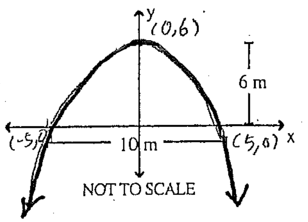
Year 11 (3 Unit) **Locus and Parabola Test** ( ST ANDREWS )

1. For the parabola $x^2 = -12y$
   (a) Find:
   (i) the co-ordinates of the focus.
   (ii) the equation of the directrix.
   (iii) the length and co-ordinates of the endpoints of the latus rectum.
   (b) Find the equation of the normal to this parabola at the point (6,–3).
   (c) The above normal meets the curve again at Q and the directrix at R. Find the midpoint of the interval QR.

2. For the parabola $x^2 - 10x - 2y + 19 = 0$
   (a) Find the focal length and the co-ordinates of the vertex.
   (b) Hence find:
   (i) the co-ordinates of the focus and the equation of the directrix.
   (ii) the length and endpoints of the latus rectum.

3. (a) Find the equation of the parabola with focus (–4,3) and directrix $y = -5$.
   (b) Determine the inequality which is satisfied by all points P(x,y) such that P is closer to the focus than the directrix.

4.




The diagram shows a bridge in the shape of a parabola.
The bridge is 6 metres high and 10 metres wide at its base.

Find the equation of the parabola.

5. A and B are the points (–3,5) and (2,–1) respectively.
   The point P(x,y) moves so that PA is perpendicular to PB.
   (a) Find the equation of the locus of P.
   (b) State, in geometric terms, the nature of this locus.

6. A is the point (12,0) and $l$ is the line $x = 3$.
   The point P(x,y) moves so that the distance of P from $l$ is half the distance of P to A.
   Show that the equation of the locus of P is $3x^2 - y^2 = 108$.

m.v.

Locus and Parabola test

1

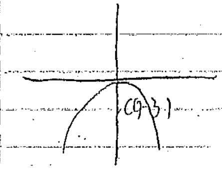
a) $x^2 = -12y$

i/

vertex $(0,0)$

$4a = 12$

$a = 3$




Focus $(0,-3)$

ii/ Directrix $y = 9$

$y = 3$

iii/ Lactus rectum

length $= 4a$

$= 4 \times 3$

$= 12$

Endpoints

$y = -3$ — ①

$x^2 = -12y$ — ②

sub ① into ②

$x^2 = 1361$

$x = \pm 6$

Endpoints $(\pm 6, -3)$

b) $x^2 = -12y$

$12y = -x^2$

$\frac{dy}{dx} = -\frac{2x}{12}$

at Point $(6,-3)$

$\frac{dy}{dx} = -\frac{2x}{12}$

$= -1$ — gradient of tangent

equation of normal:

$y_2 - y_1 = m(x - x_1)$

$y + 3 = 1(x - 6)$

$y + 3 = x - 6$

$x - y - 9 = 0$

c) $y = x - 9$ — ①

$x^2 = -12y$ — ②

sub ① into ②

$x^2 = -12(x - 9)$

$x^2 = -12x + 108$

$x^2 + 12x - 108 = 0$

$(x + 18)(x - 6) = 0$

$x = 6, -18$

sub $-18$ into ①

$y = -27$

Point $(-18, -27)$

Midpoint

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Point $(-18,-27)$ $(6,-3)$

$\frac{-18+6}{2}, \frac{-27-3}{2}$

$\frac{-12}{2}, \frac{-30}{2}$

$(-6,-15)$

2 $x^2-10x-2y+19=0$

a)

$x^2-10x+19=2y$

$y=\frac{x^2-10x+19}{2}$

$x = \frac{-b}{2a}$

$= \frac{-10}{2} \div 2$

$= -5$

$y=\frac{(-5)^2-10(-5)+19}{2}$

$y=47$

$\therefore$ vertex $(-5,47)$

2

a) $x^2-10x = 2y-19$

$x^2-10x+\left(\frac{-10}{2}\right)^2 = 2y-19+\left(\frac{-10}{2}\right)^2$

$(x-5)^2 = 2y+6$

$(x-5)^2 = 2(y+3)$

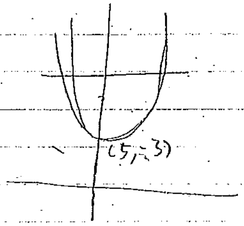
$(x-5)^2 = 4\times\frac{1}{2}(y+3)$

Focal length = ½ units

Vertex $(5,-3)$

b)

i/




Focus: $(5,-2\frac{1}{2})$

Directrix: $y=-3\frac{1}{2}$

ii/ Lactus rectum equation $y=2\frac{1}{2}$

length $= 4a$

$= 4\times\frac{1}{2}$

$= 2$ units

endpoints

$(5\pm2\times\frac{1}{2}, -2\frac{1}{2})$

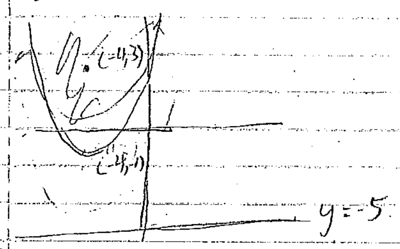
$(6,-2\frac{1}{2})$ $(4,-2\frac{1}{2})$

3

a) ~~$x^2=4ay$~~

$(x-h)^2=\pm4a(y-k)$

$(y-k)^2=\pm4a(x-h)$




Vertex $(-4,-1)$

$(x-h)^2=\pm4a(y-k)$

$(x+4)^2=4a(y+k)$

$(x+4)^2=16(y+1)$

b) Test point $(-4,3)$

$(-4+4)^2=16(3+1)$

$0 = 64$

$0<64$

$(x+4)^2\le16(y+1)$

4/ $(x-h)^2=\pm4a(y-k)$

$(x+0)^2=-4a(y-6)$

$x^2=-4a(y-6)$

$\frac{x^2}{y-6}=-4a$

$\frac{25}{-6}=-4a$

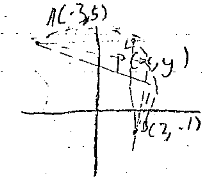
$a=\frac{25}{24}$

$x^2=-4\times\frac{25}{24}(y-6)$

$x^2=-150(y-6)$

5

a)




$m_{PA}\times m_{PB}=-1$

$m=\frac{y_2-y_1}{x_2-x_1}$

$\frac{y-5}{x+3}\times\frac{y+1}{x-2}=-1$

$(y-5)(y+1)=-1(x+3)(x-2)$

$y^2+y-5y-5=-(x^2-2x+3x-6)$

$y^2-4y-5=-x^2-x+6$

$x^2+x+y^2-4y=11$

$\left[x^2+x+\left(\frac{1}{2}\right)^2\right]+\left[y^2-4y+\left(\frac{-4}{2}\right)^2\right]=11+\left(\frac{1}{2}\right)^2+\left(\frac{-4}{2}\right)^2$

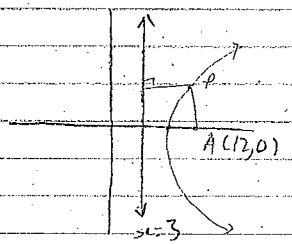
$\left(x+\frac{1}{2}\right)^2+(y-2)^2=\frac{16}{4}$

b) Geometrically AB is a circle with diameter AB

circle centre: $(-\frac{1}{2}, 2)$

radius 4 units

6




$PL = \frac{1}{2} PA$

$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$d = \sqrt{(x-3)^2 + (y-0)^2} = \frac{1}{2}\sqrt{(x-12)^2 + (y-0)^2}$

$\left[2\sqrt{(x+3)^2 + y^2}\right]^2 = \left[\sqrt{(x-12)^2 + y^2}\right]^2$

$4\left[(x+3)^2 + y^2\right] = \left[(x-12)^2 + y^2\right]$

$4\left[x^2 + 6x + 9 + y^2\right] = x^2 - 24x + 144 + y^2$

$4x^2 + 24x + 36 + 4y^2 = x^2 - 24x + 144 + y^2$

$3x^2 + 3y^2 = 108$

$LP = \frac{1}{2} PA$

$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$\sqrt{(3-x)^2 + (0-y)^2} = \frac{1}{2}\sqrt{(x-12)^2 + (y-0)^2}$

$\left[2\sqrt{(3-x)^2 + (-y)^2}\right]^2 = \left[\sqrt{(x-12)^2 + (y-0)^2}\right]^2$

$4\left[(3-x)^2 + y^2\right] = (x-12)^2 + y^2$

$4(9 - 6x + x^2 + y^2) = x^2 - 24x + 144 + y^2$

$36 - 24x + 4x^2 + 4y^2 = x^2 - 24x + 144 + y^2$

$3x^2 + 3y^2 = 108$

$\frac{|x-3|}{\sqrt{1^2}} = \frac{1}{2}\sqrt{(x-12)^2 + (y-0)^2}$

$\left[2(x-3)\right]^2 = \left[\sqrt{(x-12)^2 + (y-0)^2}\right]^2$

$4\left[x^2 - 6x + 9\right] = x^2 - 24x + 144 + y^2$

$4x^2 - 24x + 36 = x^2 - 24x + 144 + y^2$

$3x^2 - y^2 = 108$