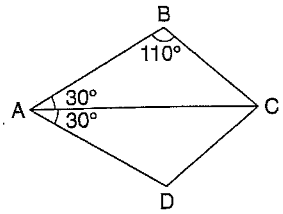


2 Two interior angles of a polygon are 155° and 165°.
Each of the remaining angles is 170°.
How many sides has the polygon?

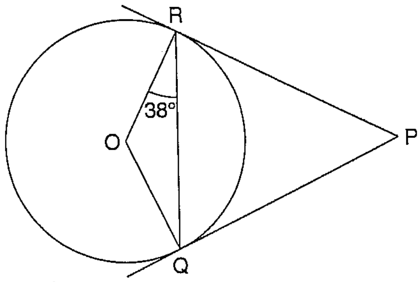
MEG

None of the diagrams is to scale.

3 In quadrilateral ABCD, AB = AD.
Calculate the sizes of angles ADC and BCD.

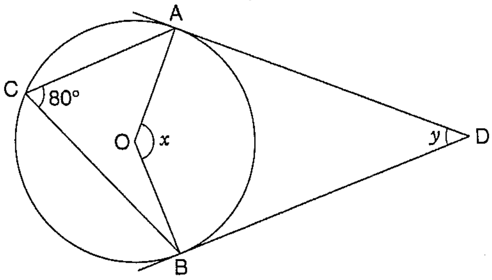


4 The diagram shows a circle, centre O.
PR and PQ are tangents to the circle at R and Q.
Showing your working clearly
(a) calculate angle ROQ,
(b) calculate angle RQP,
(c) calculate angle RPQ.



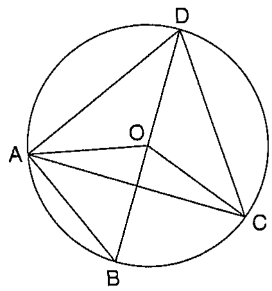
MEG

5 AD and BD are tangents to the circle.
O is the centre of the circle.
(a) Calculate the value of x.
Give a reason for your answer.
(b) Calculate the value of y.
Give a reason for your answer.



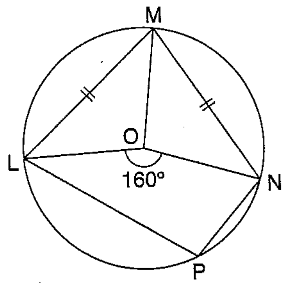
MEG (SMP)

6 ABCD are four points on the circumference of a circle, centre O.
BOD is a straight line.
Given that ∠BDC = 43° and ∠ABD = 62°, calculate the following angles, giving reasons for your answers.
(a) ∠BAC (b) ∠DAC
(c) ∠AOD (d) ∠OAC

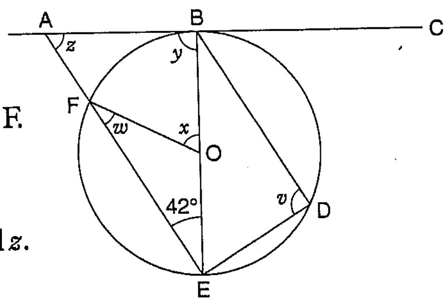


WJEC

7 LMNP is a cyclic quadrilateral, centre O.
LM is equal to MN and angle LON is 160°.
Calculate the following angles, giving reasons for your answers.
(a) ∠LPN (b) ∠LMO
(c) ∠OLM (d) ∠MOL



8 In the diagram, the straight line ABC is a tangent to the circle with centre at O.
The circle passes through the points B, D, E and F.
BOE is a diameter of the circle and AFE is a straight line. ∠BEF = 42°.
Find the sizes of the angles marked v, w, x, y and z.

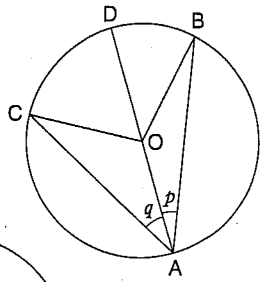


WJEC

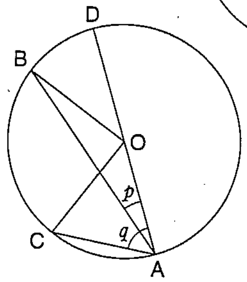
9 (a) In the diagram, O is the centre of the circle and AD is a diameter. Angle BAO is p and angle CAO is q.

(i) **Without using angle properties of circles,** find the size of angle BOC in terms of p and q.

(ii) What is the relationship between angle BAC and angle BOC?



(b) In this diagram, O remains the centre of the circle and AD is a diameter.
Angle BAO is again p, and angle CAO is q.
Show that the relationship between angle BAC and angle BOC is still true.



MEG/ULEAC (SMP)

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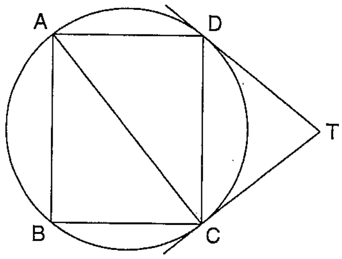


The diagram shows the circle through the vertices A, B, C and D of a rectangle.

(a) Explain why AC is a diameter of the circle.

(b) Tangents to the circle at D and C meet at T. Angle DCT = 42°.

(i) Giving a reason for each step of your working, calculate angle CTD.

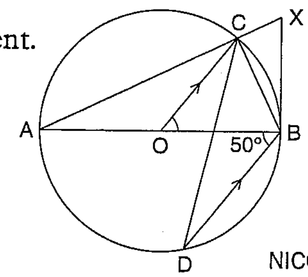
(ii) Giving a reason for each step of your working, calculate angle CAB.

MEG

11 (a) O is the centre of the circle, AB is a diameter and BX is a tangent. The lines OC and DB are parallel. Angle ABD = 50°.
Find the sizes of:

(i) angle COB (ii) angle OBC (iii) angle CBX
(iv) angle BXC (v) angle CAB

(b) Prove that OC bisects angle ACD.
State your reasons clearly.

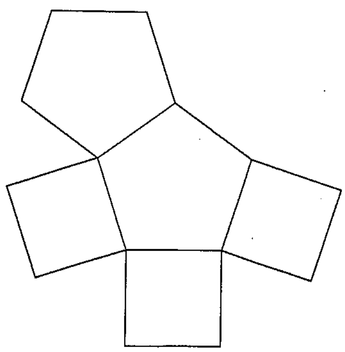


NICCEA

12 (a) Calculate the interior angle of a regular pentagon.

(b) What is the interior angle of a square?

(c) Use your answers to (a) and (b) to deduce (with reasons) whether it is possible to draw a tessellation using only squares and pentagons.



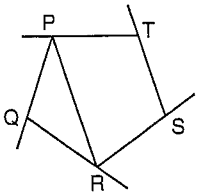
Answers and hints ► page 121

SHAPE, SPACE AND MEASURES

Angles (page 45)

Note: there are usually several ways to find an unknown angle in a diagram. So you could use different solutions to the ones here.
To help you, we have given reasons even where the question does not ask for them.

1 (a) (i) Each external angle of a regular pentagon is $360° \div 5 = 72°$.
So $\angle PQR = 180° - 72° = 108°$.



(ii) ΔPQR is isosceles, so $\angle PRQ = 72° \div 2 = 36°$.

(b) Triangles TQS and RSP are congruent and so are triangles PQR, STP, TPQ and QRS.

2 Where the interior angle is 155°, the exterior angle must be $180° - 155° = 25°$.
Where the interior angle is 165°, the exterior angle must be 15°.
So together, these two corners account for 40° of the total of the exterior angles, which sum to 360°.
So there are 320° left to account for.
Each of the other exterior angles is $180° - 170° = 10°$.
So there are another 32 ($320° \div 10°$) corners, that is a total of 34 corners in all.
So the polygon has 34 sides.

3 Triangles ABC and ADC are congruent (SAS).
So $\angle ADC = \angle ABC = 110°$ and
$\angle BCD = 360° - 110° - 110° - 60° = 80°$
(angle sum of quadrilateral).

4 (a) OR and OQ are radii, so ΔROQ is isosceles.
So $\angle RQO = 38°$.
So $\angle ROQ = 180° - 38° - 38° = 104°$.

(b) $\angle OQP = 90°$ (OQ is a radius, and QP is a tangent.)
So $\angle RQP = 90° - \angle OQR$
$= 90° - 38° = 52°$.

(c) $PQ = PR$ (equal tangents)
So ΔPQR is isosceles.
So $\angle PRQ = \angle RQP = 52°$.
So $\angle RPQ = 180° - 52° - 52° = 76°$.

5 (a) $x = 160°$
(angle at centre = twice that at circumference)

(b) $\angle OAD = \angle OBD = 90°$
(each is an angle between a radius and the tangent)
So $y = 360° - 90° - 90° - 160° = 20°$
(angle sum of quadrilateral DAOB).

6 (a) $\angle BAC = \angle BDC = 43°$ (angles in same segment)

(b) $\angle DAB = 90°$ (angle in a semicircle)
So $\angle DAC = 90° - \angle BAC = 47°$.

(c) $\angle ACD = \angle ABD = 62°$ (angles in same segment)
$\angle AOD = 2 \times \angle ACD$
(angle at centre = twice angle at circumference)
So $\angle AOD = 124°$.

(d) ΔAOD is isosceles (OA and OD radii).
So $\angle OAD = \angle ODA$.
But $\angle ODA = 90° - 62° = 28°$ (angle sum of triangle ABD).
So $\angle OAD = 28°$.
But $\angle OAC + \angle OAD + \angle BAC = 90°$.
So $\angle OAC = 90° - 28° - 43° = 19°$.

7 (a) $\angle LPN = 80°$ (half angle at centre)

(b) Triangles MLO and MNO are congruent (SSS).
So $\angle LMO = \angle OMN = \frac{1}{2} \angle LMN = 40°$

(c) $\angle OLM = \angle LMO = 40°$
(base angles of isosceles triangle)

(d) $\angle MOL = 180° - 40° - 40° = 100°$
(angle sum of triangle is 180°)

8 $v = 90°$ (angle in a semicircle)
$w = 42°$ (base angle of isosceles ΔOFE)
$x = 84°$ ($\angle BOF$, angle at centre = twice $\angle BEF$, angle at circumference)
$y = 90°$ (angle between tangent and diameter)
$z = 48°$ (angle sum of ΔABE)

9 (a) (i) OA and OB are radii, so ΔOAB is isosceles.
So $\angle OBA = \angle OAB = p$.
So $\angle AOB = 180° - 2p$.
$\angle DOB = 180° - AOB$ (angles on a straight line)
$= 180° - (180° - 2p) = 2p$
Similarly $\angle DOC = 2q$.
So $\angle BOC = 2p + 2q$.

(ii) $\angle BAC = p + q$, so $\angle BOC = 2\angle BAC$.

(b) $\angle DOB = 2p$ (proof exactly as before) and $\angle DOC = 2q$.
$\angle BOC = 2q - 2p$ and $\angle BAC = q - p$.
So we see that $\angle BOC = 2\angle BAC$.

10 (a) $\angle ADC = 90°$, since ABCD is a rectangle.
The line AC subtends $\angle ADC$ at the circumference of the circle, that is it subtends 90°.
Therefore ΔADC must lie in a semicircle, so AC is a diameter.

(b) (i) TC = TD (equal tangents)
So ΔTDC is isosceles.
Therefore $\angle TCD = \angle TDC = 42°$.
$\angle CTD = 180° - 42° - 42°$ (angle sum of triangle)
so $\angle CTD = 96°$.

(ii) $\angle ACT = 90°$ (angle between tangent and diameter AC)
Therefore $\angle ACD = 90° - 42° = 48°$.
AB is parallel to DC (opposite sides of rectangle).
So $\angle CAB = \angle ACD = 48°$ (alternate angles).

11 (a) (i) $\angle COB = \angle OBD = 50°$
(alternate angles, OC parallel to DB).

(ii) OB = OC (radii), so ΔOCB is isosceles.
So $\angle OCB = \angle OBC$.
So $50° + 2\angle OBC = 180°$ (angle sum of ΔOCB)
so $2\angle OBC = 130°$ and $\angle OBC = 65°$.

(iii) OB is perpendicular to BX (tangent and radius), so
$\angle CBX = 90° - \angle CBO = 25°$.

(iv) $\angle ACB = 90°$ (lies in a semicircle)
So $\angle BCX = 90°$.
So $\angle BXC = 180° - 90° - 25° = 65°$.

(v) $\angle CAB = \frac{1}{2}\angle COB$
(angle at centre twice that at circumference)
So $\angle CAB = 25°$.

(b) Suppose $\angle CAB = a$.
Now OA = OC (radii),
so ΔOAC is isosceles.
So $\angle OAC = \angle OCA = a$.
Also, $\angle BDC = \angle CAB = a$
(angles in the same segment).
OC is parallel to DB,
so $\angle OCD = \angle CDB$ (alternate angles),
and $\angle OCD = a$.
Thus OC bisects $\angle ACD$.

Note that you could prove that both $\angle OCA$ and $\angle OCD$ are 25° in this example; but the proof will hold whatever the size of the given angle in the diagram.

12 (a) Each external angle is $360° \div 5 = 72°$.
So each internal angle is $180° - 72° = 108°$.

(b) 90°

(c) Where the pentagons and squares meet, the interior angles must add up to 360° (angles at a point).
It is obvious that no combination of 108° and 90° can add up to 360° (except 90° + 90° + 90° + 90°, which is the equivalent of four squares tessellating without any pentagons!).

More help or practice

Review of angle relationships ► Book Y4 pages 25 to 27
Calculating angles of regular polygons ► Book Y4 pages 29 to 32
Angle properties of a circle ► Book YX1 pages 1 to 11
Deduction (proof) ► Book YX1 pages 12 to 16
Technical terms of a circle ► Book YX1 pages 81 to 82
Properties of tangents to a circle ► Book YX1 pages 83 to 87
Conditions for congruent triangles ► Book YR+ pages 14 to 18, Book YX1 pages 63 to 69