SYDNEY BOYS HIGH SCHOOL

MOORE PARK, SURRY HILLS

Year 9

Yearly Examination 2010

Mathematics

General Instructions

- Working time – 90 minutes
- Write using black or blue pen.
- Approved Calculators may be used.
- All necessary working MUST be shown in every question if full marks are to be awarded.
- Marks may not be awarded for untidy or badly arranged work.
- Write all answers in simplest exact form unless specified otherwise.
- If more space is required, clearly write the number of the QUESTION on one of the back pages and answer it there. Indicate that you have done so.
- Clearly indicate your class by placing an X in the space provided.

Examiner: *A. Fuller*

NAME:

Class	Teacher	
9MaA	Mr McQuillan	
9MaB	Ms Roessler	
9MaC	Ms Ward	
9MaD	Ms Kilmore	
9MaE	Ms Evans	
9MaF	Mr Gainford	
9MaG	Mr Hespe	

Question	Mark
1	/15
2	/15
3	/15
4	/14
5	/15
6	/14
7	/13
Total	/101

Question 1 (15 marks)

(a) Evaluate $6 - (3 - 8)$ [1]

(b) Write the following in ascending order: π, 3.1, $3.\dot{1}\dot{4}$, $\sqrt{11}$ [1]

(c) Convert $6\frac{1}{4}\%$ to a decimal. [1]

(d) Find 35% of 400 m. [1]

(e) Simplify the ratio 1000 : 150 [1]

(f) 12 L/h is equivalent to how many mL/s? [2]

(g) How many significant zeros are there in each of the following numerals? [2]

(i) 201000 (to the nearest hundred)

(ii) 0.0120

(h) Simplify the following: [4]

(i) $6a - a$

(ii) $a^6 \div a^2$

(iii) $(1\frac{3}{4})^2$

(iv) $\sqrt{5} + \sqrt{80} - 4\sqrt{3}$

(i) Convert the following: [2]

(i) 31 mm to cm.

(ii) 1.6 m^2 to cm^2.

Question 2 (15 marks)

(a) Express $\frac{2}{7}$ as a decimal. [1]

(b) Expand and simplify the following: [6]

(i) $2x - x(x - 3)$

(ii) $(2x + 1)(7 - x)$

(iii) $(2x^3)^3$

(iv) $(a - 3b)^2$

(c) Express the following in scientific notation: [2]

(i) 9310000

(ii) 0.00507

(d) Write an algebraic expression for 'a less than the square of b'. [1]

(e) Solve the following: [3]

(i) $3(x-2)=18$

(ii) $\frac{1-2x}{5} \geq -3$

(f) Write $52°29'54''$ in degrees, correct to 3 decimal places. [1]

(g) If $\sin\theta = 0.7$, and θ is acute. Find θ to the nearest degree. [1]

Question 3 (15 marks)

(a) Evaluate the following expressions if $a=3$, $b=\frac{1}{2}$ and $c=-2$ [2]

(i) $\frac{ac}{b}$

(ii) $(b+c)^2$

(b) Find the rule connecting x and y. [2]

x	1	2	3	4
y	-2	1	4	7

(c) If 10% of s is t, then what does s equal? [1]

(d) Simplify the following: [3]

(i) $10\sqrt{3} \times 5\sqrt{11}$

(ii) $(4a)^0 - 4a^0$

(iii) $\sqrt{81a^{36}}$

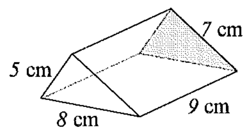
(e) Express the following with a rational denominator: [3]

(i) $\dfrac{2+\sqrt{3}}{3\sqrt{3}}$

(ii) $\dfrac{3\sqrt{3}}{2+\sqrt{3}}$

(f) Write $0.1\dot{6}$ as a fraction. [2]

(g) The area of the shaded end of this triangular prism is 17.5 cm^2. [2]



(i) Find the volume of the prism.

(ii) Find the surface area of the closed prism.

Question 4 (14 marks)

(a) The equation of a straight line is given by $4x - 2y + 12 = 0$. [2]

(i) What is the gradient of the line?

(ii) What is the x - intercept of the line?

(b) Fully factorise the following: [5]

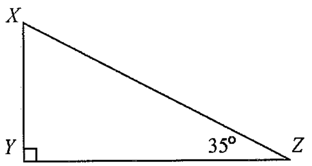
(i) $2a^2 + 6ab$

(ii) $x^2 + 3x - 28$

(iii) $(2x+1)^2 - (x+4)^2$

(c) Ronald has a jar containing jellybeans. Each jelly bean is either red, yellow or black. The ratio of red to yellow to black is 4 : 5 : 3. Ronald chooses a jelly bean at random. What is the probability that it is black? [2]

(d) Evaluate $\frac{YZ}{XZ}$, correct to 2 decimal places. [1]

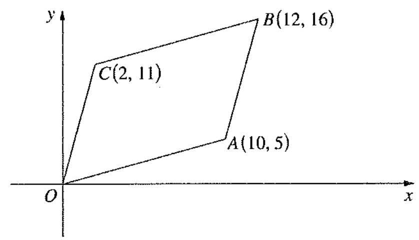


(e) In a draw there are four socks. Two are red and two are white. Two socks are taken out at random. What is the probability that two of the same colour are selected? [2]

(f) Calculate the volume of a cylinder given the height is 15cm and the area of the curved surface is 105 cm^2. [2]

Question 5 (15 marks)

(a) In the diagram below, A, B and C are the points $(10, 5)$, $(12, 16)$ and $(2, 11)$ respectively. [7]



(i) Calculate the distance AC giving your answer in exact form.

(ii) Find the midpoint of AC.

(iii) Show that $OB \perp AC$.

(iv) Find the midpoint of OB and hence explain why the quadrilateral $OABC$ is a rhombus.

(v) Hence, or otherwise, find the area of $OABC$.

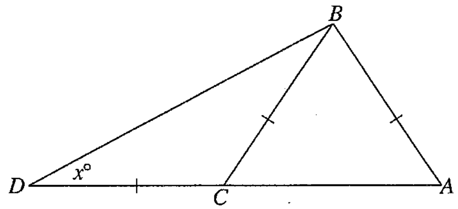
(b) B is 844 metres due south of A. C is due west of A. The bearing of C from B is 295°. 4

(i) Draw a diagram to represent this information.

(ii) Find the distance BC, to the nearest metre.

(iii) Find the bearing from C to B.

(c) In the diagram below, $AB = BC = CD$ and $\angle BDC = x°$ 4



(i) Prove that $\angle CAB = 2x°$

(ii) If $\angle ABD = 120°$, find the value of x.

Question 6 (14 marks)

(a) If $3^x = 5$. Evaluate the following: [4]

(i) 3^{x+2}

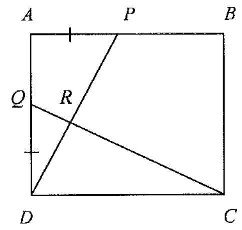
(ii) 3^{-x}

(iii) 27^x

(b) Show that the radius of a semicircle whose perimeter is numerically equal to its area is $\dfrac{2\pi + 4}{\pi}$. [3]

(c) On an island there are two types of inhabitants: Heros who always tell the truth and Villains who always lie. Four inhabitants are seated around a table. When each is asked "Are you a Hero or a Villain?", all four reply "Hero". When asked "Is the person on your right a Hero or a Villain?", all four reply "Villain". How many Heros are present? [2]

(d) $ABCD$ is a square. P lies on AB and Q lies in AD such that $AP = DQ$. [5]



(i) Prove $\triangle APD \equiv \triangle DQC$.

(ii) Show that $\angle PDC = \angle DQC$.

Question 7 (13 marks)

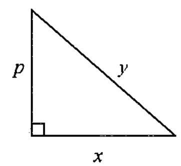
(a) On Monday Steven drove to work at an average speed of 70 km/h and arrived 1 minute late. On Tuesday, he left at the same time and drove an average speed of 75km/h and arrived 1 minute early. How long is his route to work? **3**

(b) If $\frac{1}{X} = \frac{1}{a} + \frac{1}{b}$, where $a, b > 0$ **4**

(i) Show that $X = \frac{ab}{a+b}$

(ii) Hence, find $\sqrt{\frac{a-X}{b-X}}$ in its simplest form.

(c) Larry wants to form some Pythagorean Triads (Three positive integers which satisfy Pythagoras' Theorem). He lets p be an odd prime and wants to find x and y (in terms of p) such that p, x and y form a Pythagorean Triad. **6**



(i) Prove that $x = y - 1$ and $x = p^2 - y$.

(ii) Larry used the two equations from part (i) and found that

$x = \frac{p^2 - 1}{2}$ (Do not prove this).

Prove that x is even.

(iii) Form a Pythagorean Triad which includes $p = 11$.

End of paper

Question 1 (15 marks)

(a) Evaluate $6 - (3 - 8)$ [1]

11

(b) Write the following in ascending order: π, 3.1, $3.\dot{1}\dot{4}$, $\sqrt{11}$ [1]

$3.1,\ 3.\dot{1}\dot{4},\ \pi,\ \sqrt{11}$

(c) Convert $6\frac{1}{4}\%$ to a decimal. [1]

0.0625

(d) Find 35% of 400 m. [1]

140 m

(e) Simplify the ratio 1000 : 150 [1]

20 : 3

(f) 12 L/h is equivalent to how many mL/s? [2]

$\frac{12\,000 \text{ mL}}{60 \times 60 \text{ s}} = 3\frac{1}{3} \text{ mL/s}$

(g) How many significant zeros are there in each of the following numerals? [2]

(i) 201000 (to the nearest hundred)

2

(ii) 0.0120

1

(h) Simplify the following: [4]

(i) $6a - a$

$5a$

(ii) $a^6 \div a^2$

a^4

(iii) $(1\frac{3}{4})^2$

$\frac{49}{16} = 3\frac{1}{16}$

(iv) $\sqrt{5} + \sqrt{80} - 4\sqrt{3}$

$\sqrt{5} + 4\sqrt{5} - 4\sqrt{3} = 5\sqrt{5} - 4\sqrt{3}$

(i) Convert the following: [2]

(i) 31 mm to cm.

3.1 cm

(ii) 1.6 m^2 to cm^2.

$1.6 \times 100^2 = 16\,000 \text{ cm}^2$

Question 2 (15 marks)

(a) Express $\frac{2}{7}$ as a decimal. [1]

$0.\dot{2}8571\dot{4}$ 1

(b) Expand and simplify the following: [6]

(i) $2x - x(x-3)$ $= 2x - x^2 + 3x$

$= 5x - x^2$ 1

(ii) $(2x+1)(7-x)$ $= 14x - 2x^2 + 7 - x$

$= 13x - 2x^2 + 7$ 1

(iii) $(2x^3)^3$ $= 8x^9$ 2

(iv) $(a-3b)^2$ $= a^2 - 6ab + 9b^2$ 2

(c) Express the following in scientific notation: [2]

(i) 9310000 9.31×10^6

(ii) 0.00507 5.07×10^{-3}

(d) Write an algebraic expression for 'a less than the square of b'. [1]

$b^2 - a$

(e) Solve the following: [3]

(i) $3(x-2) = 18$

$3x - 6 = 18$

$3x = 24$

$x = 8$ 1

(ii) $\frac{1-2x}{5} \geq -3$

$1 - 2x \geq -15$

$-2x \geq -16$

$x \leq 8$

$x \leq 8$ 2

(f) Write 52°29′54″ in degrees, correct to 3 decimal places. [1]

52.498

(g) If $\sin\theta = 0.7$, and θ is acute. Find θ to the nearest degree. [1]

44°

Question 3 (15 marks)

(a) Evaluate the following expressions if $a = 3$, $b = \frac{1}{2}$ and $c = -2$ [2]

(i) $\frac{ac}{b}$ $= \frac{3 \times (-2)}{1/2} = -12$

(ii) $(b+c)^2$ $(1/2 - 2)^2 = (-3/2)^2 = \frac{9}{4}$

(b) Find the rule connecting x and y. [2]

x	1	2	3	4
y	−2	1	4	7

$y = 3x - 5$

(c) If 10% of s is t, then what does s equal? [1]

$0.1s = t$ $\therefore$ $s = 10t$

(d) Simplify the following: [3]

(i) $10\sqrt{3} \times 5\sqrt{11}$ $50\sqrt{33}$

(ii) $(4a)^0 - 4a^0$ $1 - 4 = -3$

(iii) $\sqrt{81a^{36}}$ $9a^{18}$

(e) Express the following with a rational denominator: [3]

(i) $\frac{2+\sqrt{3}}{3\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}+3}{9}$

(ii) $\frac{3\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{6\sqrt{3}-9}{4-3} = 6\sqrt{3}-9$

(f) Write $0.1\dot{6}$ as a fraction. [2]

$x = 0.1\dot{6}$

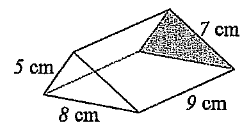
$10x = 1.\dot{6}$

$100x = 16.\dot{6}$

$\therefore 90x = 15$

$x = \frac{15}{90} = \frac{1}{6}$

(g) The area of the shaded end of this triangular prism is 17.5 cm². [2]



(i) Find the volume of the prism.

$V = 17.5 \times 9 = 157.50$

(ii) Find the surface area of the closed prism.

$SA = 17.5 \times 2 + 8 \times 9 + 7 \times 9 + 5 \times 9$

$= 215$

Year 9 Exam

Question 4 (14 marks)

(a) The equation of a straight line is given by $4x - 2y + 12 = 0$. [2]

(i) What is the gradient of the line?

$2y = 4x + 12$
$y = 2x + 6$
$m = 2$ ✓

(ii) What is the x - intercept of the line?

When $y = 0 \Rightarrow 2x + 6 = 0$
$x = \frac{-6}{2}$
$x = -3$ ✓

(b) Fully factorise the following: [5]

(i) $2a^2 + 6ab = 2a(a + 3b)$ ✓

(ii) $x^2 + 3x - 28 = (x - 4)(x + 7)$ ✓✓

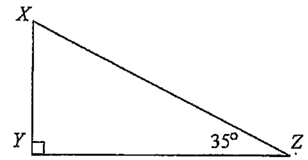
(iii) $(2x + 1)^2 - (x + 4)^2 = (2x+1 + x+4)(2x+1 - (x+4))$ ✓
$= (3x+5)(x-3)$ ✓

(c) Ronald has a jar containing jellybeans. Each jelly bean is either red, yellow or black. The ratio of red to yellow to black is $4 : 5 : 3$ (R Y B). Ronald chooses a jelly bean at random. What is the probability that it is black? [2]

12

$P(B) = \frac{3}{12} = \frac{1}{4}$ ✓

(d) Evaluate $\frac{YZ}{XZ}$, correct to 2 decimal places. [1]



$\cos 35^\circ = \frac{YZ}{XZ}$

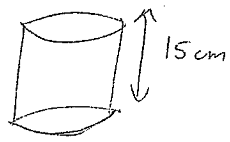
$\frac{YZ}{XZ} = 0.82$

(e) In a draw there are four socks. Two are red and two are white. Two socks are taken out at random. What is the probability that two of the same colour are selected? [2]

2R, 2W

$P(\text{2 red or 2 White}) = \frac{2}{4} \times \frac{1}{3} + \frac{2}{4} \times \frac{1}{3}$ ✓
$= \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$. ✓

(f) Calculate the volume of a cylinder given the height is 15cm and the area of the curved surface is 105 cm². [2]

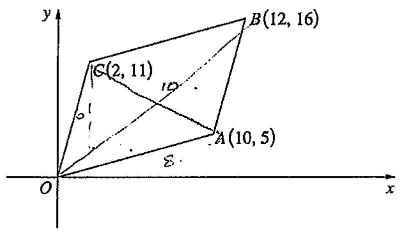


$2\pi rh = 105 \text{ cm}^2$
$2\pi r \times 15 = 105$
$r = \frac{105}{30\pi} = \frac{7}{2\pi}$ ✓

Then $V = \pi r^2 h$
$= \pi \times \frac{7}{2\pi} \times \frac{7}{2\pi} \times 15$
$= 58.49 \text{ cm}^3$. ✓

Question 5 (15 marks)

(a) In the diagram below, A, B and C are the points $(10, 5)$, $(12, 16)$ and $(2, 11)$ respectively. [7]



(i) Calculate the distance AC giving your answer in exact form.

$AC = \sqrt{(10-2)^2 + (5-11)^2}$

$= \sqrt{8^2 + (-6)^2}$

$= 10$

(1)

(ii) Find the midpoint of AC.

$\left(\frac{10+2}{2}, \frac{5+11}{2}\right)$

$= (6, 8)$

(1)

(iii) Show that $OB \perp AC$.

$m_{OB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{16-0}{12-0} = \frac{4}{3}$

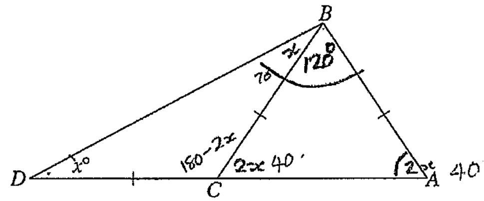
$m_{AC} = \frac{5-11}{10-2} = \frac{-6}{8} = \frac{-3}{4}$

(1)

$m_{OB} \times m_{AC} = \frac{4}{3} \times -\frac{3}{4} = -1$

$\therefore OB \perp AC$

(c) In the diagram below, $AB = BC = CD$ and $\angle BDC = x°$ [4]



(i) Prove that $\angle CAB = 2x°$

$\angle CBD = \angle BDC = x°$ (isos $\triangle BCD$)

$\angle BCA = \angle CBD + \angle BDC$ (external $\angle$, $\triangle$)

$= 2x$

$\therefore \angle CAB = 2x$ (isos $\triangle ABC$)

(3)

(ii) If $\angle ABD = 120°$, find the value of x.

$2x + 2x + (120 - x) = 180$

$3x = 180 - 120$

$x = 20$ ✓

(1)

(iv) Find the midpoint of OB and hence explain why the quadrilateral $OABC$ is a rhombus.

$mP_{OB} = \left(\frac{12-0}{2}, \frac{16-0}{2}\right) = (6,8)$ ①

OB + OC cross at right angles at the point (6,8) ∴ diagonals cross at right ∠'s + sides are equal ∴ rhombus ①

(v) Hence, or otherwise, find the area of $OABC$.

Area $= \frac{1}{2}xy$

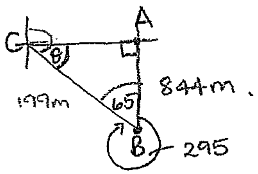
$AC = 10 = x$

$OB = \sqrt{12^2 + 16^2}$
$= \sqrt{400}$
$= 20$

$A = \frac{1}{2}(10)(20)$
$= 100 \text{ units}^2$ ②

(b) B is 844 metres due south of A. C is due west of A. The bearing of C from B is 295°. [4]

(i) Draw a diagram to represent this information.



①

(ii) Find the distance BC, to the nearest metre.

$\cos 65 = \frac{AB}{BC}$

$BC = \frac{AB}{\cos 65} = 1997m$ ①

$= \frac{84}{\cos 65°}$

(iii) Find the bearing from C to B.

$180 - 65 - 90 = 25°$ (∠ sum △)

Bearing B from C $= 90 + 25 = 115°T$ ②

Question 6 (14 marks)

(a) If $3^x = 5$. Evaluate the following: [4]

(i) 3^{x+2}

$5 \times 3^2 = 45$ 1

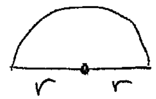
(ii) 3^{-x}

$(3^x)^{-1} = 5^{-1} = \frac{1}{5}$ 1

(iii) 27^x

$(3^x)^3 = 5^3 = 125$ 2

(b) Show that the radius of a semicircle whose perimeter is numerically equal to its area is $\frac{2\pi + 4}{\pi}$. [3]



$P = \pi r + 2r$

$A = \frac{\pi r^2}{2}$

$\frac{\pi r^2}{2} = \pi r + 2r$

$\pi r^2 = 2\pi r + 4r$

$\pi r = 2\pi + 4$

$r = \frac{2\pi + 4}{\pi}$

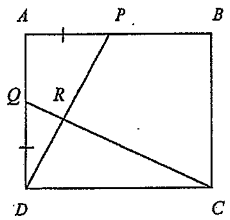
(c) On an island there are two types of inhabitants: Heros who always tell the truth and Villains who always lie. Four inhabitants are seated around a table. When each is asked "Are you a Hero or a Villain?", all four reply "Hero". When asked "Is the person on your right a Hero or a Villain?", all four reply "Villain". How many Heros are present? [2]

H

V V 2 heroes

H

(d) $ABCD$ is a square. P lies on AB and Q lies in AD such that $AP = DQ$. [5]



(i) Prove $\triangle APD \equiv \triangle DQC$.

AD = DC (given)

AP = DQ (given)

$\angle PAD = \angle QDC = 90°$ (given)

$\triangle APD \equiv \triangle DQC$ (SAS)

3

(ii) Show that $\angle PDC = \angle DQC$.

from (i)

$\angle PAD = \angle QDC$

$\angle DPA = \angle DQC$

$\angle QDP = \angle QCD$

$\angle PDC = 90° - \angle QDP$

$\angle DQC = 90° - \angle QCD$ 2

$\therefore \angle PDC = \angle DQC$

Question 7 (13 marks)

(a) On Monday Steven drove to work at an average speed of 70 km/h and arrived 1 minute late. On Tuesday, he left at the same time and drove an average speed of 75km/h and arrived 1 minute early. How long is his route to work? [3]

$S = \frac{D}{T}$

$T = \frac{D}{S}$

$\frac{d}{70} - \frac{d}{75} = \frac{2}{60}$

$d\left(\frac{75-70}{70 \times 75}\right) = \frac{1}{30}$

$d = \frac{1}{30} \times \frac{70 \times 75}{5}$

$= 35$km.

(b) If $\frac{1}{X} = \frac{1}{a} + \frac{1}{b}$, where $a, b > 0$ [4]

(i) Show that $X = \frac{ab}{a+b}$

2

$\frac{1}{X} = \frac{b+a}{ab}$

$X = \frac{ab}{a+b}$

(ii) Hence, find $\sqrt{\dfrac{a-X}{b-X}}$ in its simplest form.

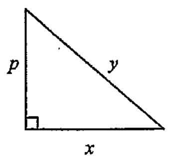
$$\sqrt{\frac{a-\frac{ab}{a+b}}{b-\frac{ab}{a+b}}} = \sqrt{\frac{\frac{a^2+ab-ab}{a+b}}{\frac{ab+b^2-ab}{a+b}}}$$

$$= \sqrt{\frac{a^2}{b^2}}$$

$$= \frac{a}{b}$$

2

(c) Larry wants to form some Pythagorean Triads (Three positive integers which satisfy Pythagoras' Theorem). He lets p be an odd prime and wants to find x and y (in terms of p) such that p, x and y form a Pythagorean Triad. 6



(i) Prove that $x = y - 1$ and $x = p^2 - y$.

$$y^2 = p^2 + x^2$$

$$p^2 = y^2 - x^2$$

$$p^2 = (y-x)(y+x).$$

3

Factors of p^2 $\{1, p, p^2\}$

$(y-x) < (y+x)$

So $y - x = 1$ and $y + x = p^2$

$x = y - 1$ $\quad$ $x = p^2 - y$.

(ii) Larry used the two equations from part (i) and found that $x = \dfrac{p^2-1}{2}$ (Do not prove this).

Prove that x is even.

2

Since p is odd it has the form $2n+1$ where $n \in \{0, 1, 2, 3, 4, \ldots\}$.

$$x = \frac{p^2-1}{2} = \frac{(2n+1)^2-1}{2} = \frac{4n^2+4n+1-1}{2} = 2(n^2+n).$$

which is even

(iii) Form a Pythagorean Triad which includes $p = 11$.

1

$$x = \frac{p^2-1}{2}$$

$$= \frac{121-1}{2}$$

$$= 60.$$

11, 60, 61.

End of paper