SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2011
YEAR 11
HALF YEARLY EXAMINATION

Mathematics Accelerated

General Instructions

- Reading Time – 5 Minutes
- Working time – 90 Minutes
- Write using black or blue pen.
- Pencil may be used for diagrams.
- Board approved calculators maybe used.
- Each section is to be returned in a separate bundle.
- All necessary working should be shown in every question if full marks are to be awarded.
- Marks may not be awarded for untidy or badly arranged work.
- Answer in simplest form unless otherwise stated.

Total Marks – 120

- Attempt questions 1 – 8

Examiner: *E Choy*

Start a new answer booklet

Question One (15 marks)

(a) (i) Evaluate $(-2)-(-5)$ 1

(ii) Evaluate $x^4 - x^3 + 2$ when $x = -2$ 1

(b) (i) Solve $(2x+3)(x-4)=0$ 2

(ii) Solve $2x = -7(500-x)$ 1

(c) Factorise completely $3a^2 - 12$ 2

(d) (i) Write down the exact value of $\cos 45^\circ$ 1

(ii) Solve $\tan x = 1$ for $0^\circ \le x \le 360^\circ$ 2

(e) Simplify $\frac{x}{4} + \frac{3x-1}{3}$ 2

(f) If $m_1 = 34$, $m_2 = 7, M = 53$ and $g = 9.8$, find correct to 4 significant figures, the value of 2

$$\left(\frac{m_1 - m_2}{M + m_1 + m_2}\right)g$$

(g) Solve $7 - 4x > 12$ 1

End of Question One

Start a new answer booklet

Question Two (20 Marks)

(a) Factorise completely each of the following:

(i) $5x^2y-10xy^2-5xy$ 1

(ii) $8a^3-27$ 1

(iii) $3x^2+4x-7$ 1

(iv) $x^2-4z^2+9y^2-6xy$ 1

(v) m^4+m^2-2 1

(vi) $a^{12}+b^{12}$ 2

(b) Simplify:

(i) $\frac{2}{m^2-4}-\frac{1}{m^2-3m+2}$ 2

(ii) $\left(\frac{a}{b}-\frac{a-b}{a+b}\right)\div\left(\frac{b}{a}-\frac{a+b}{a-b}\right)$ 2

(c) If $x+\frac{1}{x}=3$, find:

(i) $x^2+\frac{1}{x^2}$ 2

(ii) $x^3+\frac{1}{x^3}$ 2

(d) If $\frac{1}{u}+\frac{1}{v}=\frac{1}{a}$ and $u+v=k$ express in terms of a and k

(i) uv 1

(ii) u^2+v^2 1

(iii) $(u-v)^2$ 1

(e) Express $\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}$ in the form of $a+b\sqrt{6}$ 2

End of Question Two

Start a new answer booklet

Question Three (12 Marks)

(a) On a number plane mark the points $L(-2,-1)$, $M(0,3)$ and $N(4,0)$. 1

(b) Find the gradient of MN. 1

(c) Show that the equation of MN is $3x+4y-12=0$. 2

(d) Show, algebraically, that the midpoint of LN is $\left(1,-\frac{1}{2}\right)$. Call this point D. 1

(e) Find the point K such that D is the midpoint of MK. 1

(f) What type of quadrilateral is $KLMN$? Give a reason for your answer 2

(g) Find the perpendicular distance from L to MN. 2

(h) Find the area of $KLMN$. 2

End of Question Three

Start a new answer booklet

Question Four (15 Marks)

(a) Sketch, on one diagram, the region of the plane defined by the following inequations: 3

$$y<|x|,$$
$$y>0,$$
$$1<x<2$$

(b) The points A and B have coordinates $A(4,2)$ and $B(-2,-8)$. Find the locus of a point $P(x,y)$ that moves so that $\angle APB$ is a right angle. 3

(c) Find the equation of a straight line which passes through the intersection of the lines $L_1: 5x-7y+12=0$ and $L_2: 2x+3y-1=0$ and has an inclination of 135°. 4

(d) L_1 is a straight line with slope $-\frac{3}{4}$ passing through the point $A(3,0)$.

(i) Find the equation of L_1 2

(ii) Find the coordinates of the points P and Q on the line $L_2: x-y=0$ with a perpendicular distance 1 unit from L_1. 3

End of Question Four

Start a new answer booklet

Question Five (15 Marks)

(a) For which of the following equations is y a function of x (give reasons)

(i) $x^2+y=3$ 1

(ii) $\sqrt{y+1}=\frac{x}{1-x}$ 2

(b) Find the domain and range of:

(i) $a(x)=\frac{1}{\sqrt{3-x^2}}$ 2

(ii) $h(x)=\sqrt{x}+\sqrt{x-4}$ 2

(c) If $f(x)=ax+b$ show that $f(x^2)-2f(xy)+f(y^2)=a(x-y)^2$ 3

(d) Sketch $y=\begin{Bmatrix} 0, \text{ if } x<-1 \\ 0\le y\le 2 \text{ if } x=-1 \\ 2, \text{ if } x>-1 \end{Bmatrix}$ 2

Is this relation a function? Explain

(e) Sketch the graph of $y=x^2+3|x|+2$ 2

(f) If $f(x-1)=x^2-1$, find $f(x)$ for all real values of x. 1

End of Question Five

Start a new booklet

Question Six (15 Marks)

(a) The sum of the first n terms of a certain series is n^2+n. Find a formula for the n^{th} term of the series. 1

(b) $a-2,\ a,\ a+8$ are the first three terms of a geometric series. Find a and the sum of 5 terms. 2

(c) Find an expression for the sum $a_1+a_2+a_3+....+a_n$ where the term a_k is defined by: $a_k=1+2^k+2k$, for $k=1,2,3,....,n$ 3

(d) Find the value of r if $\sum_{k=1}^{\infty} 3r^{k-1}=\frac{64}{9}$ 2

(e) The Holden Car Company offers a loan of $50,000 on any of their cars purchased before 31st May, 2003. The loan attracts an interest of just $\frac{1}{2}$% per month, and to celebrate Holden's 75 years in Australia the Company also offers an interest free period for the first six months. However, the first repayment is due at the end of the first month.

A customer takes out the loan and agrees to repay the loan over ten years by making 120 equal monthly repayments of M. Let A_n be the amount owing at the end of the n^{th} repayment (in $), then:

(i) Show that $A_6=50,000-6M$. 1

(ii) Show that $A_8=(50,000-6M)\times 1.005^2-M(1.005+1)$. 2

(iii) Hence, show that $A_{120}=(50,000-6M)\times 1.005^{114}-M\times\frac{(1.005^{114}-1)}{1.005-1}$ 2

(iv) Hence, show that $M=\dfrac{50,000\times 1.005^{114}}{6\times 1.005^{114}+\dfrac{1.005^{114}-1}{0.005}}$ 1

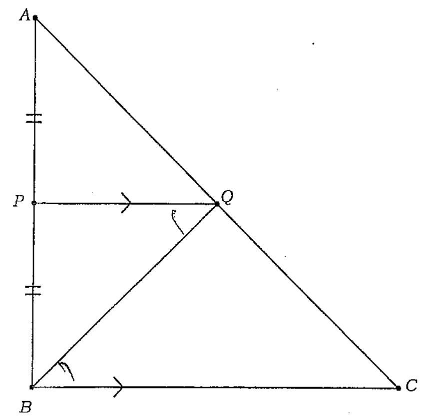
(v) Finally, find the value of the monthly repayment to the nearest cent. 1

End of Question Six

Start a new booklet

Question Seven (13 Marks)

(a)



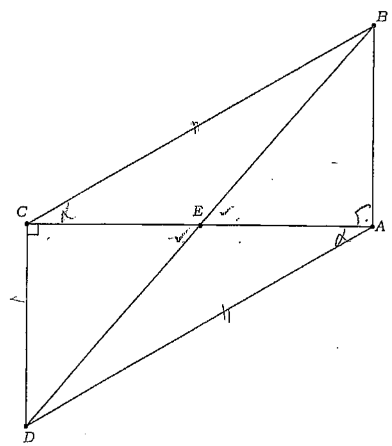
ΔABC has a right angle at B. P is the mid-point of AB and PQ is parallel to BC.

(i) Prove that $\angle APQ=90^\circ$. 2

(ii) Prove $\Delta AQP\equiv\Delta BQP$. 3

(iii) Prove $BQ=QC$ 2

(b)



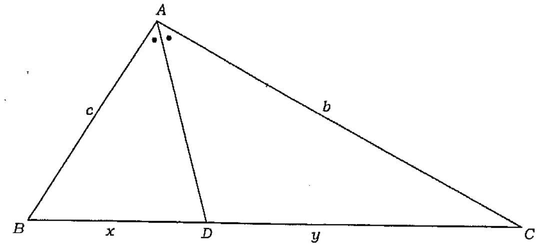
$ABCD$ is a parallelogram with diagonal AC perpendicular to CD. The two diagonals intersect at E. Use Pythagoras's theorem to show that $DE^2+3EA^2=AD^2$ 6

End of Question Seven

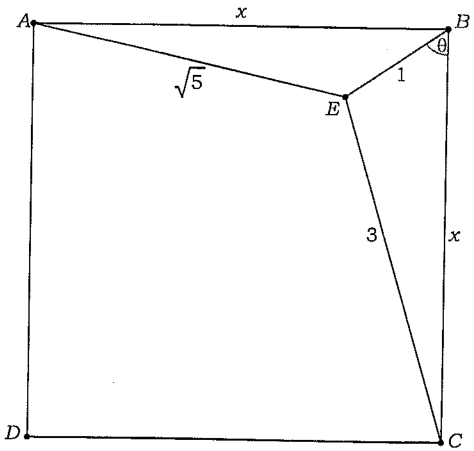
Start a new answer booklet

Question Eight (15 Marks)

(a) In ΔABC, AD bisects $\angle BAC$. If $BD = x$ and $DC = y$, prove that $bx = cy$. 4



(b)



$ABCD$ is a square of side $x$cm. E is a point inside the square such that $AE = \sqrt{5}, BE = 1, CE = 3$ and $\angle EBC = \theta$.

(i) By considering ΔCBE, show that $\cos\theta = \dfrac{x^2 - 8}{2x}$ 2

(ii) By considering ΔABE, show that $\sin\theta = \dfrac{x^2 - 4}{2x}$ 2

(iii) Hence or otherwise find x. 4

(iv) Find the exact value of ED. 3

End of Question Eight
End of Examination.

S.B.H.S – Accelerated Yr 11 Half Yrly 2011 – Solutions.

Q1 (a) (i) $(-2)-(-5) = 3$

(ii) $x^4 - x^3 + 2$

$= (-2)^4 - (-2)^3 + 2$

$= 16 - (-8) + 2$

$= 26$

(b) (i) $(2x+3)(x-4)=0$

$x = 4, -\frac{3}{2}$

(ii) $2x = -7(500-x)$

$2x = -3500 + 7x$

$3500 = 5x$

$x = 700$

(c) $3a^2 - 12 = 3(a^2-4)$

$= 3(a+2)(a-2)$

(d) (i) $\cos 45° = \frac{1}{\sqrt{2}}$

(ii) $\tan x = 1 \qquad 0 \le \theta \le 360$

$x = 45, 225$

(e) $\frac{x}{4} + \frac{3x-3}{3}$

$= \frac{3x + 12x - 4}{12}$

$= \frac{15x-4}{12}$

(f) $\left(\frac{m_1 - m_2}{m + m_1 + m_2}\right) g = \left(\frac{34-7}{53+7+34}\right) 9.8$

$= \left(\frac{27}{94}\right) 9.8$

$= 2.815$

(g) $7 - 4x > 12$

$-4x > 5$

$x < -\frac{5}{4}$

$x < -1\frac{1}{4}$

2(a) (i) $5x^2y - 10xy^2 - 5xy$

$= 5xy(x - 2y - 1)$ 1

(ii) $8a^3 - 27 = (2a-3)((2a)^2 + 2a \times 3 + 3^2)$ 1

$= (2a-3)(4a^2 + 6a + 9)$

(iii) $3x^2 + 4x - 7$ $\quad \times\ -21 \quad +\ 4 \quad 7, -3$

$= 3x^2 + 7x - 3x - 7$

$= x(3x+7) - 1(3x+7)$ 1

$= (3x+7)(x-1)$

(iv) $x^2 - 4z^2 + 9y^2 - 6xy$

$= x^2 - 6xy + 9y^2 - 4z^2$ 1

$= (x-3y)^2 - 4z^2$

$= (x - 3y + 2z)(x - 3y - 2z)$

(v) $m^4 + m^2 - 2$ $\quad \times\ -2 \quad +\ 1 \quad 2, -1$

$= (m^2+2)(m^2-1)$ 1

$= (m^2+2)(m-1)(m+1)$

(vi) $a^{12} + b^{12} = (a^4)^3 + (b^4)^3$

$= (a^4+b^4)(a^8 - a^4b^4 + b^8)$ 2

(b) (i) $\dfrac{2}{m^2-4} - \dfrac{1}{m^2-3m+2}$

$= \dfrac{2}{(m-2)(m+2)} - \dfrac{1}{(m-2)(m-1)}$

$= \dfrac{2(m-1) - 1(m+2)}{(m-2)(m+2)(m-1)}$

$= \dfrac{2m-2-m-2}{(m-2)(m+2)(m-1)}$

$= \dfrac{m-4}{(m-2)(m+2)(m-1)}$ 2

(ii) $\left(\dfrac{a}{b} - \dfrac{a-b}{a+b}\right) \div \left(\dfrac{b}{a} - \dfrac{a+b}{a-b}\right)$

$= \dfrac{a(a+b) - b(a-b)}{b(a+b)} \times \dfrac{a(a-b)}{b(a-b) - a(a+b)}$

$= \dfrac{a^2+ab-ab+b^2}{b(a+b)} \times \dfrac{a(a-b)}{ab-b^2-a^2-ab}$

$= \dfrac{a^2+b^2}{b(a+b)} \times \dfrac{a(a-b)}{-b^2-a^2}$

$= -\dfrac{a(a-b)}{b(a+b)} = \dfrac{ab-a^2}{ab+b^2}$ 2

(c) $x + \dfrac{1}{x} = 3$

(i) $\left(x+\dfrac{1}{x}\right)^2 = x^2+2+\dfrac{1}{x^2}$

$\therefore 9 = x^2 + \dfrac{1}{x^2} + 2$

$\therefore x^2 + \dfrac{1}{x^2} = 7$ 2

(ii) $\left(x+\dfrac{1}{x}\right)^3 = x^3 + 3x + 3\left(\dfrac{1}{x}\right) + \dfrac{1}{x^3}$

$\therefore 27 = x^3 + \dfrac{1}{x^3} + 3\left(x+\dfrac{1}{x}\right)$

$\therefore 27 = x^3 + \dfrac{1}{x^3} + 9$ 2

$\therefore x^3 + \dfrac{1}{x^3} = 18$

(d) $\dfrac{1}{u} + \dfrac{1}{v} = \dfrac{1}{a}$ $\quad u+v=k$

(i) $\dfrac{v+u}{uv} = \dfrac{1}{a}$.

$\therefore \dfrac{uv}{v+u} = a$

$\therefore \dfrac{uv}{k} = a$ 1

$\therefore uv = ak$

(ii) $\left(\dfrac{v+u}{uv}\right)^2 = \dfrac{1}{a^2}$

$\therefore \dfrac{v^2+2uv+u^2}{u^2v^2} = \dfrac{1}{a^2}$.

$\therefore \quad v^2+2uv+u^2 = \dfrac{u^2v^2}{a^2}$

$\therefore \quad v^2+u^2 = \dfrac{u^2v^2}{a^2} - 2uv$

$= \dfrac{a^2k^2}{a^2} - 2ak$

$= k^2 - 2ak$

(iii) $(u-v)^2 = \cancel{u}^2 - 2uv + v^2$

$= u^2+v^2 - 2uv$

$= k^2-2ak - 2ak$

$= k^2 - 4ak$

(e) $\dfrac{3\sqrt{2} - 2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} = \dfrac{(3\sqrt{2}-2\sqrt{3})^2}{9\times 2 - 4\times 3}$

$= \dfrac{18 - 12\sqrt{6} + 12}{6}$

$= \dfrac{30 - 12\sqrt{6}}{6}$

$= 5 - 2\sqrt{6}$, (or $5 + -2\sqrt{6}$)

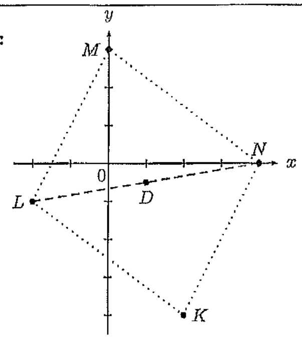
2

/20

2011 Accelerated Mathematics Half-Yearly: Solutions— Question 3

3. (a) On a number plane mark the points $L(-2, -1)$, $M(0, 3)$ and $N(4, 0)$. $\boxed{1}$

Solution:



(b) Find the gradient of MN. $\boxed{1}$

Solution: Gradient of $MN = \dfrac{0-3}{4-0}$,
$= -\dfrac{3}{4}$.

(c) Show that the equation of MN is $3x + 4y - 12 = 0$. $\boxed{2}$

Solution: $y - 0 = -\frac{3}{4}(x-4)$,
$4y = -3x + 12$,
i.e., $3x + 4y - 12 = 0$ is the equation of MN.

(d) Show, algebraically, that the midpoint of LN is $\left(1, -\frac{1}{2}\right)$. Call this point D. $\boxed{1}$

Solution: Midpoint of $LN = \left(\dfrac{-2+4}{2}, \dfrac{-1+0}{2}\right)$,
$= \left(1, -\frac{1}{2}\right)$.

(e) Find the point K such that D is the midpoint of MK. $\boxed{1}$

Solution: K is the point $\left(1 + (1-0), -1/2 + (-1/2 - 3)\right) = (2, -4)$.

(f) What type of quadrilateral is $KLMN$? Give a reason for your answer. $\boxed{2}$

Solution: $KLMN$ is a parallelogram because the diagonals bisect each other.

(g) Find the perpendicular distance from L to MN. $\boxed{2}$

Solution: $$\text{Distance} = \frac{|3(-2)+4(-1)-12|}{\sqrt{3^2+4^2}}, \quad = \frac{22}{5}.$$

(h) Find the area of $KLMN$. $\boxed{2}$

Solution:
$$\text{Length of } MN = \sqrt{3^2+4^2}, = 5.$$
$$\therefore \text{ Area of parallelogram } KLMN = 5 \times \frac{22}{5}, \text{ (base×height)} = 22.$$

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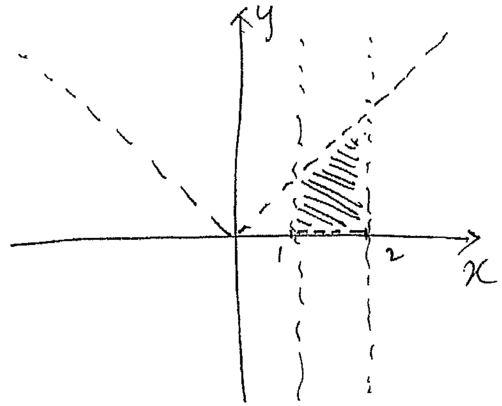
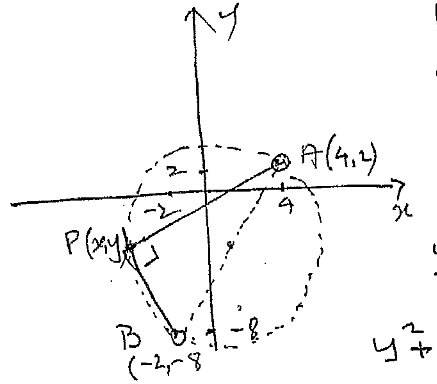
Question 4

(a)



[3]

(b)



Let $P(x,y)$ lie on the locus.

$A\hat{P}B = 90^\circ$

$\therefore m_{PA} \cdot m_{PB} = -1$

$\frac{y-2}{x-4} \cdot \frac{y+8}{x+2} = -1$

$y^2+6y-16 = -(x^2-2x-8)$

$(x-1)^2+(y+3)^2 = 34$

[3] Centre $(1,-3)$ $r = \sqrt{34}$

(c) Line is of the form $5x-7y+12+\lambda(2x+3y-1)=0$

$\alpha = 135^\circ$, $\therefore m = \tan\alpha = -1$ so $-A/B = -1$

ie $\frac{-(5+2\lambda)}{-7+3\lambda} = -1$

$\therefore \lambda = 12$

$\therefore$ Line is

$5x-7y+12+12(2x+3y-1)=0$

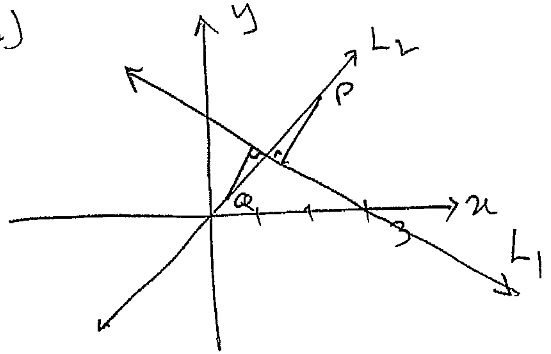
$29x+29y=0$

$x+y=0$ [4]

$(-1,1) = [2]$

2

(d)



(i) $y = mx + b$

$y = -\frac{3}{4}x + b$

But $(3, 0)$ lies on it.

$0 = -\frac{3}{4} \times 3 + b$

$b = \frac{9}{4}$

$\therefore L_1: y = -\frac{3}{4}x + \frac{9}{4}$

or $3x + 4y - 9 = 0$

[2]

(ii) P, Q are such that $x = y$.

$\therefore d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$

$1 = \frac{|3x_1 + 4x_1 - 9|}{5}$

$1 = \frac{|7x_1 - 9|}{5}$

$|7x_1 - 9| = 5$

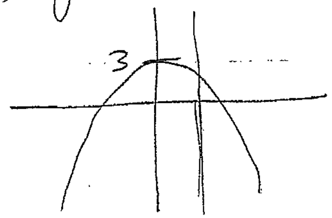
$x_1 = 2$ or $\frac{4}{7}$

$\therefore P(2, 2)\quad Q\left(\frac{4}{7}, \frac{4}{7}\right)$. [3]

Solutions

Yr 11 - 2011 - Half Yearly. (Q5)

(a) (i) $y = 3 - x^2$



Yes - for each x value there is at most one y value. ✓ 1

or passes vertical line test

(ii) $\sqrt{y+1} = \frac{x}{1-x}$

$y + 1 = \frac{x^2}{(1-x)^2}$

$y = \left(\frac{x}{1-x}\right)^2 - 1$ ✓

Yes - again, for each x value there is at most one y value. 2

(b) (i) $a(x) = \frac{1}{\sqrt{3 - x^2}}$

Domain $3 - x^2 > 0$

$-x^2 < -3$

$x^2 > 3$

$-\sqrt{3} < x < \sqrt{3}$ ✓

Range $a(x) = \frac{1}{+ve}$ $\therefore y > 0$.

Also, As $x \to 0$, $a(x)$

When $x = 0$, $a(x)$ is a min and $y = \frac{1}{\sqrt{3}}$

$\therefore y \geq \frac{1}{\sqrt{3}}$ ✓ 2

5 (b) (ii) $h(x) = \sqrt{x} + \sqrt{x-4}$

Domain $x \geq 4$ ✓

Range When $x = 4$, $h(x) = 2$ 2

$\therefore h(x) \geq 2$ ✓

(c) $f(x) = ax + b$

show $f(x^2) - 2f(xy) + f(y^2) = a(x-y)^2$

LHS. $f(x^2) - 2f(xy) + f(y^2)$

$= ax^2 + b - 2(axy + b) + ay^2 + b$ ✓

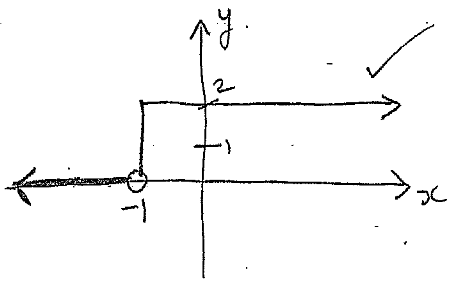
$= ax^2 + \cancel{b} - 2axy - \cancel{2b} + ay^2 + \cancel{b}$

$= ax^2 - 2axy + ay^2$

$= a(x^2 - 2xy + y^2)$ ✓ 3

$= a(x-y)^2 = RHS$ # ✓

(d) Sketch $y = \begin{cases} 0 \text{ if } x < -1 \\ 0 \leq y \leq 2 \text{ if } x = -1 \\ 2 \text{ if } x > -1 \end{cases}$



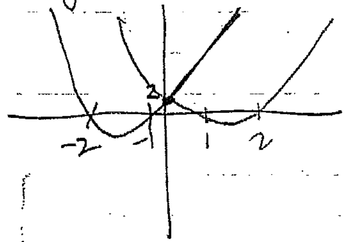
✓

No – not a function as there are many values of y for $x = -1$. ✓

2

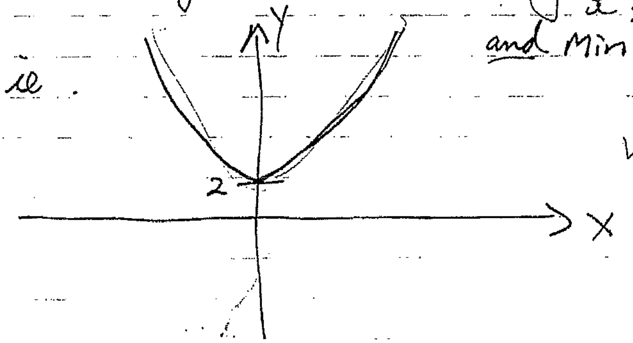
(e) For $y = x^2 + 3x + 2 = (x+2)(x+1)$ If $x > 0$,

and $y = x^2 - 3x + 2 = (x-2)(x-1)$ if $x < 0$ ✓



But for $y = x^2 + 3|x| + 2$ all y values are +ve and will be a reflection in y axis of +ve y values for $x \geq 0$ and Min $y = 2$.

ie.



✓ 2

(f) $f(x-1) = x^2 - 1$

$= (x-1)(x+1)$

$= (x-1)(x-1+2)$

$\therefore f(x) = x(x+2)$. ✓ 1

[YR 11 ACCN. 2011] (1)

Q6. (a) $T_n = S_n - S_{n-1}$

$= n^2+n - [(n-1)^2 + (n-1)]$

$= n^2+n - [n^2-2n+1 + n-1]$

$= n^2+n - [n^2-n]$

$= n^2+n-n^2+n$

$\boxed{= 2n.}$

(b) now $a^2 = (a-2)(a+8)$

$a^2 = a^2+6a-16$

$6a = 16$

$\boxed{a = 8/3.}$ (NB $a = T_2$)

now $r = \frac{T_2}{T_1} = \frac{8/3}{2/3} = 4.$ (NB $T_1 = \frac{2}{3}$)

$\therefore S_5 = \frac{\frac{2}{3}(4^5-1)}{4-1}$

$= \frac{2}{3} \times \frac{1023}{3}$

$\therefore \boxed{S_5 = \frac{682}{3}}$

(c) $\sum_{k=1}^{n}(1+2^k+2k) = \sum_{k=1}^{n}(2k+1) + \sum_{k=1}^{n}2^k$

$= \frac{n}{2}(3+2n+1) + \frac{2(2^n-1)}{2-1}$

$= \frac{n}{2}(2n+4) + 2(2^n-1)$

$= n(n+2) + 2(2^n-1)$

$= \boxed{n^2+2n+2^{n+1}-2.}$

Q6 (CONTD) (2)

(d) $\frac{3}{1-r} = \frac{64}{9}$

$\therefore \frac{1-r}{3} = \frac{9}{64}$

$64-64r = 27$

$1-r = \frac{27}{64}$

$r = 1-\frac{27}{64}$

$\boxed{r = \frac{37}{64}}$

(e) (i) $A_6 = 50000 - 6M.$

$A_7 = (50\,000-6M)1.005 - M.$

(ii) $A_8 = A_7 \times 1.005 - M$

$= (50\,000-6M)1.005^2 - M \times 1.005 - M$

$= (50\,000-6M)1.005^2 - M(1+1.005)$

(iii) $A_{120} = (50000-6M)1.005^{114} - M(1+1.005+\ldots+1.005^{113})$

$= (50\,000 - 6M)(1.005)^{114} - M\frac{(1.005^{114}-1)}{1.005-1}$

(iv) now $A_{120} = 0 \therefore (50\,000-6M)1.005^{114} = M\frac{(1.005^{114}-1)}{0.005}$

$\therefore 50\,000(1.005)^{114} - 6M(1.005)^{114} = M\frac{(1.005^{114}-1)}{0.005}$

$\Rightarrow M\left[\frac{(1.005^{114}-1)}{0.005} + 6(1.005)^{114}\right] = 50\,000(1.005)^{114}$ $\Rightarrow$

Q6 (CONTD)

(3)

$$\therefore \boxed{M = \frac{50\,000(1.005)^{114}}{6(1.005)^{114} + \frac{1.005^{114}-1}{0.005}}}$$

[a lot of work for 1 mark]

[NOT MANY were able to do this part]

(v) $\boxed{\$539.18}$ (calculator question)

QUESTION 7.

(a) (i) $\angle APQ = 90°$ (corres. $\angle$s $PQ \parallel BC$) 2

(ii) In Δs AQP & BQP 3

S: $AP = PB$ (given)

A: $\angle APQ = \angle BPQ = 90°$

S: PQ is common

$\therefore \Delta AQP \equiv \Delta BQP$ (SAS).

(iii) Let $\angle PBQ = \alpha$ and $\angle QBC = \beta$. 2

$\alpha + \beta = 90°$ (complementary $\angle$s)

$\angle PQB = \beta$ ($\angle$ sum Δ)

$\angle PAQ = \alpha$ (corres. $\angle$s in congruent Δs).

$\angle AQP = \beta$ (corres. $\angle$s in congruent Δs).

$\therefore \angle BQC = 180 - 2\beta$ (supplementary $\angle$s).

$\angle QCB = 180 - (180 - 2\beta) - \beta$ ($\angle$ sum Δ).

$= \beta$.

Thus ΔBQC is isosceles (base $\angle$s equal)

$\therefore BQ = CQ$.

(b) $DE^2 = CD^2 + CE^2$ ①

$DA^2 = CD^2 + CA^2$ ②   6

sub ① into ② for CD^2

$DA^2 = DE^2 - CE^2 + CA^2$

$DA^2 = DE^2 - CE^2 + (CE + EA)^2$ since $CA = CE + EA$.

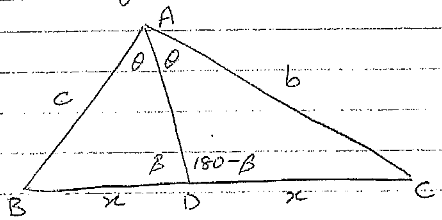
$= DE^2 - \cancel{CE^2} + \cancel{CE^2} + 2CE \cdot EF + EA^2$

$DA^2 = DE^2 + EA(2CE + EA)$ CE = EA since diagonals of a parallelogram bisect each other.

$DA^2 = DE^2 + 3EA^2$.

Question Eight

a)



let $B\hat{A}D = \theta$

$D\hat{A}C = \theta$ (given AD bisects $B\hat{A}C$)

let $A\hat{D}B = \beta$

$A\hat{D}C = 180 - \beta$ (angles on a straight line)

In ΔABD

$$\frac{\sin\theta}{x} = \frac{\sin\beta}{c}$$

$$\frac{\sin\theta}{\sin\beta} = \frac{x}{c}$$

In ΔADC

$$\frac{\sin\theta}{y} = \frac{\sin(180-\beta)}{b}$$

$$\frac{\sin\theta}{\sin(180-\beta)} = \frac{y}{b}$$

$$\frac{\sin\theta}{\sin\beta} = \frac{y}{b}$$

$$\therefore \frac{x}{c} = \frac{y}{b}$$

$bx = cy$.

b) i) In ΔCBE $\cos\theta = \dfrac{(1)^2 + (x)^2 - (3)^2}{2(1)(x)}$

$$= \frac{x^2 - 8}{2x}$$

ii) In ΔABE $\cos(90-\theta) = \dfrac{(1)^2 + (x)^2 - (\sqrt{5})^2}{2(1)(x)}$

$$\sin\theta = \frac{x^2 - 4}{2x}$$

iii) $\sin^2\theta + \cos^2\theta = 1$

$$\left(\frac{x^2-8}{2x}\right)^2 + \left(\frac{x^2-4}{2x}\right)^2 = 1$$

$$\frac{x^4-16x^2+64+x^4-8x^2+16}{4x^2} = 1$$

$$2x^4-24x^2+80 = 4x^2$$

$$2x^4-28x^2+80 = 0$$

$$x^4-14x^2+40 = 0$$

$\times$ | 40
$+$ | -14

$$(x^2-4)(x^2-10) = 0$$

$x^2 = 4$ $\quad$ $x^2 = 10$

$x = \pm 2$ $\quad$ $x = \pm\sqrt{10}$

using the triangle inequality

in $\triangle BEC$

$$(3)-(1) < x < (3)+(1)$$
$$2 < x < 4$$

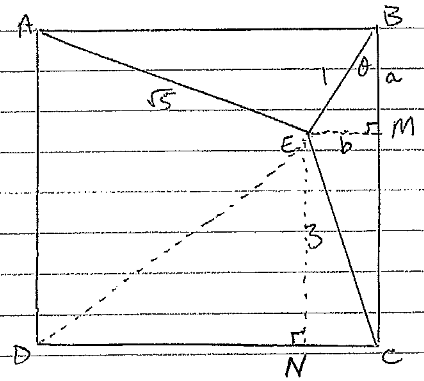
in $\triangle ABE$

$$(\sqrt{5})-(1) < x < (\sqrt{5})+(1)$$
$$\sqrt{5}-1 < x < \sqrt{5}+1$$
$$1{\cdot}24 < x < 3{\cdot}24$$

$\therefore \quad 2 < x < 3{\cdot}24$

The only possibility is $\underline{\underline{x = \sqrt{10}}} \approx 3{\cdot}16$

iv)



In $\triangle BEM$

$\cos\theta = \frac{a}{1}$ $\quad$ $\sin\theta = \frac{b}{1}$

$$a = \frac{(\sqrt{10})^2-8}{2(\sqrt{10})} \qquad b = \frac{(\sqrt{10})^2-4}{2(\sqrt{10})}$$

$$a = \frac{1}{\sqrt{10}} \qquad b = \frac{3}{\sqrt{10}}$$

In $\triangle DEN$

$$DN = \sqrt{10} - \frac{3}{\sqrt{10}} \qquad EN = \sqrt{10} - \frac{1}{\sqrt{10}}$$

$$= \frac{7}{\sqrt{10}} \qquad = \frac{9}{\sqrt{10}}$$

$$ED^2 = \left(\frac{9}{\sqrt{10}}\right)^2 + \left(\frac{7}{\sqrt{10}}\right)^2$$

$$ED^2 = \frac{81+49}{10}$$

$$ED^2 = 13$$

$$\underline{\underline{ED = \sqrt{13}}}$$