Stationary Points, local extreme points and absolute extreme points

Quick Review 7.6 (d)

1 For each of the following functions, find all critical values, relative extreme values, inflection values, and the largest open interval on which the given function is increasing, decreasing, concave up and concave down.
(a) $f(x) = x^2e^x$
(b) $f(x) = 2x^3 - 6x^2 + 4$
(c) $f(x) = -3x^4 + 8x^3 + 5$
(d) $f(x) = 4x^5 - 5x^4 + 1$
(e) $f(x) = xe^x$
(f) $f(x) = -x^3 + 6x^2 - 9x + 1$

2 Locate the value(s) at which the function $f(x) = 2x^3 - 6x + 2$ attains an absolute maximum and the value(s) at which f attains an absolute minimum on each interval.
(a) $[-2, 2]$ (b) $[0, 2]$ (c) $[2, 4]$

3 On each interval locate the value(s) (if they exist) at which the function $f(x) = \frac{1 + 3x^4}{x^3}$ at which f attains an absolute extremum.
(a) $(-\infty, 0)$ (c) $(0, 3]$
(b) $[-1, 3]$ (d) $[0.5, 2]$

4 Locate the values(s) at which each of the following functions attains an absolute maximum and the values at which the function attains an absolute minimum (if they exist) on the given interval:
(a) $f(x) = -2x^3 + 3x^2$ on $[-2, 2]$
(b) $f(x) = 2x^2 - 8x + 2$ on $[-1, 1]$
(c) $f(x) = -2x^2 + 8x - 4$ on $[0, 3]$
(d) $f(x) = x^4 - 8x^2 + 3$ on $[-3, 3]$
(e) $f(x) = \frac{16 + x^4}{x^2}$ on $[1, 8]$
(f) $f(x) = 2x^4 + 12x^2 - 4$ on $(-\infty, \infty]$

5 If the revenue function for a firm is given by $R(x) = 16x - 0.2x^2$, and the cost function for the firm is $C(x) = 5 + 8x$ where x is the volume of sales, find:
(a) the values of x at which the revenue is maximum.
(b) the values of x for which the profit is maximum.

6 If the revenue function for a firm is given by $R(x) = -x^3 + 48x$ and the cost function for the firm is $C(x) = 21x + 30$, where x is the number of units of products sold, find
(a) the value of x at which the revenue is maximized.
(b) the value of x for which the profit is maximized.

7 A scientist shows that the plant responses to the phosphorus fertilizer were approximated by $f(x) = -0.057 - 0.417x + 0.852\sqrt{x}$, where $f(x)$ is the function of yield which depends on the units of nitrogen, x. Find the number of units of nitrogen that maximize the yield.

8 Agricultural research shows that the yield response of corn in a certain place to soil moisture is approximated by $y = 2x^3 - 210x^2 + 7200x - 7800$, where y is measured in bushels per hectare and x is a soil moisture index.
Find the value of soil moisture index that maximises yield, and find the maximum yield.

9 The cost of running a truck at an average speed of V km h^{-1} is $64 + \frac{V^2}{100}$ ringgit per hour. Show that the cost of running a truck from town A to town B, a distance of 100 km is

$$V + \frac{6400}{V}, V > 0$$

Calculate the average speed for which this total cost will be a minimum. What is the minimum total cost?

10 A company manufactures a toy at a cost of 2 ringgit per unit and sells them for RMx per unit. If the number sold is $\frac{800}{x^2}$ per month, find the value of x for which the company could expect to maximise its monthly profit.

11 A curve has the equation $y = xe^{-x}$ for $-1 \leqslant x \leqslant 5$. Find all stationary points and global maximum or minimum points on the curve, where they exist.

12 A curve has the equation $y = \frac{e^x}{x}$ for $0.1 \leqslant x \leqslant 2$. Find all stationary points and global maximum or minimum points on the curve, where they exist.

13 Given the curve $y = ax^3 + bx^2 + cx + d$, where a, b, c and d are constants and where the gradients at the points $(0, 3)$ and $(1, 0)$ are -7 and 0 respectively.
(a) Find the values of a, b, c and d.
(b) Show that the curve cuts the x-axis at the point $(3, 0)$.
(c) Find the local maximum of this curve.

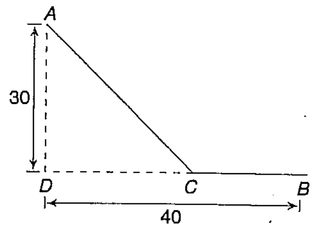
14 A right circular cone is cut from a solid sphere of unit radius, the vertex and the circumference of the base being on the surface of the sphere.
If x is the distance of the base from the centre of the sphere, prove that the volume of the cone is

$$\frac{1}{3}\pi(1 + x - x^2 - x^3)$$

Hence find the value of x so as to get the maximum cone. What fraction of the volume of the sphere is occupied by the maximum cone?

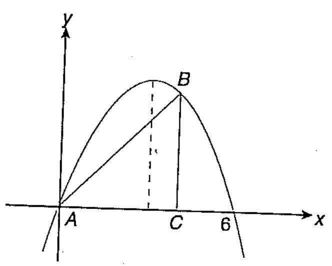
15 A circular cyclinder open at the top is to be made so as to have a volume of 1 cu. m. If r m is the radius of the base, prove that the total outside surface area is $(\pi r^2 + \frac{2}{r})$ sq. m.
Hence prove that this surface area is a maximum when the height equals the radius of the base.

16 Pipe is to be laid connecting A with B in the figure.



The cost along the level stretch from B to C is RM10 per foot, whereas the cost along the difficult (away from the road) stretch from C to A is RM20 per foot. The distance from B to D is 40 feet, and the distance from A to D is 30 feet.
At what point C is the cost of laying pipe from A to B through C minimum?

17 ABC is a triangle with A and B on the curve with equation $y = -x^2 + 6x$.
A and C are on the x-axis with $\angle ACB = 90°$.
Find the maximum area of triangle ABC.



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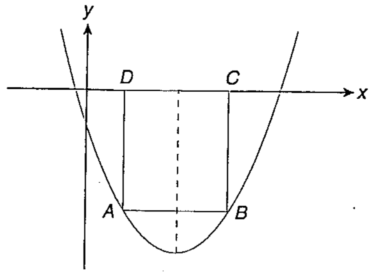

ABCD is a rectangle with A and B on the curve with equation $y = x^2 - 4x$. C and D are on the x-axis with AB parallel to DC. Find the maximum area of rectangle ABCD.

19 Find the maximum and minimum values of y given by the equation $x^3 + y^3 - 3xy = 0$.

20 B is a point l km due north of A, while C is $3l$ km due east of B; P is a variable point on BC at a distance x km from B.
A man walks straight from A to P at 4 km h^{-1} and then straight from P to C at 5 km h^{-1}. Prove that the time for the whole journey is, in hours,

$$\frac{1}{4}\sqrt{l^2 + x^2} + \frac{1}{5}(3l - x)$$

Find what value of x must be for the time taken on the whole journey to be a minimum, and also find this minimum time in hours.

Quick Review 7.6 (d)

1 (a) Critical values at $x = -2, 0$
relative maximum at $x = -2$
relative minimum at $x = 0$
inflection values at $x = -2 \pm \sqrt{2}$
increasing on $(-\infty, -2), (0, \infty)$
decreasing on $(-2, 0)$
concave up on $(-\infty, -2 - \sqrt{2}), (-2 + \sqrt{2}, \infty)$
concave down on $(-2 - \sqrt{2}, -2 + \sqrt{2})$

(b) Critical values at $x = 0, 2$
relative maximum at $x = 0$
relative minimum at $x = 2$
inflection values at $x = 1$
increasing on $(-\infty, 0), (2, \infty)$
decreasing on $(0, 2)$
concave up on $(1, \infty)$
concave down on $(-\infty, 1)$

(c) Critical values at $x = 0, 2$
relative maximum at $x = 2$
inflection values at $x = 0, \frac{4}{3}$
increasing on $(-\infty, 2)$
decreasing on $(2, \infty)$
concave up on $(0, \frac{4}{3})$
concave down on $(-\infty, 0), (\frac{4}{3}, \infty)$

(d) Critical values at $x = 0, 1$
relative maximum at $x = 0$
relative minimum at $x = 1$
inflection values at $x = \frac{3}{4}$
increasing on $(-\infty, 0), (1, \infty)$
decreasing on $(0, 1)$
concave up on $(\frac{3}{4}, \infty)$
concave down on $(-\infty, \frac{3}{4})$

(e) Critical values at $x = -1$
relative minimum at $x = -1$
inflection values at $x = -2$
increasing on $(-1, \infty)$
decreasing on $(-\infty, -1)$
concave up on $(-2, \infty)$
concave down on $(-\infty, -2)$

(f) Critical values at $x = 1, 3$
relative maximum at $x = 3$
relative minimum at $x = 1$
inflection values at $x = 2$
increasing on $(1, 3)$
decreasing on $(-\infty, 1), (3, \infty)$
concave up on $(-\infty, 2)$
concave down on $(2, \infty)$

2 (a) absolute minimum at $x = -2, 1$
absolute maximum at $x = -1, 2$
(b) absolute minimum at $x = 1$
absolute maximum at $x = 2$
(c) absolute minimum at $x = 2$
absolute maximum at $x = 4$

3 (a) no absolute minimum
absolute maximum at $x = -1$
(b) no absolute minimum
no absolute maximum
(c) absolute minimum at $x = 1$
no absolute maximum
(d) absolute minimum at $x = 1$
absolute maximum at $x = 0.5$

4 (a) absolute minimum at $x = 2$,
absolute maximum at $x = -2$
(b) absolute minimum at $x = 1$
absolute maximum at $x = -1$
(c) absolute minimum at $x = 0$
absolute maximum at $x = 2$
(d) absolute minimum at $x = -2, 2$
absolute maximum at $x = -3, 3$
(e) absolute minimum at $x = 2$
absolute maximum at $x = 8$
(f) absolute minimum at $x = 0$
no absolute maximum

5 (a) 40 (b) 20

6 (a) 4 (b) 3

7 1.044

8 30, 73 200

9 V = 80 km h^{-1}, RM160

10 4

11 Local and global max. (1, 0.37)
Global min. (–1, –2.72)

12 Global max. (0.1, 11.05)
Global and local min (1, 2.72)

13 (a) $a = -1, b = 5, c = -7, d = 3$
(c) (2.33, 1.19)

14 $x = \frac{1}{3}, \frac{8}{27}$ of the sphere.

16 C is $10\sqrt{3}$ feet to the right of D.

17 16

18 6.1584

19 $(2^{\frac{1}{3}}, 2^{\frac{2}{3}})$ max. (0, 0) min.

20 $x = \frac{4}{3} l$ time $= \frac{3}{4} l$ hours.