Student Name: .....
Teacher Name: .....

Year 11 Extension I Mathematics Assessment Task 1 3rd May 2011

Time allowed: 1 hour
Reading Time 5 minutes

General Instructions

- Attempt ALL questions
- Write your NAME in all the examination booklets used
- Start each question on a new page.

Questions	Marks
Q.1	/9
Q.2	/9
Q.3	/9
Q.4	/9
Q.5	/9
Total	/45

Question One (9marks) — Marks

Solve for $0 \leq \theta \leq 360°$:

a) $sin\theta = 0$ — 1

b) $cos2\theta = \frac{\sqrt{3}}{2}$ — 2

c) $2sin^2\theta + cos\theta - 1 = 0$ — 3

d) $sin2\theta = cos\theta$ — 3

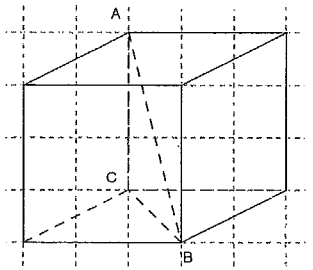
Question Two (9 marks)

a) Solve $\frac{x}{x+3} \leq 2$ — 3

b) i) On the same set of axes draw the graphs of $y = |x + 3|$ and $y = 3 - x$. — 2

ii) Hence or otherwise solve $|x + 3| < 3 - x$ — 1

c) AB is the diagonal of a cube and the point C is a vertex on the base of the cube as shown in the diagram. — 3



Determine the size of $\angle ABC$ correct to the nearest minute.

Question Three (9 marks)

a) Given $sinx = \frac{2}{7}$ and $cosy = \frac{1}{3}$, where x and y are acute, find the exact value of:

i) $sin(x + y)$ — 3

ii) $cos2x$ — 2

b) i) Express $\sqrt{3}sinx - cosx$ in the form $rsin(x - \alpha)$ 2

ii) Hence or otherwise solve for $0 \leq x \leq 360°$ the equation 2

$\sqrt{3}sinx - cosx = 1.$

Question Four (9 marks)

a) Solve $k^{-\frac{3}{4}} = 64$ 2

b) Use $t = tan\frac{\theta}{2}$ to prove that 3

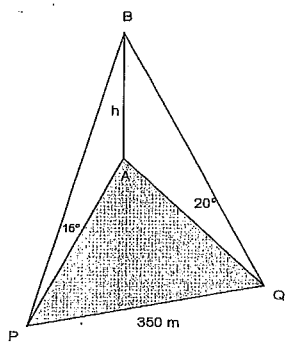
$$\frac{cos\theta + sin\theta - 1}{cos\theta - sin\theta + 1} = tan\frac{\theta}{2}$$

c) From a point P on a bearing of $230°$ from a tower, the angle of elevation to the top of the tower is $15°$. From another point Q, on a bearing of $160°$ from the tower, the angle of elevation to the top of the tower is $20°$. If P and Q are 350 m apart:

i) Show that $\angle PAQ = 70°$ 1

ii) Show that $AP = htan75°$ 1

iii) Find the height, h, of the tower to the nearest metre. 2



Question 5 (9 marks)

a) i) Show that $(sinA - cosA)^2 = 1 - sin2A$ 2

ii) Hence find the exact value of $sin15° - cos15°$ 2

b) Prove that $\frac{sin3\theta}{sin\theta} - \frac{cos3\theta}{cos\theta} = 2$ (for $sin\theta \neq 0, cos\theta \neq 0$) 2

c) Eliminate θ from the equations $x = 3sec\theta,\ y = 2tan\theta,$ 3

leaving an equation in terms of x and y.

End of Test

SOLUTIONS

Q1

a) $\sin\theta = 0$

$\theta = 0^\circ, 180^\circ, 360^\circ$

b) $\cos 2\theta = \frac{\sqrt{3}}{2}$

$2\theta = 30^\circ, 330^\circ, 390^\circ, 690^\circ$

$\theta = 15^\circ, 165^\circ, 195^\circ, 345^\circ$

c) $2\sin^2\theta + \cos\theta - 1 = 0$

$2(1-\cos^2\theta) + \cos\theta - 1 = 0$

$2 - 2\cos^2\theta + \cos\theta - 1 = 0$

$2\cos^2\theta - \cos\theta - 1 = 0$

$(2\cos\theta + 1)(\cos\theta - 1) = 0$

$\cos\theta = -\frac{1}{2} \quad \cos\theta = 1$

$\theta = 120^\circ, 240^\circ, 0^\circ, 360^\circ$

d) $\sin 2\theta = \cos\theta$

$2\sin\theta\cos\theta - \cos\theta = 0$

$\cos\theta(2\sin\theta - 1) = 0$

$\cos\theta = 0 \quad \sin\theta = \frac{1}{2}$

$\theta = 90^\circ, 270^\circ, 30^\circ, 150^\circ$

Q2

a) $\frac{(x+3)^2 x}{x+3} \leq 2(x+3)^2$

$x(x+3) - 2(x+3)^2 \leq 0$

$(x+3)[x - 2(x+3)] \leq 0$

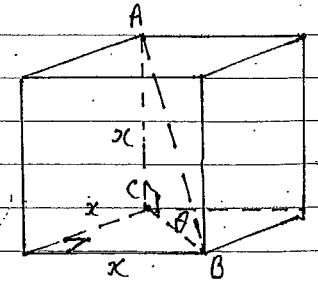
$(x+3)(-x-6) \leq 0$

$x \leq -6, \quad x > -3$

b) i)

ii) $x < 0$

c)



Let side length = x

$BC^2 = x^2 + x^2$

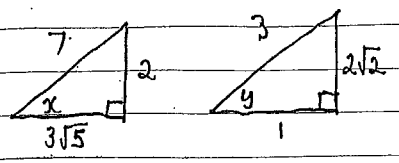
$BC = \sqrt{2x^2} = \sqrt{2}x$

In $\triangle ABC$, $\tan\theta = \frac{x}{\sqrt{2}x}$

$= \frac{1}{\sqrt{2}}$

$\theta = 35^\circ 16'$

Q3



a) i) $\sin(x+y) = \sin x\cos y + \cos x\sin y$

$= \left(\frac{2}{7}\right)\left(\frac{1}{3}\right) + \left(\frac{3\sqrt{5}}{7}\right)\left(\frac{2\sqrt{2}}{3}\right)$

$= \frac{2 + 6\sqrt{10}}{21}$

ii) $\cos 2x = 2\cos^2 x - 1$

$= 2\left(\frac{3\sqrt{5}}{7}\right)^2 - 1$

$= \frac{41}{49}$

b) i) $\sqrt{3}\sin x - \cos x = r\sin(x-\alpha)$

$r = \sqrt{a^2+b^2}$ $\quad \tan\alpha = \frac{b}{a}$

$= \sqrt{(\sqrt{3})^2 + (1)^2}$ $\quad = \frac{1}{\sqrt{3}}$

$= 2$ $\quad \alpha = 30^\circ$

$\therefore \sqrt{3}\sin x - \cos x = 2\sin(x - 30^\circ)$

ii) $\sqrt{3}\sin x - \cos x = 1$

$2\sin(x-30^\circ) = 1$

$\sin(x-30^\circ) = \frac{1}{2}$

$x - 30^\circ = 30^\circ, 150^\circ$

$x = 60^\circ, 180^\circ$

Q4

a) $\left(k^{-\frac{3}{4}}\right)^{-\frac{4}{3}} = 64^{-\frac{4}{3}}$

$k = \frac{1}{\sqrt[3]{64^4}}$

$= \frac{1}{256}$

b) $\frac{\cos\theta + \sin\theta - 1}{\cos\theta - \sin\theta + 1} = \tan\frac{\theta}{2}$

$\text{L.H.S.} = \frac{\frac{1-t^2}{1+t^2} + \frac{2t}{1+t^2} - 1}{\frac{1-t^2}{1+t^2} - \frac{2t}{1+t^2} + 1} \times \frac{1+t^2}{1+t^2}$

$= \frac{1 - t^2 + 2t - 1 - t^2}{1 - t^2 - 2t + 1 + t^2}$

$= \frac{2t - 2t^2}{2 - 2t}$

$= \frac{2t(1-t)}{2(1-t)}$

$= t$

$= \tan\frac{\theta}{2} = \text{RHS}.$

b) $\frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} = 2$

$LHS = \frac{\cos\theta\sin 3\theta - \sin\theta\cos 3\theta}{\sin\theta\cos\theta}$

$= \frac{\sin(3\theta - \theta)}{\sin\theta\cos\theta}$

$= \frac{\sin 2\theta}{\sin\theta\cos\theta}$

$= \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta}$

$= 2 = RHS$

c) $x = 3\sec\theta \quad y = 2\tan\theta$

$\frac{x}{3} = \sec\theta \quad \frac{y}{2} = \tan\theta$

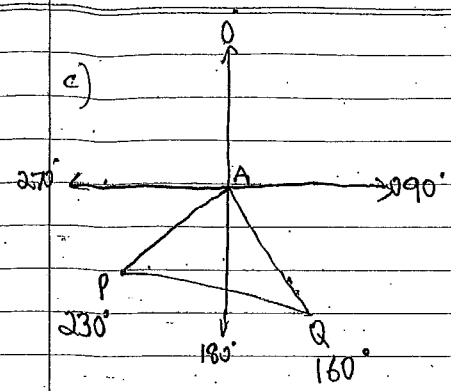
SQUARING BOTH SIDES

$\frac{x^2}{9} = \sec^2\theta \quad \frac{y^2}{4} = \tan^2\theta$

$\frac{x^2}{9} - \frac{y^2}{4} = \sec^2\theta - \tan^2\theta$

ie $\frac{x^2}{9} - \frac{y^2}{4} = 1$

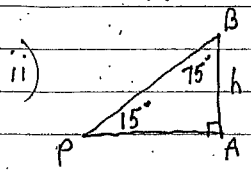
c)



$\angle PAQ = 230° - 160°$

$= 70°$

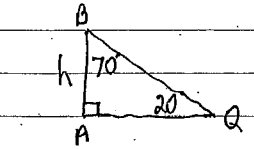
ii)



$\tan 75° = \frac{AP}{h}$

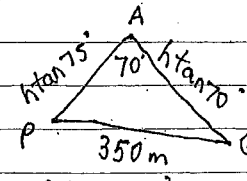
$AP = h\tan 75°$

iii)



$\tan 70° = \frac{AQ}{h}$

$AQ = h\tan 70°$



$c^2 = a^2 + b^2 - 2ab\cos C$

$350^2 = h^2\tan^2 75 + h^2\tan^2 70 - 2(h\tan 75)(h\tan 70)\cos 70°$

$350^2 = h^2(\tan^2 75 + \tan^2 70 - 2\tan 75\tan 70\cos 70)$

$350^2 = 14.46\ldots h^2$

$h^2 = 8469.9\ldots$

$h \doteq 92m$ (NEAREST m)

(5)

a) i) $(\sin A - \cos A)^2 = 1 - \sin 2A$

$LHS = \sin^2 A - 2\sin A\cos A + \cos^2 A$

$= 1 - 2\sin A\cos A$

$= 1 - \sin 2A = RHS.$

ii) NOTE:

$(\sin 15 - \cos 15)^2 = 1 - \sin 30°$

$= \frac{1}{2}$

$\sin 15 - \cos 15 = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

(Since $\sin 15 < \cos 15$)