ASCHAM

Ext 2 Mathematics Test on Polynomials Term 1 Week 4 2011

1. Resolve into partial fractions: $\dfrac{2x^2+x+6}{x^2-4}$ [4]

2. The polynomial $P(x)=x^4+ax^3+bx+21$ has a double root at $x=1$. Find the values of a and b. [4]

3. The roots of $x^3+5x^2+11=0$ are α,β,λ
   - (i) Find the equation whose roots are $\alpha^2,\beta^2,\lambda^2$ [3]
   - (ii) Find the value of $\alpha^2+\beta^2+\lambda^2$ [1]

4. You may assume that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$
   - (i) Using the above identity find the roots of $8x^3-6x-1=0$ in the form $x=\cos\theta$ [2]
   - (ii) Hence evaluate $\cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{4\pi}{9}$ (explain your work) [3]

5. (i) Find the three cube roots of unity by solving $z^3=1$ [2]
   - (ii) Show that the three cube roots of unity can be expressed as 1, ω and ω^2 where ω is one of the complex roots found in (i). [2]
   - (iii) With ω defined as above, and a, b, c real numbers,
     - (i) prove that $(b\omega+c\omega^2)(b\omega^2+c\omega)=b^2-bc+c^2$ [2]
     - (ii) show that $(b+c)^3-3bc(b+c)-(b^3+c^3)=0$ [2]
     - (iii) hence prove that $(b+c)$ is a root of the equation $x^3-3bcx-(b^3+c^3)=0$ and deduce the other two roots of this equation. [2]

End of Test **27 marks**

$\frac{20}{21}$:)!!

Polynomials

1. $\dfrac{2x^2+x+6}{x^2-4} \equiv \dfrac{A}{x+2} + \dfrac{B}{x-2}$

$2x^2+x+6 = A(x-2)+B(x+2)$

Let $x=2$ $16=4B$

$B=4$

$x=-2$ $12=-4A$

$A=-3$

$\therefore \dfrac{2x^2+x+6}{x^2-4} = \dfrac{-3}{x+2} + \dfrac{4}{x-2}.$

$\frac{1}{4}$

2. $P(x)=x^4+ax^3+bx+21$

$P'(x)=4x^3+3ax^2+b.$

$P'(1)=0$ $P(1)=0$

$4+3a+b=0$

$3a+b=-4$ ①

$1+a+b=-21$

$a+b=-22$ ②

①-② $2a=18$

$\therefore a=9 \rightarrow$ ① ✓

$27+b=-4$

$\therefore b=-31$ ✓

$\therefore P(x)=x^4+9x^3-31x+21.$

$\frac{4}{4}.$

$x^2-4 \overline{)2x^2+x+6}$ 2

$2x^2+0x-8$

$x+14.$

$\dfrac{2x^2+x+6}{x^2-4} = 2+\dfrac{x+14}{x^2-4}$

$= 2+\dfrac{A}{x+2}+\dfrac{B}{x-2}.$

$2x^2+x+6 = A(x-2)+B(x+2).$

let $x=2$ $8+2+6=4B$ $x=-2$ $8-2+6=-4A$

$16=4B$ $A=0.$

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3. $x^3+5x^2+11=0$.

Roots α, β, γ.

a) $\alpha^2, \beta^2, \gamma^2$ let $x \rightarrow x^{1/2}$.

$(x^{1/2})^3+5(x^{1/2})^2+11=0$

$(x^{1/2})^3+5x+11=0$.

$(x^{3/2})^2=(-5x\textcircled{+}11)^2$ ✗

$x^3=25x^2-110x+121$.

$\therefore x^3-25x^2+110x-121=0$ ✓

b) $\alpha^2+\beta^2+\gamma^2=\frac{-b}{a}$

$=25$ ✓

$\frac{2}{3}$

$+1$

4. $\cos 3\theta=4\cos^3\theta-3\cos\theta$

i) $8x^3-6x-1=0$.

~~When $\cos 3\theta=0$, $\cos\theta=0$~~ $\cdot$ $2\cos 3\theta=8\cos^3\theta-6\cos\theta$

~~Let $x=\cos\theta$.~~ $\therefore 2\cos 3\theta=1$. when $8x^3-6x-1=0$

~~$\cos^3\theta-6\theta 6\cos\theta=1$.~~ $(x=\cos\theta)$.

$\cos 3\theta=\frac{1}{2}$

$3\theta=\frac{\pi}{3}+2k\pi$, $k=0,\pm1$.

$\theta=$ ~~$\frac{\pi+2k\pi}{3}$~~ $\frac{\pi}{9}+\frac{2k\pi}{3}$

$=\frac{\pi}{9}$, ~~$\frac{3\pi}{9}$~~ $\frac{7\pi}{9}$, $-\frac{5\pi}{9}$. ✓

$=\frac{\pi}{9}$, $\frac{-2\pi}{9}$, $-\frac{5\pi}{9}$.

2?

$\therefore$ Solutions $x=\cos\frac{\pi}{9}$, $\cos(\frac{-2\pi}{9})$, $\cos(\frac{-5\pi}{9})$

$=\cos\frac{\pi}{9}$, $\cos\frac{2\pi}{9}$, $\cos(\frac{-4\pi}{9})=\cos\frac{4\pi}{9}$

ii) $\cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{4\pi}{9}$ is the product of roots of $8x^3-6x-1$,

so $\cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{4\pi}{9}=-(-\frac{1}{8})$

$=\frac{1}{8}$ ✓

$\frac{4}{5}$

needs to be clearer why.

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5. i) $z^3=1$

~~Let $\text{cis}(0)=1$.~~

~~$z^3=1^3\text{cis}0$~~

~~$z=\text{cis}0$~~

~~$\text{cis}0=1$~~

~~$\cos$~~

$z^3=\text{cis}0$.

$z=\text{cis}0+\frac{2k\pi}{3}$, where $k=0,\pm1$

~~cis~~

$k=0$ $z=\text{cis}0=1$ ✓

$=1$ $z=\text{cis}\frac{2\pi}{3}=\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}$

$=-\frac{1}{2}+i\frac{\sqrt{3}}{2}$

$=-1$ $z=\text{cis}\frac{-2\pi}{3}=-\frac{1}{2}-i\frac{\sqrt{3}}{2}$.

$\therefore z=1, -\frac{1}{2}+i\frac{\sqrt{3}}{2}, -\frac{1}{2}-i\frac{\sqrt{3}}{2}$

2

2

ii) $\text{cis}0=1$.

Let $w=\text{cis}\frac{2\pi}{3}$.

$\therefore w=\text{cis}\frac{2\pi}{3}$ is a root

$w^2=\text{cis}\frac{4\pi}{3}$

$=\text{cis}\frac{-2\pi}{3}$ which is a root.

$\therefore 1, w$ and w^2 are roots of unity.

careful.

2

2.

(5) (iii) Using $1+\omega+\omega^2=0$ & $\omega^3=1$

(i) $LHS=(b\omega+c\omega^2)(b\omega^2+c\omega)$

$=b^2\omega^3+bc\omega^2+bc\omega^4+c^2\omega^3$

$=b^2+c^2+bc(\omega^2+\omega)$ since $\omega^3=1$

$=b^2+c^2-bc$ since $\omega^2+\omega=-1$

$=RHS$.

(ii) $LHS=(b+c)[(b+c)^2-3bc-(b^2-bc+c^2)]$

$=(b+c)[b^2+c^2+2bc-3bc-b^2+bc-c^2]$

$=(b+c)(0)=0=RHS$

(iii) Sub $x=b+c$

$LHS=(b+c)^3-3bc(b+c)-(b^3+c^3)$

$=0$ (using (ii)) $=RHS$

Let the other two roots be α, β

Using product of roots

$\alpha.\beta.(b+c)=b^3+c^3$

$=(b+c)(b^2-bc+c^2)$

$\therefore \alpha\beta=b^2-bc+c^2=(b\omega+c\omega^2)(b\omega^2+c\omega)$

$\therefore \alpha=(b\omega+c\omega^2)$; $\beta=b\omega^2+c\omega$.