

South Sydney High School
Extension Mathematics
Coordinate Geometry

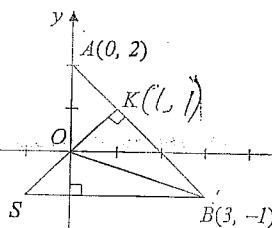
Term: Term 3 Week 1
Time allowed: 100 minutes

Date: Today July 2012
Assessment: Class Stuff

1) Yr11-2U\linfunc.hsc Qn2) 2U85-4ii

The vertices of the triangle OAB are the points $O(0, 0)$, $A(0, 2)$ and $B(3, -1)$.

- Draw a sketch diagram of the triangle.
- The point K on AB is such that OK is perpendicular to AB . Find the coordinates of K , and show the point K on your diagram.
- Find the area of the triangle OAB .
- The line through the point K perpendicular to OA , meets KO produced at S . Find the coordinates of S .
- Verify that AS is perpendicular to OB . \square



«1)→ a)

b) $K(1, 1)$ c) 3 units² d) $S(-1, -1)$ e) Proof »

2) Yr11-2U\linfunc.hsc Qn1) 2U84-4i

The lines L_1 , L_2 have equations

$$L_1: x - 2y - 1 = 0,$$

$$L_2: 2x + y + 18 = 0.$$

- Find the shortest distance from $P(8, 4)$ to L_1 .
- Show that the lines intersect at $Q(-7, -4)$.
- Find the distance PQ . \square

«2)→ a) $\frac{1}{\sqrt{5}}$ or $\frac{\sqrt{5}}{5}$ units b) Proof c) 17 units »

3) Yr11-2U\linfunc.hsc Qn3) 2U86-2

The coordinates of the points A , B , C are $(0, 2)$, $(4, 0)$ and $(6, -4)$ respectively.

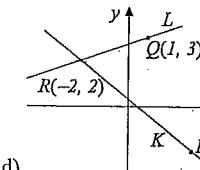
- Find the length AB , and the gradient of AB .
- Show that the equation of the line L , drawn through C parallel to AB is $x + 2y + 2 = 0$.
- Find the coordinates of D , the point where L intersects the x -axis.
- Find the perpendicular distance of the point A from the line L .
- Find the area of the quadrilateral $ABCD$. \square

«3)→ a) $2\sqrt{5}$ units, $-\frac{1}{2}$ b) Proof c) $D(-2, 0)$ d) $\frac{6}{\sqrt{5}}$ or $\frac{6\sqrt{5}}{5}$ units e) 18 units² »

4) Yr11-2U\linfunc.hsc Qn4) 2U87-2

The points P and Q have coordinates $(3, -2)$ and $(1, 3)$ respectively.

- The line K has equation $4x + 5y - 2 = 0$. Verify that P lies on K .
- The line L through Q has gradient $\frac{1}{3}$. Show that the equation of L is $x - 3y + 8 = 0$.
- The point of intersection of K and L is R . Find the coordinates of R .
- Draw a neat sketch on a number plane showing P , Q , R , K and L .
- Find the perpendicular distance of P from L . Leave your answer as a surd.
- Find the area of the triangle PQR . \square



«4)→ a) b) Proof c) $(-2, 2)$ d)

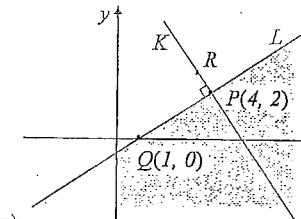
e) $\frac{17}{\sqrt{10}}$ or $\frac{17\sqrt{10}}{10}$ units f) 8.5 units² »

5)

Yr11-2U\linfunc.hsc Qn5) 2U88-3

Q is the point of intersection of the x -axis and the line L with equation $2x - 3y = 2$.

- On a number plane draw the line L , marking on it the point Q .
- On your diagram, indicate the point $P(4, 2)$ which lies on L . Draw the line K through P perpendicular to L .
- Find the equation of the line K .
- Without calculating its co-ordinates, indicate a point R on K which is one unit from P . Mark the right angle RPQ on your diagram.
- Find the distance PQ .
- Find the area of the triangle QPR .
- On your diagram shade the region given by $x \geq 0$, $2x - 3y \geq 2$. \square



«5)→

c) $3x + 2y - 16 = 0$ or $y = -\frac{3}{2}x + 8$ d) Note: Only one of the two

possible positions of R is shown on the diagram. e) $\sqrt{13}$ units f) $\frac{\sqrt{13}}{2}$ units² »

6)

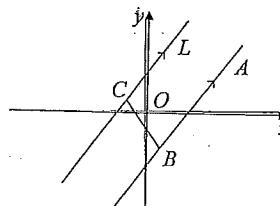
Yr11-2U\linfunc.hsc Qn6) 2U89-1c

Find the co-ordinates of the midpoint of the interval AB , where $A = (2, 5)$ and $B = (-4, 4)$.

$$\langle(6)\rightarrow(-1, \frac{9}{2})\rangle$$

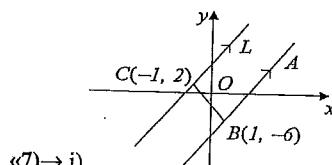
7)

Yr11-2U\linfunc.hsc Qn7) 2U89-2



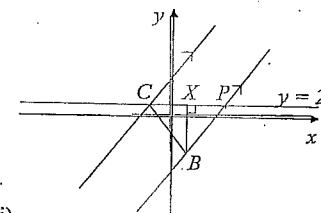
The line L passes through $C(-1, 2)$ and has equation $y = 2x + 4$. The point B has coordinates $(1, -6)$ and the line AB is parallel to L .

- Copy the diagram into your examination booklet and next to the points B and C write their co-ordinates.
- Find the length of the interval BC .
- Write down the slope of the line L and use your calculator to find the angle L makes with the x -axis. Give your answer to the nearest degree.
- Show that AB has equation $y = 2x - 8$.
- If P is a point which lies on AB and on the line $y = 2$, find the coordinates of P .
- Find the length of PC .
- On your diagram, draw the line PC and the perpendicular from B to PC .
- Find the area of the triangle PBC .



$\langle(7)\rightarrow i\rangle$

ii) $2\sqrt{17}$ units iii) $2, 63^\circ$ iv) Proof v) $P(5, 2)$ vi) 6 units



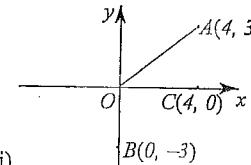
vii)

viii) $24 \text{ units}^2 \rangle$

8)

Yr11-2U\linfunc.hsc Qn8) 2U90-2b

- Plot on a number plane the point $A(4, 3)$. Draw the interval AO where O is the origin.
- Plot the points $B(0, -3)$ and $C(4, 0)$ on your diagram.
- Show that the line BC has equation $4y = 3x - 12$.
- Show that $OACB$ is a parallelogram. Give reasons.
- Find the area of the parallelogram $OACB$.
- Calculate the length of the diagonal AB .



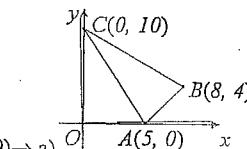
$\langle(8)\rightarrow i, ii\rangle$

iii) iv) Proof, $OA \parallel BC$ and $OB \parallel AC$. v) 12 units^2 vi) $2\sqrt{13}$ units

9)

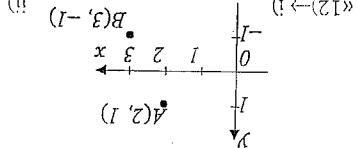
Yr11-2U\linfunc.hsc Qn9) 2U912

- On a number plane, mark the origin O and $A(5, 0)$, $B(8, 4)$, and $C(0, 10)$. Join A to B , B to C , and, C to A .
- Show that the line AB has equation $3y = 4x - 20$.
- Show that the gradient of the line BC is $-\frac{3}{4}$.
- Show that AB and BC are perpendicular.
- Show that the length of AB is 5 units.
- Show that triangles ACO and ACB are congruent. Give reasons.
- Find the area of the quadrilateral $ABCO$.
- If D is the point $(8, 0)$, calculate the perpendicular distance of D from the line AB .



$\langle(9)\rightarrow a\rangle$

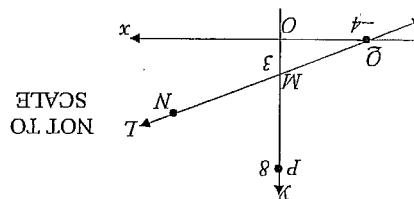
c) d) e) f) Proof g) 50 units^2 h) $\frac{12}{5}$ units



- i. Find the gradients m_1 of OA and m_2 of AB .
ii. Find the equation of the line AB .
iii. Show that OA is perpendicular to AB .
iv. Show that OA is a bisector of $\angle ABC$.
v. Calculate the length of BC and AC , show that $\triangle ABC$ is isosceles.
vi. Find the midpoints of the interval BC and AC .
vii. Find the shape best describes the geometric figure $OABC$.

- «12)» i) $m_1 = \frac{1}{2}$, $m_2 = -2$ ii) $\text{Proof } v) D\left(\frac{3}{2}, -\frac{1}{2}\right)$ vi) $C(1, -2)$ vii) Square »
a. Copy the diagram into your Writing Booklet.
b. Show that the point $(16, 15)$ lies on the line L .
c. By considering the lengths of MG and MP , show that $\triangle GMP$ is isosceles.
d. Calculate the gradient of the line PQ .
e. M is the midpoint of the interval GN . Find the coordinates of the point N .
f. Show that $\angle NPG$ is a right angle.
g. Find the equation of the circle passing through the points N , P and Q .

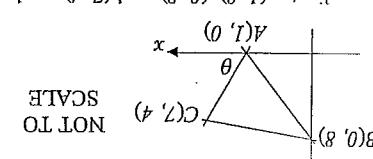
The line L cuts the x -axis at $(-4, 0)$ and the y -axis at $M(0, 3)$ as shown. N is a point on the line L , and P is the point $(0, 8)$.



g) 26 units^2 h) $E(8, -4)$ »

«13)» a) $\frac{3}{2}$ b) $33^\circ 41'$ (to the nearest minute) c) $2x - 3y - 2 = 0$ d) $D(4, 2)$ e) $\text{Proof } f)$ $\triangle ABC$ is isosceles

- a. Calculate the size of angle θ in degrees.
b. Calculate the size of angle θ in degrees.
c. Find the equation of the line AC .
d. Find the coordinates of D , the midpoint of AC .
e. Show that AC is perpendicular to BD .
f. Find the area of $\triangle ABC$.
g. What does part (e) show about $\triangle ABC$?
h. Write down the coordinates of a point E such that $ABCE$ is a rhombus.

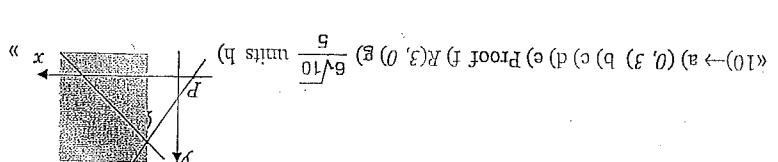


The points A , B , and C have coordinates $(1, 0)$, $(0, 8)$, and $(7, 4)$, as shown in the diagram. The angle

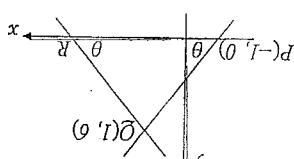
13) YR11-2 Mathematics Q1(T0) 2U94-2

«11)» k = 7 »

The line $6x - 4y = 4$ passes through the point $(3, 2)$. Find the value of k .
i. On a number plane, mark the origin O and the points $A(2, 1)$ and $B(3, -1)$.



- «10)» a) (a, 3) b) (c, d) e) $\text{Proof } f) R(3, 0)$ g) $\frac{5}{6\sqrt{10}}$ units h) »
i. On your diagram, shade in the region satisfying both the inequalities: $y \leq 3x + 3$ and $3x + y - 9 \geq 0$.
j. Find the perpendicular distance from P to QR .
k. Find the coordinates of R .
l. Show that the equation of QR is $3x + y - 9 = 0$.
m. Show that the gradient of QR is -3 .
n. Show that $\tan \theta = 3$.
o. Show that PQ has equation $y = 3x + 3$.
p. Find the coordinates of the midpoint of PQ .
q. Find the coordinates of R .
r. Show that the gradient of QR is -3 .
s. Calculate the size of angle θ in degrees.



In the diagram P and Q have coordinates $(-1, 0)$ and $(1, 6)$ and $\angle QPR = \angle QPQ = \theta$. Copy the diagram into your examination booklet.

10) YR11-2 Mathematics Q1(T0) 2U92-2

YR11-2 Mathematics Q1(T1) 2U93-2a

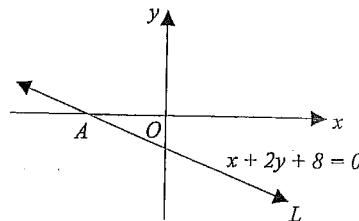
- i. On a number plane, mark the origin O and the points $A(2, 1)$ and $B(3, -1)$.
ii. Find the gradients m_1 of OA and m_2 of AB .
iii. Show that OA is perpendicular to AB .
iv. Find the mid-point D of the interval OB .
v. M is the midpoint of the interval OG . Find the coordinates of the point N .
vi. Calculate the length of GN and MP , show that $\triangle GMP$ is isosceles.
vii. Find the equation of the circle passing through the points N , P and Q .

«14)» a. Copy the diagram into your Writing Booklet.
b. Find the equation of the line L .
c. Show that the point $(16, 15)$ lies on the line L .
d. Calculate the length of MG and MP , show that $\triangle GMP$ is isosceles.
e. M is the midpoint of the interval GN . Find the coordinates of the point N .
f. Show that $\angle NPG$ is a right angle.
g. Find the equation of the circle passing through the points N , P and Q .

YR11-2 Mathematics Q1(T1) 2U93-2b

- i. On a number plane, mark the origin O and the points $A(2, 1)$ and $B(3, -1)$.
ii. Find the gradients m_1 of OA and m_2 of AB .
iii. Show that OA is perpendicular to AB .
iv. Find the mid-point D of the interval OB .
v. M is the midpoint of the interval OG . Find the coordinates of the point N .
vi. Calculate the length of GN and MP , show that $\triangle GMP$ is isosceles.
vii. Find the equation of the circle passing through the points N , P and Q .

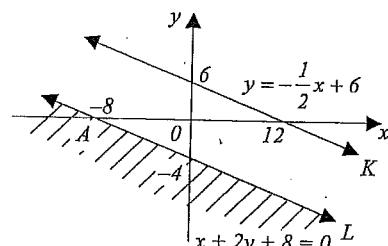
YR11-2 Mathematics Q1(T1) 2U93-2c



The line L is shown in the diagram. It has equation $x + 2y + 8 = 0$ and cuts the x -axis at A . The line k has equation $y = -\frac{1}{2}x + 6$, and is not shown on the diagram.

Copy or trace the diagram into your Writing Booklet.

- Find the coordinates of A .
- Explain why k is parallel to L .
- Draw the graph of k on your diagram, indicating where it cuts the axes.
- Shade the region $x + 2y + 8 \leq 0$ on your diagram.
- Write down a pair of inequalities which define the region between k and L .
- Show that $P(8, 2)$ lies on k .
- Find the perpendicular distance from P to L .
- $Q(4, -6)$ lies on L . Show that Q is the point on L which is closest to P . \square



«15)→

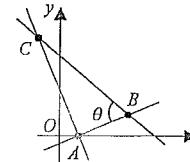
- a) $A(-8, 0)$ b) The gradient of both lines is the same, ie. $-\frac{1}{2}$.

e) $x + 2y - 12 \leq 0$ and $x + 2y + 8 \geq 0$ f) Proof g) $4\sqrt{5}$ units h) Proof »

Let A and B be the points $(0, 1)$ and $(2, 3)$ respectively.

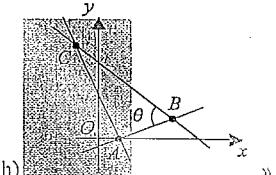
- Find the coordinates of the midpoint of AB .
- Find the slope of the line AB .
- Find the equation of the perpendicular bisector of AB .
- The point P lies on the line $y = 2x - 9$ and is equidistant from A and B . Find the coordinates of P . \square

«16)→ i) $(1, 2)$ ii) 1 iii) $y = -x + 3$ iv) $(4, -1)$ »



The diagram shows points $A(1, 0)$, $B(4, 1)$ and $C(-1, 6)$ in the Cartesian plane. Angle ABC is θ . Copy or trace this diagram into your Writing Booklet.

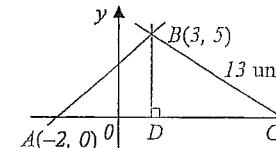
- Show that A and C lie on the line $3x + y = 3$.
- Show that the gradient of AB is $\frac{1}{3}$.
- Show that the length of AB is $\sqrt{10}$ units.
- Show that AB and AC are perpendicular.
- Find $\tan \theta$.
- Find the equation of the circle with centre A that passes through B .
- The point D is not shown on the diagram. The point D lies on the line $3x + y = 3$ between A and C , and $AD = AB$. Find the coordinates of D .
- On your diagram, shade the regions satisfying the inequality $3x + y \leq 3$. \square



«17)→ a) b) c) d) Proof e) 2 f) $(x - 1)^2 + y^2 = 10$ g) $D(0, 3)$ h) »

The points A and B have coordinates $(3, -4)$ and $(7, 2)$ respectively. Find the coordinates of the midpoint of AB . \square

«18)→ (5, -1) »

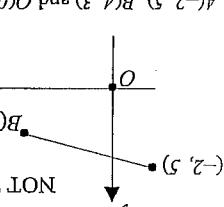


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The diagram shows the points $A(-2, 0)$, $B(3, 5)$ and the point C which lies on the x axis. The point D also lies on the x axis such that BD is perpendicular to AC .

- Show that the gradient of AB is 1.
- Find the equation of the line AB .
- What is the size of $\angle BAC$?
- The length of BC is 13 units. Find the length of DC .
- Calculate the area of $\triangle ABC$.
- Calculate the size of $\angle ABC$, to the nearest degree. \square

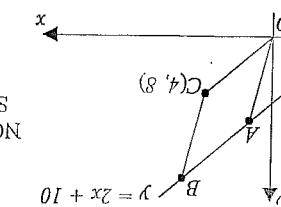
«19)→ i) Proof ii) $y = x + 2$ iii) 45° iv) 12 units v) 42.5 units² vi) 112° »



Y11-2-U1Infinite-hse Q123) 2U01-2b

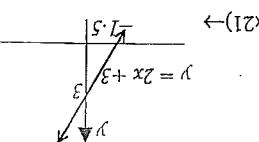
The diagram shows the points $A(-2, 5)$, $B(4, 3)$ and $C(4, 8)$. The point C is the fourth vertex of the parallelogram $OABC$.

$$\text{The equation of } AB \text{ is } y = 2x + 10. \text{ The point } C \text{ is } (4, 8). \text{ Copy or trace the diagram into your writing booklet.}$$



The diagram shows the points $P(0, 2)$ and $G(4, 0)$. The point M is the midpoint of PG . The line MN is perpendicular to PG and meets the x -axis at G and the y -axis at N .

- a. Show that the gradient of PG is $-\frac{1}{2}$.
- b. Find the coordinates of M .
- c. Find the equation of the line MN .
- d. Show that N has coordinates $(0, -3)$.
- e. Find the distance NO .
- f. Find the equation of the circle with centre N and radius NO .
- g. Hence show that the circle in part (f) passes through the point P .
- h. The point R lies in the first quadrant and $PNGR$ is a rhombus. Find the coordinates of R .
- $\left(22 \leftrightarrow a\right) \text{ Proof b) } M(2, 1) \quad c) \quad 2x - y - 3 = 0 \quad d) \quad N(0, -3) \quad e) \quad 5 \text{ units} \quad f) \quad x^2 + (y + 3)^2 = 25 \quad g) \quad \text{Proof h) } R(4, 5) \quad \left(22 \leftrightarrow a\right)$



Sketch the line $y = 2x + 3$ in the Cartesian Plane.

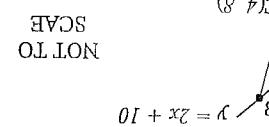
$\left(20 \leftrightarrow i\right) \text{ Proof ii) Alternate angles, } AB \parallel OC \text{ iii) Proof }$

iii. The line OB divides the quadrilateral $OABC$ into two congruent triangles. Prove that $OABC$ is a parallelogram.

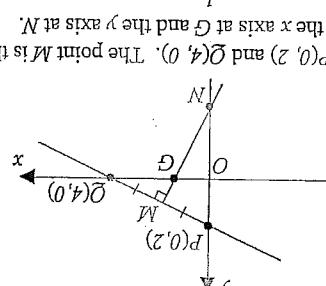
ii. State why $\angle ABO = \angle BOC$.

i. Show that OC and AB are parallel.

The equation of AB is $y = 2x + 10$. The point C is $(4, 8)$. Copy or trace the diagram into your writing booklet.



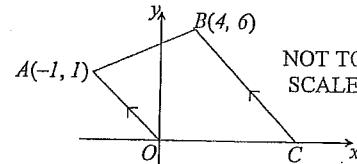
$\left(21 \leftrightarrow 2-U1Infinite-hse Q121\right) 2U00-1g$



The diagram shows the points $P(0, 2)$ and $Q(4, 0)$. The point M is the midpoint of PQ . The line MN is perpendicular to PQ and meets the x -axis at G and the y -axis at N .

- a. Show that the area of triangle ABD is 15 units 2 .
- b. Find the coordinates of the point P .
- c. Hence show that the circle in part (f) passes through the point P .
- d. Find the equation of the line MN .
- e. Find the distance NO .
- f. Find the equation of the circle with centre N and radius NO .
- g. The point R lies in the first quadrant and $PNGR$ is a rhombus. Find the coordinates of R .
- h. The point R lies in the first quadrant and $PNGR$ is a rhombus. Find the coordinates of R .
- $\left(22 \leftrightarrow a\right) \text{ Proof b) } M(2, 1) \quad c) \quad 2x - y - 3 = 0 \quad d) \quad N(0, -3) \quad e) \quad 5 \text{ units} \quad f) \quad x^2 + (y + 3)^2 = 25 \quad g) \quad \text{Proof h) } R(4, 5) \quad \left(22 \leftrightarrow a\right)$

25) Yr11-2U\linfunc.hsc Qn25) 2U03-2b

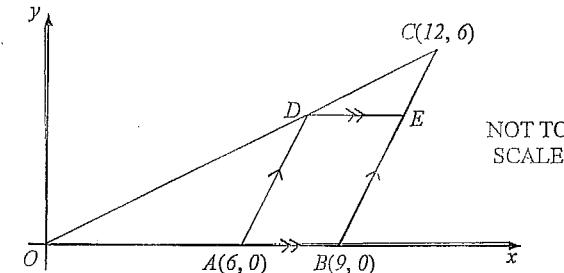


In the Diagram, $OABC$ is a trapezium with $OA \parallel CB$. The coordinates of O , A and B are $(0, 0)$, $(-1, 1)$ and $(4, 6)$ respectively.

- Calculate the length of OA .
- Write down the gradient of the line OA .
- What is the size of $\angle AOC$?
- Find the equation of the line BC , and hence find the coordinates of C .
- Show that the perpendicular distance from O to the line BC is $5\sqrt{2}$.
- Hence, or otherwise, calculate the area of the trapezium $OABC$. \square

«25)→ i) $\sqrt{2}$ units ii) -1 iii) 135° iv) $x + y - 10 = 0$, $C(10, 0)$ v) Proof vi) 35 units 2 »

27) Yr11-2U\linfunc.hsc Qn27) 2U05-3c



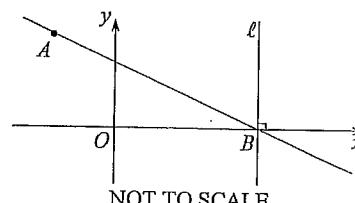
In the diagram, A , B and C are the points $(6, 0)$, $(9, 0)$ and $(12, 6)$ respectively. The equation of the line OC is $x - 2y = 0$. The point D on OC is chosen so that AD is parallel to BC . The point E on BC is chosen so that DE is parallel to the x -axis.

- Show that the equation of the line AD is $y = 2x - 12$.
- Find the coordinates of the point D .
- Find the coordinates of the point E .
- Prove that $\triangle OAD \sim \triangle DEC$.
- Hence, or otherwise, find the ratio of the lengths AD and EC . \square

«27)→ i) Proof ii) $D(8, 4)$ iii) $E(11, 4)$ iv) Proof v) $2:1$ »

26) Yr11-2U\linfunc.hsc Qn26) 2U04-2a

The diagram shows the points $A(-1, 3)$ and $B(2, 0)$. The line ℓ is drawn perpendicular to the x -axis through the point B .

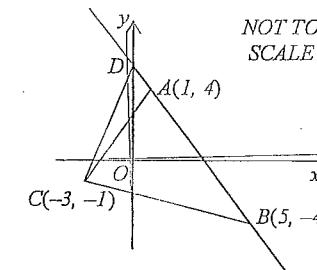


- Calculate the length of the interval AB .
- Find the gradient of the line AB .
- What is the size of the acute angle between the line AB and the line ℓ ?
- Show that the equation of the line AB is $x + y - 2 = 0$.
- Copy the diagram into your writing booklet and shade the region defined by $x + y - 2 \leq 0$.
- Write down the equation of the line ℓ .
- The point C is on the line ℓ such that AC is perpendicular to AB . Find the coordinates of C . \square



«26)→ i) $3\sqrt{2}$ units ii) -1 iii) 45° iv) Proof v) »

28) Yr11-2U\linfunc.hsc Qn28) 2U06-3a



In the diagram, A , B and C are the points $(1, 4)$, $(5, -4)$ and $(-3, -1)$ respectively. The line AB meets the y -axis at D .

- Show that the equation of the line AB is $2x + y - 6 = 0$.
- Find the coordinates of the point D .
- Find the perpendicular distance of the point C from the line AB .
- Hence, or otherwise, find the area of the triangle ADC . \square

«28)→ i) Proof ii) $D(0, 6)$ iii) $\frac{13}{\sqrt{5}}$ units iv) 6.5 units 2 »