SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2010
Year 11 Yearly
Examination

Mathematics Extension Continuers

(3 Unit)

General Instructions

- Reading Time – 5 Minutes
- Working time – 90 Minutes
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators maybe used.
- All necessary working should be shown in every question.
- All answers to be given in simplified exact form unless otherwise stated.
- Hand in your answers in 3 separate bundles:
  Section A (Question 1 and Question 2),
  Section B (Question 3 and Question 4) and
  Section C (Question 5 and Question 6)

Total Marks – 82

- Attempt questions 1-6
- All questions are **NOT** of equal value.

Examiner: *P. Bigelow*

Section A – Start a new booklet.

Question 1 (14 marks). **Marks**

a) Solve $x^2+2x-8=0$. 1

b) Find T_{10} of $5+9+13+17+...$ 1

c) If $f(y)=9-y^2$, find: 2

(i) $f(-2)$

(ii) $f(y+1)$

d) State the domain of $f(x)=\sqrt{3-x}$. 1

e) Write down the equation of the parabola with focus (0,5) and directrix $y+5=0$. 1

f) Find x in the following: 2

(i) $\log_x 36=2$

(ii) $\log_8 128=x$

g) Find the exact value of $\tan 15^\circ$. 2

h) Solve: 4

(i) $|3x-1|>7$

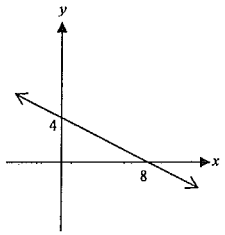
(ii) $\frac{4}{x}\leq\frac{3}{x+1}$

End of Question 1

Question 2 (12 Marks). **Marks**

a) Find the sum of the first 9 terms of $2-1+\frac{1}{2}-\frac{1}{4}+\ldots$ 1

b) Write down the equation of the line represented in the diagram: 2



c) State whether the following functions are ODD, EVEN or NEITHER: 3

(i) $f(x)=\frac{4}{9+x^2}$

(ii) $f(x)=\frac{x}{9+x}$

(iii) $f(x)=\frac{-x}{9+x^2}$

d) Solve $2^{3x+2}=64$. 2

e) By considering $0.\dot{5}\dot{4}$ as an infinite geometric series, express $0.\dot{5}\dot{4}$ as a fraction in simplest form. 2

f) Simplify: 2

(i) $\sin 5A\cos 2A-\cos 5A\sin 2A$

(ii) $2\sin 3\theta\cos 3\theta$

End of Question 2

End of Section A

Section B – Start a new booklet.

Question 3 (11 marks). **Marks**

a) Sketch the following on separate diagrams (showing essential features): 4

(i) $xy=-4$

(ii) $y=2^{-x}$

(iii) $y=\sqrt{4-x^2}$

(iv) $x^2+y^2-6y=0$

b) State whether the following quadratics are INDEFINITE, POSITIVE DEFINITE or NEGATIVE DEFINITE: 3

(i) $2x^2+3x+7$

(ii) $6-x-x^2$

(iii) x^2-9x-8

c) Solve: $4^x-9(2)^x+8=0$. 2

d) An interval PQ is divided externally in the ratio 4:3 by the point S. Find S, if P is (4,3) and Q is (-1,9). 2

End of Question 3

Question 4 (16 marks). **Marks**

a) Graph the region defined by the intersection of $y \geq x^2$ and $x+y \leq 2$. 2

b) Find the acute angle between the lines: $2x-y-5=0$ and $x-3y+3=0$. 2

c) Find $f'(x)$ in the following: 6

(i) $f(x)=3x^2-x+1$

(ii) $f(x)=(1-5x)^6$

(iii) $f(x)=x\sqrt{1+x}$

(iv) $f(x)=\dfrac{3x-1}{3x+2}$

d) If $\sin x=\dfrac{3}{4}$ and $\dfrac{\pi}{2} \leq x \leq \pi$, find the exact value of $\sin 2x$. 2

e) If α and β are the roots of $2x^2-6x-1=0$, find: 4

(i) $\alpha+\beta$

(ii) $\alpha\beta$

(iii) $\dfrac{1}{\alpha}+\dfrac{1}{\beta}$

(iv) $\alpha^2+\beta^2$

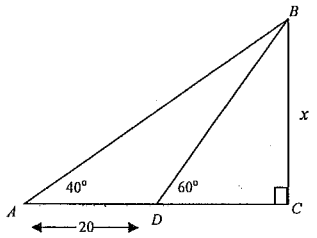
End of Question 4

End of Section B

Section C – Start a new booklet.

Question 5 (15 marks). **Marks**

a) Find the value of x correct to 3 significant figures. 3



b) Sketch $(x-2)^2=8(y+1)$, showing vertex, focus and directrix. 2

c) (i) Express $\sin\theta+\cos\theta$ in the form $R\sin(\theta+\alpha)$ where $R>0$ and $0^\circ<\alpha<90^\circ$. 4

(ii) Hence, solve $\sin\theta+\cos\theta=1$ for $0<\theta<2\pi$.

d) Find the general solution of $\tan x=1$ 2

e) (i) Find the axis of symmetry of $y=4+x-x^2$ 2

(ii) Hence, or otherwise, find the maximum value of $4+x-x^2$ 2

End of Question 5

Question 6 (14 marks). **Marks**

a) Prove the following: 4

(i) $\cos^4 x - \sin^4 x = \cos 2x$

(ii) $\dfrac{\sin 2A}{1-\cos 2A} = \cot A$

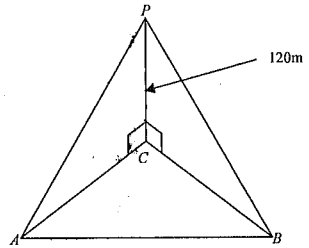
b) Given $5^x = 13$, find x correct to two decimal places. 2

c) A couple wish to save for a deposit on a home. They need to save \$20,000 over a 5 year period. They deposit \$$P$, every month, into an account which is paying 9%p.a., compounding monthly. 4

(i) Show that $20000 = P\left(1.0075 + 1.0075^2 + \ldots + 1.0075^{60}\right)$.

(ii) Find P to the nearest dollar.

d) Two boats at A and B are observed from the top P of a vertical cliff CP of height 120 metres. A is on a bearing of 195°T from the cliff and its angle of depression from P is 22°. B is on a bearing of 161°T from the cliff and its angle of depression from P is 27°. 4



(i) Find $\angle ACB$.

(ii) Use the cosine rule to find the distance between the boats (to the nearest metre).

End of Question 6.

End of Section C.

End of Examination.

Mathematics Extension Continuers 2010 – Section A:

Question 1:

a) $x^2 + 2x - 8 = 0$
$(x+4)(x-2) = 0$
$x = -4, \quad 2$

b) $a = 5,\ d = 4$
$T_{10} = 5 + 9 \times 4$
$T_{10} = 41$

c) $f(y) = 9 - y^2$
(i) $f(-2) = 9 - (-2)^2$
$f(-2) = 5$
(ii) $f(y+1) = 9 - (y+1)^2$
$= 9 - (y^2 + 2y + 1)$
$= 9 - y^2 - 2y - 1$
$= 8 - 2y - y^2$

d) $f(x) = \sqrt{3-x}$
Domain: $x \leq 3$

e) $x^2 = 20y$

f)
(i) $log_x 36 = 2$
$36 = x^2$
$x = 6$
(ii) $\log_8 128 = x$
$\log_8 8^{7/3} = x$
$\frac{7}{3}\log_8 8 = x$
$x = \frac{7}{3}$

g) $\tan 15° = \tan(45° - 30°)$
$= \dfrac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°}$
$= \dfrac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \times \frac{1}{\sqrt{3}}}$
$= \dfrac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}$
$= \dfrac{\sqrt{3}-1}{\sqrt{3}+1}$
$= 2 - \sqrt{3}$

Question 2:

a) $2-1+\frac{1}{2}-\frac{1}{4}+\cdots$

$a=2$

$r=-\frac{1}{2}$

$S_9=\frac{2\left(1-\left(-\frac{1}{2}\right)^9\right)}{1+\frac{1}{2}}$

$S_9=\frac{171}{128}$

b) $m=-\frac{1}{2}$

$y-0=-\frac{1}{2}(x-8)$

$2y=-x+8$

$x+2y-8=0$

c)

(i) $f(x)=\frac{4}{9+x^2}$

$f(-x)=\frac{4}{9+(-x)^2}$

$=\frac{4}{9+x^2}$

Since $f(x)=f(-x)$

$\therefore f(x)$ is even

(ii) $f(x)=\frac{x}{9+x}$

$f(-x)=\frac{-x}{9-x}$

$-f(x)=-\frac{x}{9+x}$

Since $f(x)\neq f(-x)\neq -f(x)$

$\therefore f(x)$ is neither.

(iii) $f(x)=\frac{-x}{9+x^2}$

$f(-x)=\frac{-(-x)}{9+(-x)^2}$

$=\frac{x}{9+x^2}$

$-f(x)=-\frac{-x}{9+x^2}$

$=\frac{x}{9+x^2}$

Since $f(-x)=-f(x)$

$\therefore f(x)$ is odd.

d) $2^{3x+2}=64$

$2^{3x+2}=2^6$

$3x+2=6$

$x=\frac{4}{3}$

QUESTION THREE

a)

i) $xy=-4$

ii) $y=2^{-x}$

iii) $y=\sqrt{4-x^2}$

iv) $x^2+y^2-6y=0$

$x^2+y^2-6y+9=9$

$\therefore x^2+(y-3)^2=9$

v) Positive Definite

vi) $y=-x^2-x+6$, $\Delta=25$

Indefinite

vii) $y=x^2-9x-8$, $\Delta=113$

Indefinite

b) $4^x-9(2^x)+8=0$

$(2^x)^2-9(2^x)+8=0$

let $u=2^x$

$u^2-9u+8=0$

$(u-8)(u-1)=0$

$u=8,1$

$x=3,0$

c) $P(4,3)$ $Q(-1,9)$

$m:n=4:-3$

$X=\frac{-3\times4+4\times-1}{1}$, $Y=\frac{-3\times3+4\times9}{1}$

$=-16$ $=27$

$(X,Y)=(-16,27)$

QUESTION FOUR

a) $(-2,4)$

$y\geq x^2$, $x+y\leq 2$

b) $y=2x-5$ $\tan A=2$

$y=\frac{1}{3}x+1$ $\tan B=\frac{1}{3}$

$\tan(A-B)=\frac{2-\frac{1}{3}}{1+\frac{2}{3}}=1$

$A-B=45^\circ=\frac{\pi}{4}$

c) i) $y=3x^2-x+1$ $y'=6x-1$

ii) $y=(1-5x)^6$ $y'=-30(1-5x)^5$

iii) $y=x(1+x)^{1/2}$, $y'=x.\frac{1}{2}(1+x)^{-1/2}+(1+x)^{1/2}$

$y'=\frac{x}{2\sqrt{1+x}}+\sqrt{1+x}$

$=\frac{x+2(1+x)}{2\sqrt{1+x}}=\frac{3x+2}{2\sqrt{1+x}}$

iv) $y=\frac{3x-1}{3x+2}$, $y'=\frac{(3x+2)3-3(3x-1)}{(3x+2)^2}$

$=\frac{7}{(3x+2)^2}$

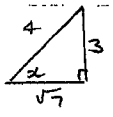
d) $\sin x=\frac{3}{4}$ $\frac{\pi}{2}\leq x\leq\pi$

$x\approx 131^\circ$, $2x\approx 263^\circ$ Hence $\sin 2x<0$

$\sin 2x=2\sin x\cos x$

$=2\times\frac{3}{4}\times\frac{-\sqrt{7}}{4}$

$=-\frac{3\sqrt{7}}{8}$



e) $2x^2-6x-1=0$, $\alpha+\beta=3$, $\alpha\beta=-\frac{1}{2}$

$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{3}{-\frac{1}{2}}=-6$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=9-2(-\frac{1}{2})$

$=10$

SECTION C

Q5

(a) $\tan 60° = \frac{x}{DC}$.

$DC = \frac{x}{\tan 60°}$

$\tan 40° = \frac{x}{20+DC}$.

So $(20+DC) = \frac{x}{\tan 40°}$.

$20 + \frac{x}{\tan 60°} = \frac{x}{\tan 40°}$

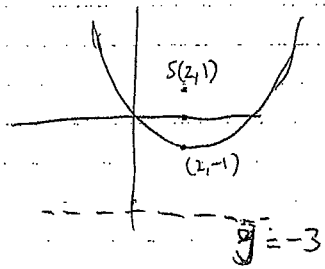
$20 = x\left(\frac{1}{\tan 40°} - \frac{1}{\tan 60°}\right)$.

$x = \frac{20}{\frac{1}{\tan 40°} - \frac{1}{\tan 60°}}$.

$= \frac{20(\tan 40 \tan 60)}{\tan 60 - \tan 40}$

$= 32.6$.

(b)



(c)(i) $R\sin(\theta+\alpha) = R\sin\theta\cos\alpha + R\sin\alpha\cos\theta$

$R\sin\alpha = 1$

$R\cos\alpha = 1$

$\tan\alpha = 1$.

$\alpha = 45°$.

$R^2 = 2$

$R = \sqrt{2}$.

$\sqrt{2}\sin(\theta+45°)$.

(ii) $\sqrt{2}\sin(\theta+45°) = 1$

$\sin(\theta+45°) = \frac{1}{\sqrt{2}}$.

~~$\theta+45° = 45°$~~)

$\theta + \frac{\pi}{4} = \frac{\pi}{4}, \frac{3\pi}{4}$)

$\theta = \frac{\pi}{2}$.

(d) $\tan x = 1$

$x = \frac{\pi}{4} + \pi n \qquad n \in \mathbb{Z}$

(e) (i) $y = 4 + x - x^2$.

$x = \frac{-b}{2a}$.

$x = \frac{-1}{-2}$.

$x = \frac{1}{2}$.

(ii) $4 + \frac{1}{2} - \frac{1}{4}$.

$= \$4\frac{3}{4}$.

Q6 (a) (i) $\cos^4 x - \sin^4 x = (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)$.

$= \cos^2 x - \sin^2 x$

$= \cos 2x$.

(ii) $\frac{\sin 2A}{1 - \cos 2A} = \frac{2\sin A \cos A}{\cos^2 A + \sin^2 A - \cos^2 A + \sin^2 A}$.

$= \frac{2\sin A \cos A}{2\sin^2 A}$

$= \frac{\cos A}{\sin A}$

$= \cot A$.

(b) $5^x = 13$.

$x = \frac{\log 13}{\log 5}$.

$= 1.60$.

(c) (i) $A_1 = P \times 1.0075$.

$A_2 = (A_1 + P)\,1.0075$.

$= P \times 1.0075^2 + P \times 1.0075$

$A_3 = P \times 1.0075^3 + P \times 1.0075^2 + P \times 1.0075$.

$A_{60} = P(1.0075^{60} + 1.0075^{59} + \ldots + 1.0075)$.

Sum $A_{60} = 20000$.

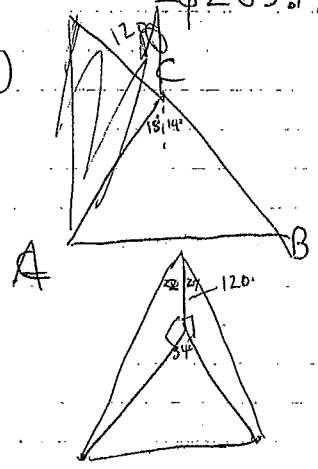
Then $20000 = P(1.0075 + \ldots + 1.0075^{60})$.

(ii) $20000 = P\left(\frac{1.0075(1 - (1.0075)^{60})}{1 - 1.0075}\right)$

$P = \frac{20000\,(-0.0075)}{1.0075(1 - 1.0075^{60})}$.

$P = \$263.19$.

(d)



(i) $34°$.

(ii) $\tan 22 = \frac{AC}{120} \Rightarrow AC = 120 \tan 22$

$\tan 27 = \frac{BC}{120} \Rightarrow BC = 120 \tan 27$.

$AB^2 = AC^2 + BC^2 - 2 \times AC \times BC \cos 34$.

$AB = 34.26$.