FORT ST HIGH SCHOOL - EXT 2 - 2011
TASK 2

Question 1. Marks

a) Given that the polynomial $P(x)=x^4-6x^3+12x^2-8x$ has a root of multiplicity three (i.e. a triple root), completely factorise the polynomial and sketch the polynomial showing all roots. **3**

b) Show that $2-i$ is a root of $x^3-3x^2+x+5=0$ and find the other roots of this equation. **3**

c) If α,β,γ are the roots of the polynomial $x^3+x^2-1=0$, find the equation of a polynomial whose roots are $\alpha^2,\beta^2,\gamma^2$. **3**

d) Resolve $\dfrac{4x+10}{(2x-1)(4x^2+3)}$ into partial fractions. **3**

Question 2. Marks

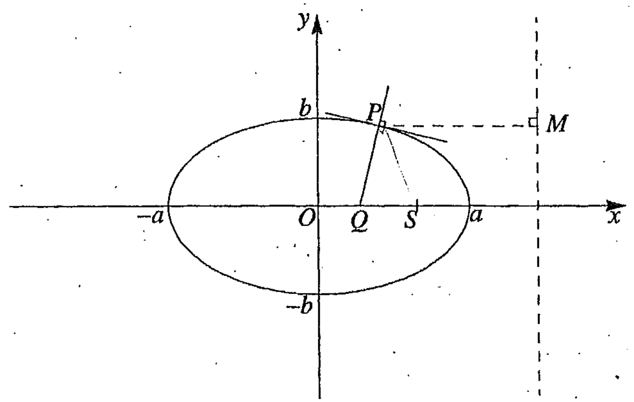
a) If the line $x=1$ is a directrix and the point (2, 0) is a focus of a conic of eccentricity $\sqrt{2}$:

i) Find the equation of the conic, showing that it is a rectangular hyperbola and sketch the curve showing its asymptotes, foci and directrices. **2**

ii) Find the equation of the normal to the curve at any point P on the curve. **2**

iii) The normal to the curve at P meets the x-axis at $(X,0)$ and the y-axis at the point $(0,Y)$. If T is the point (X,Y), show that as P varies on the curve, T always lies on the hyperbola $x^2-y^2=8$. **2**

b)



Consider the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, drawn above, with eccentricity e.

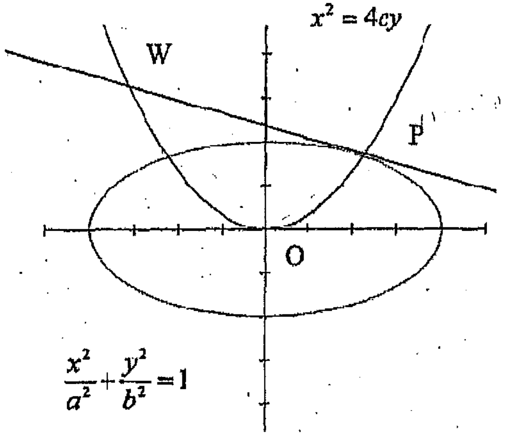
i) Write down in terms of a and e the coordinates of the focus S, and the equation of the associated directrix. **2**

ii) Show that the equation of the normal to the ellipse at the point $P(x_1,y_1)$ is given by

$$\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$$ **2**

iii) Let Q be the x-intercept of the normal and let M be the foot of the perpendicular from P to the directrix as shown in the diagram. Show that $QS=e^2PM$. **2**

Question 3. Marks



The Parabola $x^2 = 4cy$ intersects the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the point $P(2cp, cp^2)$ in the first quadrant, where $0 < b < a$ and $c > 0$. The tangent to the ellipse at P meets the parabola again at $W(2cw, cw^2)$.

i) Show that the tangent to the ellipse at P has the equation $\frac{2cpx}{a^2}+\frac{cp^2y}{b^2}=1$. **3**

ii) If this tangent meets the parabola at $(2ct, ct^2)$ show that $\frac{p^2t^2}{b^2}+\frac{4pt}{a^2}-\frac{1}{c^2}=0$. **2**

iii) Explain why $t = p$ and $t = w$ are the roots of the equation in part ii). **1**

iv) If PW subtends a right angle at the origin, show that $pw = -4$. **1**

v) Hence show $\frac{1}{b^2}=\frac{1}{a^2}+\frac{1}{(4c)^2}$ **2**

vi) By considering the roots of the equation in part (ii), show $p=\frac{b}{2c}$. **1**

vii) Hence show that if PW subtends a right angle at the origin, then $p = 2e$, where e is the eccentricity of the ellipse. **2**

Question 4. Marks

The Hyperbola $\mathcal{H}$ has equation $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and eccentricity e, while ellipse $\mathcal{E}$ has equation $\frac{x^2}{a^2+b^2}+\frac{y^2}{b^2}=1$.

i) Show that $\mathcal{E}$ has eccentricity $\frac{1}{e}$. **2**

ii) Show that $\mathcal{E}$ passes through one focus of $\mathcal{H}$ and $\mathcal{H}$ passes through one focus of $\mathcal{E}$. **2**

iii) Sketch $\mathcal{H}$ and $\mathcal{E}$ on the same diagram, showing the foci S, S' of $\mathcal{H}$ and T, T' of $\mathcal{E}$ and the directrices of $\mathcal{H}$ and $\mathcal{E}$. Give the coordinates of the foci and the equations of the directrices in terms of a and e. **3**

iv) If $\mathcal{H}$ and $\mathcal{E}$ intersect at P in the first quadrant, show that the acute angle α between the tangents to the curves at P satisfies $\tan\alpha = \sqrt{2}\left(e+\frac{1}{e}\right)$ **4**

v) Find the acute angle between the tangents to the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$ and the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ at their points of intersection. Give you answer correct to the nearest degree. **1**

Question 5. Marks

a) Consider the quadratic polynomial equation $x^2 - x + k = 0$ where k is a real number. The equation has two distinct positive roots α and β.

i) Show that $0 < k < \frac{1}{4}$ **2**

ii) Show that $\frac{1}{\alpha^2} + \frac{1}{\beta^2} > 8$ **2**

b) A cubic polynomial is given by $P(x) = x^3 + ax + b$ where a, b are constants.

It is given that the polynomial equation $P(x) = 0$ has three roots α, β and γ.

i) Find the value of $\alpha + \beta + \gamma$. **1**

ii) Show that $\alpha^2 + \beta^2 + \gamma^2 = -2a$. **1**

iii) If the polynomial has a positive double root, show that this double root is $\frac{-3b}{2a}$. **3**

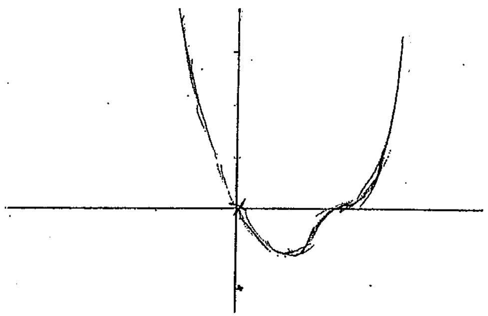
iv) If the polynomial has three distinct roots show that $4a^3 + 27b^2 < 0$. **3**

STANDARD INTEGRALS

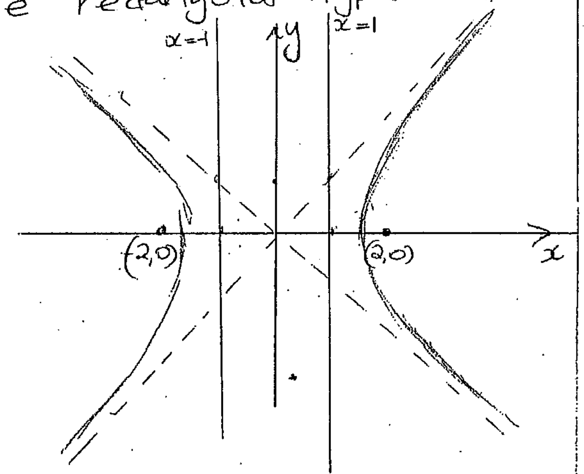
$\int x^n \, dx$	$= \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$
$\int \frac{1}{x} \, dx$	$= \ln x, \quad x > 0$
$\int e^{ax} \, dx$	$= \frac{1}{a} e^{ax}, \quad a \neq 0$
$\int \cos ax \, dx$	$= \frac{1}{a} \sin ax, \quad a \neq 0$
$\int \sin ax \, dx$	$= -\frac{1}{a} \cos ax, \quad a \neq 0$
$\int \sec^2 ax \, dx$	$= \frac{1}{a} \tan ax, \quad a \neq 0$
$\int \sec ax \tan ax \, dx$	$= \frac{1}{a} \sec ax, \quad a \neq 0$
$\int \frac{1}{a^2 + x^2} \, dx$	$= \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$
$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx$	$= \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$
$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx$	$= \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$
$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx$	$= \ln\left(x + \sqrt{x^2 + a^2}\right)$

NOTE : $\ln x = \log_e x, \quad x > 0$

EXTENSION II TASK 2 : Polynomials and Conics 2011.

SOLUTIONS	mark	COMMENTS
Question 1.		
a) $P(x) = x^4 - 6x^3 + 12x^2 - 8x$		
$P'(x) = 4x^3 - 18x^2 + 24x - 8$		• most got to $P''(x)=0$ and $x = 1, 2$
$P''(x) = 12x^2 - 36x + 24$		
$= 12(x^2 - 3x + 2)$		
$= 12(x-1)(x-2)$.		
Since $P(x)$ has root of multiplicity 3, $P''(x) = 0$,	①	• Note that $P(2)=0$ is not sufficient evidence for $x=2$ to be a tripple root! (must test $P(2)$)
So either $x = 1$ or $x = 2$		
$P(1) \neq 0$		
$P(2) = 0$.	①	
$\therefore P(x) = x(x^3 - 6x^2 + 12x - 8)$	①	
$= x(x-2)^3$.		• too many students did not know what a cubic root should look like!
	①	• positive leading term
b) $P(2-i) = (2-i)^3 - 3(2-i)^2 + 2 - i + 5$		• READ question - many did not show this!
$= 8 - 12i - 6 + i - 9 + 12i + 2 - i + 5$		
$= 0$.	①	
$\therefore 2-i$ is a factor.		
Since real coefficients		• give REASON why $2+i$ is the other root!
$2+i$ is also a factor	①	

SOLUTIONS	mark	COMMENTS
$\therefore x^3 - 3x^2 + x + 5 = (x-(2-i))(x+(2-i))(x-a)$		• READ the question! $P(x) = (x^2-4x+5)(x-1)$ is the factored form (or factors) $x=-1$ etc are the roots
$= (x^2 - 4x + 5)(x+1)$	①.	
$\therefore$ Other root is $x = -1$		
c) Required equation is:		• generally well done
$(\sqrt{x})^3 + (\sqrt{x})^2 - 1 = 0$.	①	
$x^{3/2} + x - 1 = 0$.		
$x^{3/2} = 1 - x$.		
$(x^{3/2})^2 = (1-x)^2$	①	
$x^3 = 1 - 2x + x^2$		
$\therefore x^3 - x^2 + 2x - 1 = 0$.	①	
d) $\dfrac{4x+10}{(2x-1)(4x^2+3)} = \dfrac{a}{2x-1} + \dfrac{bx+c}{4x^2+3}$		• many did not get the $\frac{bx+c}{4x^2+3}$ factor correctly!
$\therefore 4x + 10 = a(4x^2+3) + (bx+c)(2x-1)$:		• do NOT substitute complex numbers! (too lengthy)
Let $x = 1/2$		
$12 = a(4) + 0$	①	
$\therefore a = 3$		
$4x + 10 = 12x^2 + 9 + 2bx^2 + (2c-b)x - c$		
$\therefore 4 = 2c - b$		
$10 = 9 - c$	①	
$\therefore c = -1,\ b = -6$		
$\dfrac{4x+10}{(2x-1)(4x^2+3)} = \dfrac{3}{2x-1} - \dfrac{6x+1}{4x^2+3}$	①	

SOLUTIONS	mark	COMMENTS
Question 2.		
a) (i) Focus of conic is $(ae, 0)$		
$\therefore a\sqrt{2} = 2$		
$\therefore a = \sqrt{2}$.		
Since directrix is $x = 1$ conic is hyperbola		Fairly well done
$\therefore b^2 = a^2(e^2 - 1)$		
$b = \sqrt{2}$.		
$\therefore$ Equation of conic is		
$\dfrac{x^2}{(\sqrt{2})^2} - \dfrac{y^2}{(\sqrt{2})^2} = 1$	1	
ie $x^2 - y^2 = 2$.		
ie rectangular hyperbola		
	1	
(ii) Let $P = (x_1, y_1)$		
$\therefore 2x - 2y\dfrac{dy}{dx} = 0$		
$\dfrac{dy}{dx} = \dfrac{x}{y}$	1	
At P $\dfrac{dy}{dx} = \dfrac{x_1}{y_1}$.		

SOLUTIONS	mark	COMMENTS
$\therefore$ Gradient of normal at P is $-\dfrac{y_1}{x_1}$		
$\therefore$ Equation of normal is:		
$y - y_1 = \dfrac{-y_1}{x_1}(x - x_1)$	1	
ie $xy_1 + x_1 y = 2x_1 y_1$		
(iii) Normal meets x axis at $x = 2x_1$		
and y axis at $y = 2y_1$		
$\therefore (X, 0) = (2x_1, 0)$ $(0, Y) = (0, 2y_1)$	1	
$x_1 = \dfrac{X}{2}$, $y_1 = \dfrac{Y}{2}$		
$x_1^2 - y_1^2 = 2$		
$\therefore \left(\dfrac{X}{2}\right)^2 - \left(\dfrac{Y}{2}\right)^2 = 2$		
ie $X^2 - Y^2 = 8$	1	
$\therefore$ T lies on $x^2 - y^2 = 8$		

SOLUTIONS	mark	COMMENTS

Question 2

(b)

(i) The focus is $S(ae, 0)$. The directrix is $x = \frac{a}{e}$. — (1)

(ii) $$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-b^2 x}{a^2 y}$$

So the slope of the normal at $P(x_1, y_1)$ is $\frac{a^2 y_1}{b^2 x_1}$ — (1)

The equation of the normal is $y - y_1 = \frac{a^2 y_1}{b^2 x_1}(x - x_1)$

$$b^2 x_1 (y - y_1) = a^2 y_1 (x - x_1)$$

$$a^2 x y_1 - b^2 x_1 y = a^2 x_1 y_1 - b^2 x_1 y_1$$

$$\frac{a^2 x}{x_1} - \frac{b^2 y}{y_1} = a^2 - b^2 \quad \text{(divide both sides by } x_1 y_1\text{)}$$ — (1)

(iii) Q is the point $\left(\frac{a^2 - b^2}{a^2} x_1, 0\right)$ or $(e^2 x_1, 0)$ (from $a^2(1 - e^2) = b^2$).

so, $QS = |e^2 x_1 - ae|$

$= e|ex_1 - a|$ — (1)

Also, $PM = \frac{a}{e} - x_1$

so $e^2 PM = e^2\left(\frac{a}{e} - x_1\right)$ — (1)

$= e(a - ex_1)$ — (1)

Hence $QS = e^2 PM$.

SOLUTIONS	mark	COMMENTS

Question 3.

(i) $$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\therefore \frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-b^2 x}{a^2 y}$$

At P $$\frac{dy}{dx} = \frac{-b^2 \times 2c}{a^2 \times cp^2} = \frac{-2b^2}{a^2 p}$$ — (1)

• many did not put in any co-ords for P! (using $P(x_1, y_1)$ and later using $x_1 = 2cp, y_1 = cp^2$ is ok - most who did this were successful).

$\therefore$ Equation of tangent is:

$$y - cp^2 = \frac{-2b^2}{a^2 p}(x - 2cp)$$

$$a^2 p y - a^2 c p^3 = -2b^2 x + 4b^2 cp$$

$$2b^2 x + a^2 p y = 4b^2 cp + a^2 c p^3$$

$$= \frac{a^2 b^2}{cp}$$ — (1)

• many did not develop this identity (by subst P coords in eqn).

since $\frac{4c^2p^2}{a^2} + \frac{c^2 p^4}{b^2} = 1$ as P lies on ellipse.

$\left(\times \frac{cp}{a^2 b^2}\right)$ gives

$$\therefore \frac{2cpx}{a^2} + \frac{cp^2 y}{b^2} = 1.$$ — (1)

• THIS IS A STANDARD QUESTION. LEARN THE PROCESS!!

SOLUTIONS	mark	COMMENTS

(II) Since P lies on tangent $\frac{2cpx}{a^2} + \frac{cp^2y}{b^2} = 1$ then

$$\frac{2cp}{a^2}\cdot 2ct + \frac{cp^2}{b^2}\cdot ct^2 = 1.$$

① • mostly welldone

$$\frac{4c^2pt}{a^2} + \frac{c^2p^2t^2}{b^2} = 1.$$

$$\frac{4pt}{a^2} + \frac{p^2t^2}{b^2} - \frac{1}{c^2} = 0$$

①

(III) Since tangent cuts the parabola at P and W, then $t = p$ and $t = w$ are solutions to the above equation.

① • those who did not get this mark didn't associate the solutions of the eqn in (ii) with the points of intersection of the two graphs.

(IV) Gradient $OP = \frac{cp^2}{2cp} = \frac{p}{2}$

" $OW = \frac{w}{2}$

If $OP \perp OW$ $\frac{p}{2} \times \frac{w}{2} = -1$

$\therefore pw = -4$

① • mostly welldone

(V) Substitute $p = w$ into equation:

$$\frac{4pw}{a^2} + \frac{p^2w^2}{b^2} = \frac{1}{c^2}$$

$$-\frac{16}{a^2} + \frac{16}{b^2} = \frac{1}{c^2}$$

$$\therefore \frac{1}{b^2} = \frac{1}{a^2} + \frac{1}{(4c)^2}$$

• mostly welldone

SOLUTIONS	mark	COMMENTS

(VI) Since $\frac{p^2t^2}{b^2} + \frac{4pt}{a^2} - \frac{1}{c^2} = 0$ has roots p and w

• those who didn't use the $\alpha\beta = \frac{c}{a}$ identity got very lost in this part.

$$pw = \frac{-1/c^2}{\frac{p^2}{b^2}} \quad \text{(product of roots)}$$

$$-4 = \frac{-b^2}{c^2p^2}$$

①

$$\therefore p^2 = \frac{b^2}{4c^2}$$

$$p = \frac{b}{2c} \quad (p > 0 \text{ since in 1st Quadrant}).$$

(VI) If $\frac{1}{b^2} = \frac{1}{a^2} + \frac{1}{(4c)^2}$

• many did not use $e^2 = 1 - \frac{b^2}{a^2}$ identity.

$$1 = \frac{b^2}{a^2} + \frac{b^2}{(4c)^2}$$

$$1 - \frac{b^2}{a^2} = \frac{b^2}{(4c)^2}$$

$$e^2 = \frac{b^2}{(4c)^2}.$$

①

$$\therefore e = \frac{b}{4c} \quad \text{since } e > 0.$$

$$2e = \frac{b}{2c}$$

$$= p.$$

①

SOLUTIONS	mark	COMMENTS

Question 4

(i) For the hyperbola $\mathcal{H}$,

$b^2 = a^2(e^2 - 1)$

$$e^2 = \frac{b^2}{a^2} + 1 = \frac{b^2 + a^2}{a^2}$$

If the ellipse $\mathcal{E}$ has eccentricity ε,

$$b^2 = (a^2 + b^2)(1 - \varepsilon^2)$$

$$\varepsilon^2 = 1 - \frac{b^2}{a^2 + b^2}$$

$$\therefore \varepsilon^2 = \frac{a^2}{a^2 + b^2} = \frac{1}{e^2}$$

Hence the ellipse $\mathcal{E}$ has eccentricity $\frac{1}{e}$.

Mark: 1, 1

Comments: many students did not communicate what the were doing and as such made careless errors.

e.g $b^2 \neq (a^2+b^2)(1-e^2)$

$b^2 = (a^2+b^2)(1-E^2)$

where E = eccentr of the elip

(ii) Since $a^2 + b^2 = a^2e^2$, the equation of the ellipse can be rewritten as $\dfrac{x^2}{a^2e^2} + \dfrac{y^2}{b^2} = 1$.

One focus of $\mathcal{H}$ is $S(ae, 0)$, and this point clearly lies on the ellipse $\mathcal{E}$.

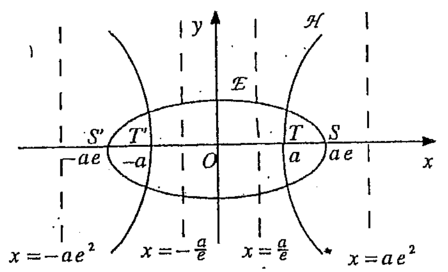
One focus of the ellipse is $T(ae \cdot \frac{1}{e}, 0) \equiv T(a, 0)$ and this point is clearly on the hyperbola $\mathcal{H}$.

Mark: 1, 1

Comments: poor setting out

(iii) Hyperbola $\mathcal{H}$ has foci $S(ae, 0)$, $S'(-ae, 0)$ and directrices $x = \dfrac{a}{e}$, $x = -\dfrac{a}{e}$.

Ellipse $\mathcal{E}$ has foci $T(a, 0)$, $T'(-a, 0)$ and directrices $x = \dfrac{ae}{\left(\frac{1}{e}\right)} = ae^2$, $x = -ae^2$.



Mark: 1 co-ords of foci; 1 eq^n of directrices; 1 diagram

Comments: many students did not read the question carefully

SOLUTIONS	mark	COMMENTS

(iv) Where the curves intersect,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad (1)$$

$$\frac{x^2}{a^2e^2} + \frac{y^2}{b^2} = 1 \quad (2)$$

$$(1)+(2) \Rightarrow \frac{x^2}{a^2e^2}(e^2+1) = 2$$

$$e^2 \times (2) - (1) \Rightarrow \frac{y^2}{b^2}(e^2+1) = e^2 - 1$$

$$b^2 = a^2(e^2-1) \Rightarrow \frac{y^2}{a^2(e^2-1)}(e^2+1) = e^2 - 1$$

$$\therefore \text{ at } P, \quad x = ae\sqrt{\frac{2}{e^2+1}}, \quad y = \frac{a(e^2-1)}{\sqrt{e^2+1}}$$

Mark: 1

For the hyperbola, at P

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{2x}{a^2} - \frac{2y}{b^2}\frac{dy}{dx} = 0$$

$$\therefore \frac{dy}{dx} = \frac{b^2}{a^2}\frac{x}{y} = (e^2-1)\frac{x}{y} = \sqrt{2}\,e$$

Mark: 1

For the ellipse, at P

$$\frac{x^2}{a^2e^2} + \frac{y^2}{b^2} = 1$$

$$\frac{2x}{a^2e^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0$$

$$\therefore \frac{dy}{dx} = -\frac{b^2}{a^2e^2}\frac{x}{y} = -\frac{(e^2-1)}{e^2}\frac{x}{y} = -\sqrt{2}\,\frac{1}{e}$$

Mark: 1

Hence the gradients of the tangents to $\mathcal{H}$ and $\mathcal{E}$ at P are $\sqrt{2}\,e$ and $-\sqrt{2}\,\frac{1}{e}$ respectively.

$$\tan\alpha = \left|\frac{\sqrt{2}\,e - \left(-\sqrt{2}\,\frac{1}{e}\right)}{1 + \sqrt{2}\,e\left(-\sqrt{2}\,\frac{1}{e}\right)}\right| = \sqrt{2}\left|\frac{e + \frac{1}{e}}{1-2}\right|$$

$$\therefore \tan\alpha = \sqrt{2}\left(e + \frac{1}{e}\right)$$

Mark: 1

(v) Hyperbola $\mathcal{H}$: $\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1$, with eccentricity e given by $9 = 16(e^2-1) \Rightarrow e = \frac{5}{4}$, and ellipse $\mathcal{E}$: $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$ are two such conics.

Using the symmetry in their graphs, at all of their points of intersection, the acute angle α between the tangents to the curves is given by $\tan\alpha = \sqrt{2}\left(\frac{5}{4} + \frac{4}{5}\right)$

Hence $\alpha \approx 71°$ (to the nearest degree)

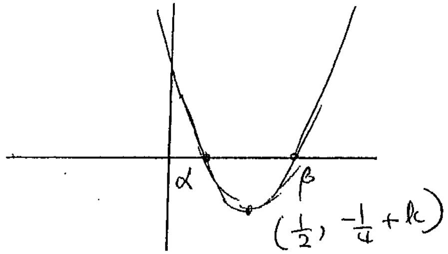
Mark: 1

SOLUTIONS	mark	COMMENTS

Question 5.

a) (i) The quadratic $P(x)=x^2-x+k$ has $P'(x)=2x-1$

ie it has a minimum turning point at $\left(\frac{1}{2}, -\frac{1}{4}+k\right)$



Since α and β are positive

$\alpha\beta > 0$

$\therefore k > 0.$ — (1)

Many students didn't show reasons for both conditions

and $-\frac{1}{4}+k<0$ or $\Delta>0$

$\therefore k < \frac{1}{4}$ $\quad 1-4k>0$

$k<\frac{1}{4}$ — (1)

ie $0 < k < \frac{1}{4}.$

(ii) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{\beta^2+\alpha^2}{\alpha^2\beta^2}=\frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}$

$=\frac{1-2k}{k^2}$ — (1)

Not very well explained by many.

Since $k<\frac{1}{4}$ $\quad 1-2k>\frac{1}{2}$

and $\frac{1}{k^2}>16$

$\therefore \frac{1-2k}{k^2} > \frac{1}{2}\times 16$ — (1)

>8

SOLUTIONS	mark	COMMENT

Question 5 (b)

$P(x)=x^3+ax+b.$

(i) $\alpha+\beta+\delta=0$ — (1)

(ii) $\alpha^2+\beta^2+\delta^2=(\alpha+\beta+\delta)^2-2(\alpha\beta+\beta\delta+\alpha\delta)$

$=0-2a$ — (1)

$=-2a$

(iii) If $P(x)$ has double root $P'(x)=3x^2+a$ has single root.

$\therefore P'(x)=0$

$\therefore 3x^2+a=0$

ie $x=\sqrt{\frac{-a}{3}}$ since positive. — (1)

Now $P\left(\sqrt{\frac{-a}{3}}\right)=\left(\sqrt{\frac{-a}{3}}\right)^3+a\sqrt{\frac{-a}{3}}+b.$

$=\frac{2a}{3}\sqrt{\frac{-a}{3}}+b.$ — (1)

Since $P\left(\sqrt{\frac{-a}{3}}\right)=0$

$\frac{2a}{3}\sqrt{\frac{-a}{3}}=-b$ — (1)

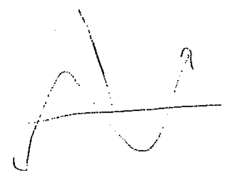
$\sqrt{\frac{-a}{3}}=\frac{-3b}{2a}.$

$\therefore$ double root is $-\frac{3b}{2a}.$

(iv) If 3 distinct roots $P\left(\sqrt{\frac{-a}{3}}\right)<0$ — (1)

$\frac{2a}{3}\sqrt{\frac{-a}{3}}+b<0$ since minimum value at $x=\sqrt{\frac{-a}{3}}$

$\frac{2a}{3}\sqrt{\frac{-a}{3}}<-b$



SOLUTIONS	Mark	COMMENT.
$\sqrt{\frac{-a}{3}} > \frac{3b}{2a}$ since $a<0$	1	
$-\frac{a}{3} > \frac{9b^2}{4a^2}$		
$-4a^3 > 27b^2$		
$-4a^3 - 27b^2 > 0$.	1	
$4a^3 + 27b^2 < 0$.		
OR $P\left(\frac{-3b}{2a}\right) < 0$		
$\left(\frac{-3b}{2a}\right)^3 + a\left(\frac{-3b}{2a}\right) + b < 0$		
$-\frac{27b^3}{8a^3} - \frac{3b}{2} + b = 0$		
$-27b^3 - 4a^3b > 0$ since $a<0$, $a^3<0$		
$4a^3b + 27b^3 < 0$		
$4a^3 + 27b^2 < 0$		