SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

Year 9

Yearly Examination 2011

Advanced Mathematics

General Instructions
- Working time – 90 minutes
- Write using black or blue pen.
- Approved calculators may be used.
- All necessary working MUST be shown in every question if full marks are to be awarded.
- Marks may not be awarded for untidy or badly arranged work.
- If more space is required, clearly write the number of the QUESTION on one of the back pages and answer it there. Indicate that you have done so.
- All answers must be given in exact simplified form unless otherwise indicated.
- Clearly indicate your class by placing an X, next to your class

Examiner: *B. Kilmore*

NAME:

Class	Teacher	
9 A	Mr Fuller	
9 B	Ms Chen	
9 C	Ms Nesbitt	
9 D	Mr Elliott	
9 E	Ms Ward	
9 F	Mr Boros	
9 G	Mr McQuillan	

| Question	Mark
1	/15
2	/15
3	/13
4	/12
5	/14
6	/13
7	/15
Total	**/97**

Section 1 (15 Marks)

1. Round 0.070283 to 3 significant figures. **(1)**

2. If 100 is divided by $\frac{1}{5}$ and then, from, this the reciprocal of $3\frac{1}{3}$ is subtracted, what is the result? **(1)**

3. Solve $-4a \geq -18$ **(1)**

4. Use your calculator to find correct to 2 decimal places **(1)**
$$\frac{8+\sqrt{2}}{8-\sqrt{2}}$$

5. Simplify: $4x - 3(x - 2)$ **(1)**

6. Expand and simplify: **(2)**

    a) $(1 - t)(8 - t)$

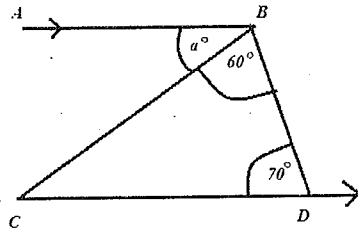   b) $(a^2 + 1)^2$

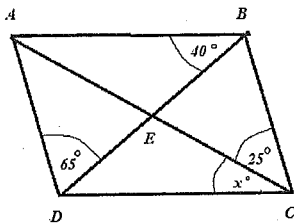7. Find the value of the pronumeral, giving reasons. **(4)**

   a)

   b)
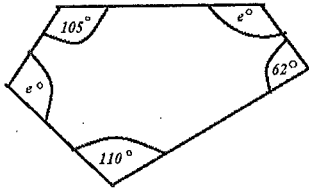
   c)   ABCD is a parallelogram.

8. Between which 2 integers does the surd $\left(\sqrt{17} + 1\right)$ lie? **(1)**

9. Simplify: **(3)**
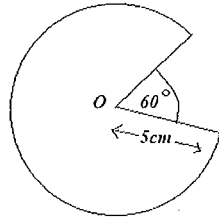
   a) $\sqrt{10} \times \sqrt{5}$

   b) $\sqrt{24} - \sqrt{6}$

   c) $2\sqrt{3}(\sqrt{2} + \sqrt{3})$

Section 2 (15 Marks)

1. Find the perimeter correct to 1 decimal place: **(1)**

2. Find the area in cm^2 of a rhombus with diagonals measuring 8cm and 16cm.

(1)

3. Solve the following equations: **(3)**

a) $\dfrac{3a}{7} + 3 = a$

b) $\dfrac{x}{4} - \dfrac{2x-1}{3} = \dfrac{x}{6} + 5$
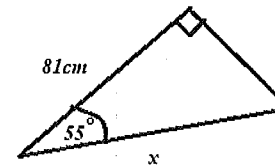
4. Make x the subject of the formula: **(3)**

a) $a = 3(x - 2)$

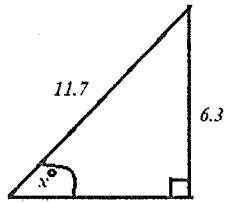b) $y = \dfrac{x}{x+3}$

5. Find the value of x in the following triangles: **(2)**

a)

Answer to 1 decimal place

b)

11.7

6.3

$x°$

Round to nearest minute.

6. If A = (3,6) and B = (-3,2), **(5)**

   a) Find the midpoint of AB

   b) Find the length of AB. (Write your answer as a simplified surd)

   c) Find the equation of the line AB and express your answer in **general form.**

Section 3 (13 Marks)

1. Factorise: **(3)**
   a) $2m^2 - 242$

   b) $3r^2 - 15r + 2ry - 10y$

   c) $5x^2 + 13x - 6$

2. Find the gradient and y-intercept of the line $x - 2y + 7 = 0$. **(1)**

3. Graph the region $y \leq 2 + x$ **(1)**

4. When a decimal point is placed between 2 digits, the result is the average of the two digits. What are they? (2)

5. Simplify: (4)

   a) $7x^6 \times 6x^7$

   b) $6^{2+w} \div 6^w$
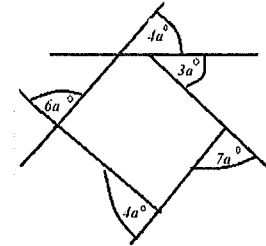
   c) $\left(\dfrac{m^4}{m^{-1}w^5}\right)^{-5}$

6. Evaluate: $(-27)^{\frac{-2}{3}}$ (2)

Section 4 (12 Marks)
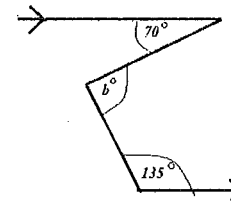
1. Find the value of the pronumeral, giving reasons:
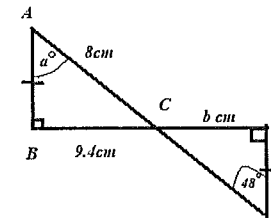
   (3)

   a)

   b)

2. Prove that the 2 triangles are congruent and then find the value of the pronumerals: (3)

3. What is the size of each of the angles in a regular octagon? (2)

4. Two dice are rolled simultaneously. Find the probability that the first dice showed a five, given that the sum is a six. (1)

5. Factorise: $x^2 + 2 + \frac{1}{x^2}$ (1)

6. Simplify: $\frac{5^{n+1} \times 5^{-2}}{5^{1-n}}$ (2)

Section 5 (14 Marks)

1. A card is selected at random from a standard pack of 52 cards. What is the probability of selecting an even numbered card which is black? (1)

2. The line whose equation is $y = x + 5$ is the perpendicular bisector of the line joining the points P(3,10) and Q (α , β). (3)

   a) Write down in terms of α and β, the co-ordinates of the mid-point of PQ.

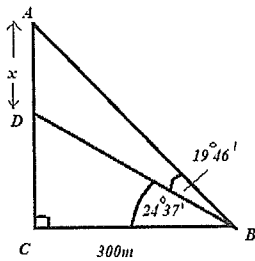   b) Show that $\beta = \alpha + 3$ and $\alpha + \beta = 13$.

3. A has twice as much money as B. If A has 10 cents more, one-fifth of A's amount would exceed one quarter of B's amount by 20 cents. How much do they each have? (2)

4. Find x and y given that

$$\frac{-12}{\sqrt{15}-3} = x - 2\sqrt{y}$$

(3)

5. A ship leaves port and sails on a bearing of 305° for 200 km. How far is the ship west of the port at this point? Answer to the nearest km.

(2)

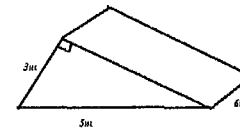6. Find x correct to 1 decimal place.

(3)

Section 6 (13 Marks)

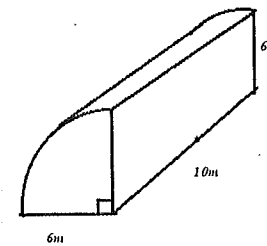1. Without evaluating each square, find the value of

$$30^2 - 29^2 + 28^2 - 27^2 + 26^2 - 25^2 + \cdots + 4^2 - 3^2 + 2^2 - 1^2$$

(3)

2. Find the surface area of these solids to the nearest whole unit:

(4)

a)

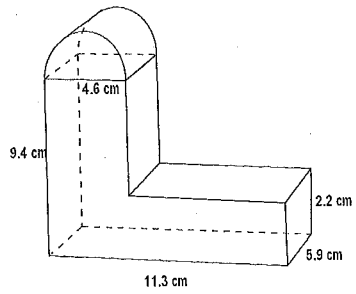

b)

3. Find the volume correct to 1 decimal place. (3)

4.6 cm
9.4 cm
2.2 cm
5.9 cm
11.3 cm

4. Show that the radius of a semi-circle whose perimeter is numerically equal to its area

is $\dfrac{2\pi + 4}{\pi}$. (3)

Section 7 (15 Marks)

1. Factorise (4)

a) $\dfrac{a^3}{b^2 c^3} - \dfrac{3a^2}{bc^2} + \dfrac{2a^3}{b^2 c^2}$
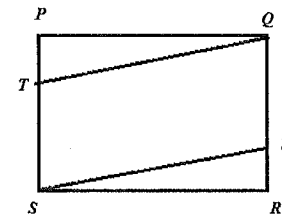
b) $4(3x-5)^2 - 49(2x-3)^2$

2. Evaluate:

$(2^x + 2^{-x})^2 - (2^x - 2^{-x})^2$ (2)

3. PQRS is a rectangle and PT=RU. Prove that TQUS is a parallelogram. (2)

4. In an exam, all questions were of equal value. Jack answered 7 of the first 12 questions correctly but only 40% of the remaining questions correctly. If he received a final mark of 50% for the whole test, How many questions were in the test?

(3)

5. A plane which is 200 km out of Sydney is flying at a speed of 900 km/h when the pilot sights Sydney airport. Ten minutes later, Sydney airport is at an angle of depression of 28° and at this point the plane begins its descent.
Assuming that its vertical rate of descent is a constant 30m/s, find how long it takes to make its descent. (4)

Section 1 (15 Marks)

1. Round 0.070283 to 3 significant figures. **(1)**

$$0.0703$$

2. If 100 is divided by $\frac{1}{5}$ and then, from, this the reciprocal of $3\frac{1}{3}$ is subtracted, what is the result? **(1)**

$$\frac{10}{3}$$

$$499.7$$

$$\text{or } \frac{4997}{10} \text{ or } 499\frac{7}{10}$$

3. Solve $-4a \geq -18$ **(1)**

$$a \leq 4.5 \text{ or } \frac{9}{2} \text{ or } 4\frac{1}{2}$$

4. Use your calculator to find correct to 2 decimal places **(1)**

$$\frac{8+\sqrt{2}}{8-\sqrt{2}}$$
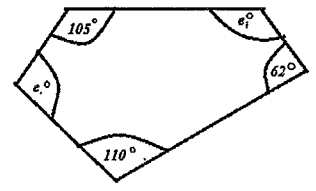
$$1.43$$

5. Simplify: $4x - 3(x - 2)$ **(1)**

$$4x - 3x + 6 = x + 6$$

6. Expand and simplify: **(2)**

a) $(1 - t)(8 - t) = 8 - t - 8t + t^2$
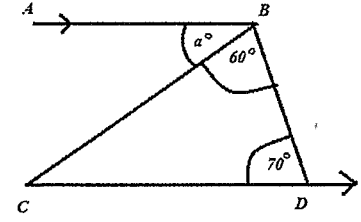
$$= 8 - 9t + t^2$$

b) $(a^2 + 1)^2 = a^4 + 2a^2 + 1$

7. Find the value of the pronumeral, giving reasons. **(4)**

a)

$$105 + 62 + 110 + 2e = 540$$
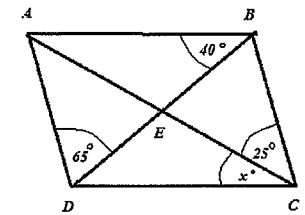(∠ sum pentagon).

$$e = 131.5$$
or
$$= 131°30'$$

$$\angle BCD = 50°$$
(∠ sum △)

$$a = 50°$$
(alt ∠s $AB \parallel CD$)

b)

c) ABCD is a parallelogram.

$$\angle OAB = 75°$$
(∠ sum △OAB).

$$\angle BCD = 75°$$
(opp ∠s parallelogram)

$$\therefore x = 50° \text{ (adj ∠s)}$$

8. Between which 2 integers does the surd $(\sqrt{17} + 1)$ lie? **(1)**

5 and 6.

9. Simplify: **(3)**

a) $\sqrt{10} \times \sqrt{5}$ = ~~50~~ $5\sqrt{2}$

b) $\sqrt{24} - \sqrt{6}$ = $2\sqrt{6} - \sqrt{6}$
$\qquad = \sqrt{6}$
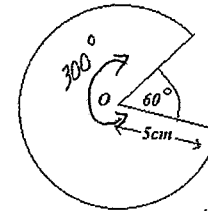
c) $2\sqrt{3}(\sqrt{2} + \sqrt{3})$ = $2\sqrt{6} + 6$

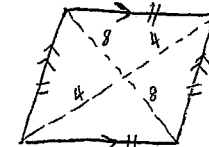## Section 2 (15 Marks)

1. Find the perimeter correct to 1 decimal place: **(1)**

$P = 5 + 5 + \dfrac{300}{360} \times 2 \times \pi \times 5$

$= 10 + 26 \cdot 1799$

$= 36 \cdot 1799$

$\doteqdot 36 \cdot 2$ cm. ①

2. Find the area in cm^2 of a ~~parallelogram~~ *rhombus* with diagonals measuring 8cm and 16cm. **(1)**

$A = \dfrac{1}{2} \times 8 \times 16$

$= 64 cm^2$ ①

3. Solve the following equations: **(3)**

a) $\dfrac{3a}{7} + \dfrac{3}{1} = \dfrac{a}{1}$

$\dfrac{3a}{7} + \dfrac{21}{7} = \dfrac{7a}{7}$

$21 = 4a$

$a = \dfrac{21}{4} = 5\frac{1}{4} \quad (5.25)$ ①

b) $\dfrac{x}{4} - \dfrac{(2x-1)}{3} = \dfrac{x}{6} + \dfrac{5}{1}$

$LCD \; \dfrac{3x - 4(2x-1) = 2x + 60}{12}$

$3x - 8x + 4 = 2x + 60$

$-5x + 4 = 2x + 60$

$-56 = 7x$ ②

$x = -8$

4. Make x the subject of the formula: **(3)**

a) $a = 3(x - 2)$

$a = 3x - 6$

$a + 6 = 3x$

$x = \dfrac{a+6}{3}$, $\left(\dfrac{a}{3} + 2\right)$ ①

b) $y = \dfrac{x}{x+3}$

$xy + 3y = x$

$xy - x = -3y$

$x(y-1) = -3y$

$x = \dfrac{-3y}{y-1}$ ②

or $\dfrac{3y}{1-y}$

5. Find the value of x in the following triangles:

(2)

a)

81cm

55°

x

Answer to 1 decimal place

$\cos 55° = \dfrac{81}{x}$

$x \cos 55° = 81$

$x = \dfrac{81}{\cos 55°}$

$x \doteq 141.2 \text{ cm}$ ①

b)

11.7

6.3 ..

x°

Round to nearest minute.

$\sin x° = \dfrac{6.3}{11.7}$

$x = \sin^{-1}\left(\dfrac{6.3}{11.7}\right)$

$x \doteq 32° \ 35'$ ①

6. If A = (3,6) and B = (-3,2),

a) Find the midpoint of AB

(5)

$M\left(\dfrac{3+^-3}{2}, \dfrac{6+2}{2}\right)$

$= M(0, 4)$ ①

b) Find the length of AB. (Write your answer as a simplified surd)

$AB = \sqrt{(-3-3)^2 + (2-6)^2}$

$= \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$ ②

c) Find the equation of the line AB and express your answer in **general form**.

$m = \dfrac{2-6}{-3-3} = \dfrac{-4}{-6} = \dfrac{2}{3}$ and $(3,6)$

$(y - y_1) = m(x - x_1)$

$(y - 6) = \dfrac{2}{3}(x - 3)$

$3y - 18 = 2x - 6$

$0 = 2x - 3y + 12$ ②

Section 3 (13 Marks)

1. Factorise: **(3)**

a) $2m^2 - 242$

$\bigcirc\!\!1$ $= 2(m^2 - 121)$ $(\sqrt{2}m + \sqrt{242})(\sqrt{2}m - \sqrt{242})$ $\left(\frac{1}{2}\right)$

$= 2(m-11)(m+11)$

b) $3r^2 - 15r + 2ry - 10y$

$\bigcirc\!\!1$ $= 3r(r - 5) + 2y(r-5)$

$= (3r + 2y)(r - 5)$

c) $5x^2 + 13x - 6$

$x - 30$.
$+ \, +13$ $\bigcirc\!\!1$ $= 5x^2 + 15x - 2x - 6$

$= 5x(x + 3) - 2(x + 3)$

$= (5x - 2)(x + 3)$

2. Find the gradient and y-intercept of the line $x - 2y + 7 = 0$. **(1)**

$2y = x + 7$

$y = \frac{x}{2} + \frac{7}{2}$

$\bigcirc\!\!1$ gradient $= \frac{1}{2}$

y-intercept $= \frac{7}{2}$

3. Graph the region $y \le 2 + x$ **(1)**

$\bigcirc\!\!1$
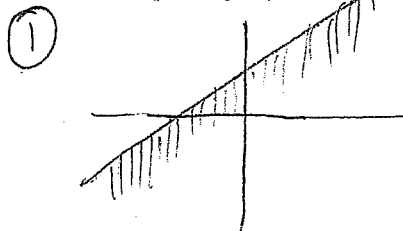
test $(0,0)$.

$0 \le 2 + 0$ ✓

4. When a decimal point is placed between 2 digits, the result is the average of the two digits. What are they? **(2)** $\bigcirc\!\!2$

$x \cdot y = \frac{x+y}{2}$ $x + \frac{y}{10} = \frac{x+y}{2}$

$\boxed{4 \cdot 5 = \frac{9}{2}}$ $\frac{10x + y}{10} = \frac{5x + 5y}{10}$

$5 \cdot 4 = \frac{5+4}{2}$

$5 4 = \frac{9}{2}$ $10x + y = 5x + 5y$

$5x = 4y$

$4 : 5$.

$x = 4$ $y = 5$

5. Simplify:

a) $7x^6 \times 6x^7 = 42x^{13}$ $\bigcirc\!\!1$ **(4)**

b) $6^{2+w} \div 6^w = 6^{2+w-w} = 36$ $\bigcirc\!\!1$

c) $\left(\frac{m^4}{m^{-1}w^5}\right)^{-5} = \frac{m^{-20}}{m^5 w^{-25}}$

$= \frac{w^{25}}{m^5 \times m^{20}} = \frac{w^{25}}{m^{25}}$ $\bigcirc\!\!2$

6. Evaluate: $(-27)^{\frac{-2}{3}}$ **(2)**

$\left(\sqrt[3]{-27}\right)^{-2} = -3^{-2}$ $\bigcirc\!\!2$
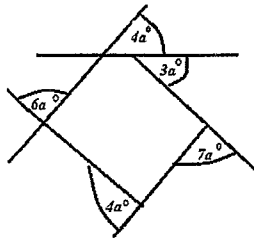
$= \frac{1}{9}$

does not work on some calculators (eg fx 100 AU)

Section 4 (12 Marks)

1. Find the value of the pronumeral, giving reasons:

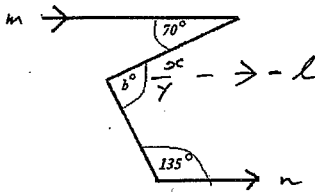(3)

a)

EXTERIOR ANGLES
OF POLYGON = $360°$

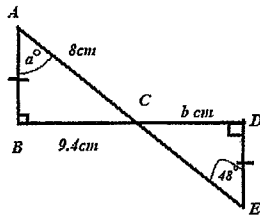$\therefore 6a + 4a + 3a + 7a + 4a = 360°$

$24a° = 360°$

$a = 15$

b)

Draw l, Parallel to m, n

$b = x + y$

$x = 70°$ Alt $\angle s$

$y = 45°$ Co-int $\angle s$

$\therefore b = 115$

2. Prove that the 2 triangles are congruent and then find the value of the pronumerals:

(3)

In $\triangle$'s ABC, EDF

$A = E$, since $AB \parallel DE$

$A\hat{C}B = E\hat{C}D$, V.O.

$AB = DE$ Given

$\therefore \triangle$'s congruent A.S.A.

$\therefore a = 48, \ b = 9.4$

3. What is the size of each of the angles in a regular octagon? (2)

Angle sum of octagon $= (2 \times 8 - 4) \times 90$

$= 1080°$

$\therefore 8x = 1080$

$x = 135°$

4. Two dice are rolled simultaneously. Find the probability that the first dice showed a five, given that the sum is a six. (1)

$\therefore n(\text{Sample space}) = 5$

$P(\text{First being a 5}) = \frac{1}{5}$

DIE 1		DIE 2
1	—	5
2	—	4
3	—	3
4	—	2
5	—	1

5. Factorise: $x^2 + 2 + \frac{1}{x^2}$ (1)

$= \left(x + \frac{1}{x} \right)^2$

6. Simplify: $\dfrac{5^{n+1} \times 5^{-2}}{5^{1-n}} = \dfrac{5^{n-1}}{5^{1-n}} = 5^{2n-2}$ (2)

Section 6 (13 Marks)

1. Without evaluating each square, find the value of

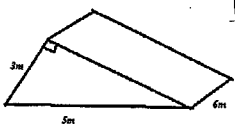$$30^2 - 29^2 + 28^2 - 27^2 + 26^2 - 25^2 + \cdots + 4^2 - 3^2 + 2^2 - 1^2$$

$= (30+29)(30-29) + (28+27)(28-27) + \cdots + (2+1)(2-1)$ **(3)**

$= 30+29+28+27+ \cdots +2+1$

$= \boxed{465}$

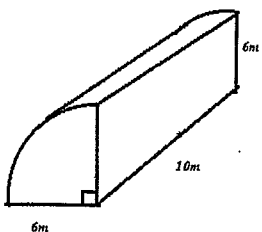**2.** Find the surface area of these solids to the nearest whole unit:

(4)

a)

$SA = \left(\tfrac{1}{2}(3)(4) \times 2\right) + (3 \times 6)+(5 \times 6)+(4 \times 6)$

$\boxed{SA = 84 \ m^2}$

b)

$SA = (6 \times 10)+(6 \times 10)+ \dfrac{\pi(6)^2}{2} + 3 \times \pi \times 10$

$= 270.796$

$\boxed{SA = 271 \ m^2}$

Section 5 (14 Marks)

1. A card is selected at random from a standard pack of 52 cards. What is the probability of selecting an even numbered card which is black? **(1)**

$\dfrac{10}{52} = \dfrac{5}{26}$

2. The line whose equation is $y = x + 5$ is the perpendicular bisector of the line joining the points $P(3,10)$ and $Q(\alpha, \beta)$. **(3)**

a) Write down in terms of α and β, the co-ordinates of the mid-point of PQ.

$M = \left(\dfrac{\alpha+3}{2}, \dfrac{\beta+10}{2}\right)$

b) Show that $\beta = \alpha+3$ and $\alpha+\beta = 13$.

M on line $y = x+5$

$\dfrac{\beta+10}{2} = \dfrac{\alpha+3}{2} +5$

$\beta+10 = \alpha+3 +10$

$\beta = \alpha+3$

eqn of line joining $(3,10)(\alpha,\beta)$ $m=-1$

$\beta-10 = -(\alpha-3)$

$\beta = -\alpha+3$

$\beta+\alpha = 13$

3. A has twice as much money as B. If A has 10 cents more, one-fifth of A's amount would exceed one quarter of B's amount by 20 cents. How much do they each have? **(2)**

$A = 2B$

$\dfrac{A+10}{5} = \dfrac{B}{4} + 20$

$4A+40 = 5B +400$

$8B+40 = 5B +400$

$3B = 360$

$B = \$120$

$A = \$240$

4. Find x and y given that **(3)**

$\dfrac{-12}{\sqrt{5}-3} = x - 2\sqrt{5}$

$\dfrac{-12}{\sqrt{5}-3} \times \dfrac{\sqrt{5}+3}{\sqrt{5}+3} = x-2\sqrt{5}$
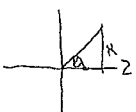
$\dfrac{12\sqrt{5}-36}{6} = x-2\sqrt{5}$

$-2\sqrt{5} - 6 = x - 2\sqrt{5}$

$x = -6 \qquad y = 15$

5. A ship leaves port and sails on a bearing of 305° for 200 km. How far is the ship west of the port at this point? Answer to the nearest km. **(2)**
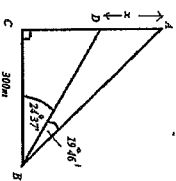
$\sin 55 = \dfrac{x}{200}$

$x = 200 \sin 55$

$164 \ km$ (nearest km)

6. Find x correct to 1 decimal place. **(3)**

$\tan 44°23 = \dfrac{AC}{300}$

$\tan 24°37 = \dfrac{DC}{300}$

$x = 300(\tan 44°23' - \tan 24°37')$

$= 156.15$

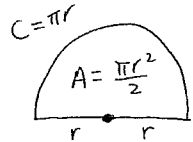$= 136.2 \ (1 \ d.p.)$

3. Find the volume correct to 1 decimal place. **(3)**
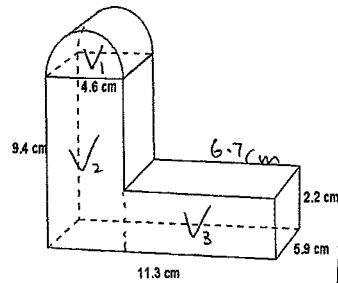
$$V = V_1 + V_2 + V_3$$
$$= \frac{\pi r^2 h}{2} + (4\cdot6 \times 9\cdot4 \times 5\cdot9) + (6\cdot7 \times 2\cdot2 \times 5\cdot9)$$
$$= \frac{\pi(2\cdot3)^2(5\cdot9)}{2} + 255\cdot116 + 86\cdot966$$
$$= 391\cdot10812$$
$$\boxed{\therefore V = 391\cdot1 \text{ cm}^3}$$

4. Show that the radius of a semi-circle whose perimeter is numerically equal to its area is $\frac{2\pi+4}{\pi}$. **(3)**

$$C = \pi r$$
$$A = \frac{\pi r^2}{2}$$

$$P = A$$
$$r + r + \pi r = \frac{\pi r^2}{2}$$
$$2r + \pi r = \frac{\pi r^2}{2}$$
$$4r + 2\pi r = \pi r^2$$
$$4 + 2\pi = \pi r$$
$$\frac{4+2\pi}{\pi} = r$$
$$\boxed{\therefore r = \frac{2\pi+4}{\pi}}$$
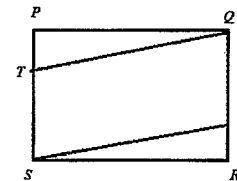
Section 7 (15 Marks)

1. Factorise **(4)**

a) $\dfrac{a^3}{b^2c^3} - \dfrac{3a^2}{bc^2} + \dfrac{2a^3}{b^2c^2} = \dfrac{a^2}{bc^2}\left(\dfrac{a}{bc} - 3 + \dfrac{2a}{b}\right)$

$$= \frac{a^2}{b^2c^3}(a - 3bc + 2ac)$$

b) $4(3x-5)^2 - 49(2x-3)^2 = [2(3x-5) - 7(2x-3)][2(3x-5) + 7(2x-3)]$

$$= [6x - 10 - 14x + 21][6x - 10 + 14x - 21]$$
$$= (11 - 8x)(20x - 31)$$

2. Evaluate:

$(2^x + 2^{-x})^2 - (2^x - 2^{-x})^2 = [(2^x + 2^{-x}) - (2^x - 2^{-x})][(2^x + 2^{-x}) + (2^x - 2^{-x})]$ **(2)**

$$= [2 \cdot 2^{-x}][2 \cdot 2^x]$$
$$= 2^{-x+1} \cdot 2^{x+1}$$
$$= 2^2$$
$$= 4$$

3. PQRS is a rectangle and PT=RU. Prove that TQUS is a parallelogram. **(2)**

PS = QR (opposite sides of rectangle)
PT = RU (given)
PS - PT = QR - RU
∴ TS = QU
PS // QR (opposite sides of rectangle)
∴ TS // QU
TQUS is a parallelogram
(opposite sides are equal & parallel)

4. In an exam, all questions were of equal value. Jack answered 7 of the first 12 questions correctly but only 40% of the remaining questions correctly. If he received a final mark of 50% for the whole test, How many questions were in the test? (3)

let x be the number of remaining questions

$$\frac{7 + 0.4x}{12 + x} = 0.5$$

$$7 + 0.4x = 6 + 0.5x$$

$$1 = 0.1x$$

$$x = 10$$

$\therefore$ Number of questions in test $= 12 + 10$

$$= 22$$

5. A plane which is 200 km out of Sydney is flying at a speed of 900 km/h when the pilot sights Sydney airport. Ten minutes later, Sydney airport is at an angle of depression of 28° and at this point the plane begins its descent.
Assuming that its vertical rate of descent is a constant 30m/s, find how long it takes to make its descent. (4)

900 km / 60 mins

(150 km) / 10 mins

30 m/s

1.8 km/min
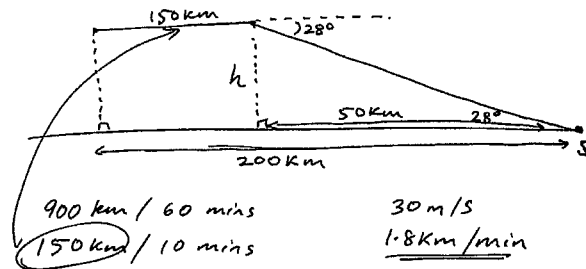
$$\tan 28° = \frac{h}{50}$$

$$h = 50 \tan 28°$$

$$h \approx 26.585 \text{ km}$$

Time for descent $= \dfrac{50 \tan 28°}{1.8}$

$$= 14.7697.... \text{ minutes}$$

$$= 14 \text{ minutes } 46 \text{ seconds}$$