SYDNEY GRAMMAR SCHOOL

2009 Annual Examination

FORM I
MATHEMATICS

Friday 30th October 2009

General Instructions

- Writing time — 1 hours 30 minutes
- Write using black or blue pen.
- Calculators are not to be used.
- All necessary working should be shown in every question.
- Start each question on a new page.

Structure of the paper

- Total marks — 110
- All ten questions may be attempted.
- All ten questions are of equal value.

Collection

- Write your name, class and master clearly on each page of your answers and on the tear-off sheet.
- Staple your answers in a single bundle.
- Bundle the tear-off sheet with the question it belongs to.
- Write your name and master on this question paper and submit it with your answers.

1BR/ADS:	SJE	1DBD/CJW:	BR	1FHB/MW:	MW
1JAG/DJM:	REP	1JFC/PKR:	JMR	1PGM/AHWD:	RCF
1RDWL/LDR:	SO	1WTR/AGY:	BR		

Checklist

- Writing paper required.
- Candidature — 184 boys

Examiner

SO

QUESTION ONE (11 marks) Start a new page.

(a) Find:

(i) $5 - 3 \times 7$

(ii) $3{\cdot}27 \times 100$

(iii) $0{\cdot}3 \times 1{\cdot}5$

(b) Calculate:

(i) $5 - (-8)$

(ii) $-42 \div 7$

(iii) -3^2

(c) Evaluate:

(i) $\frac{2}{3} + \frac{4}{9}$

(ii) $\frac{2}{3} \times \frac{6}{5}$

(d) Simplify:

(i) $2b \times 3b$

(ii) $-8y + 3y$

(e) Find the value of $7 - 3a$ when $a = -1$.

QUESTION TWO (11 marks) Start a new page.

(a) Calculate:

(i) $2{\cdot}367 + 0{\cdot}09$

(ii) $64 \div 0{\cdot}4$

(b) (i) Write 32% as a fraction in lowest terms.

(ii) Write $\frac{9}{20}$ as a decimal.

(iii) Find 48% of 600.

(iv) Write $\frac{5}{8}$ as a percentage.

(c) If 8 burgers cost \$28, how much will 5 burgers cost?

(d) Expand:

(i) $2(5 - 3x)$

(ii) $-y(3 - y)$

Exam continues next page ...

QUESTION THREE (11 marks) Start a new page.

(a) A bag contains 4 green, 7 yellow and 1 pink counter. If a counter is drawn at random, what is the probability that it is yellow?

(b) Solve:

(i) $x + 19 = 32$

(ii) $5x - 3 = 17$

(c) Simplify:

(i) $7x - 2 + x + 9$

(ii) $\frac{x}{5} + \frac{x}{4}$

(iii) $48a^5 \div 16a^3$

(d) (i) Copy and fill in the following table using the rule $y = -2x + 3$:

x	-1	0	1	2	3
y					

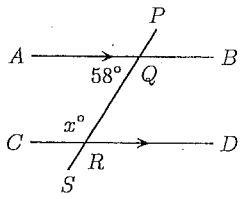
(ii) Plot the resulting points on a number plane. Use a scale of one centimetre per unit on each axis. Use a ruler to draw your axes.

Exam continues overleaf ...

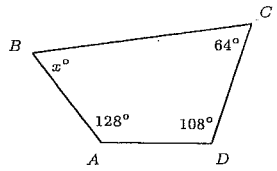
QUESTION FOUR (11 marks) Start a new page.

(a) Find the value of x in each diagram below, giving reasons:

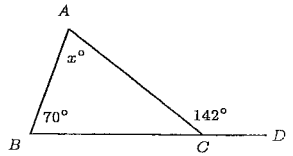
(i)



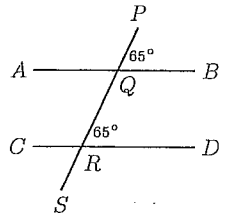
(ii)



(iii)



(b)



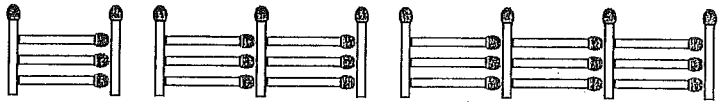
Give a reason to explain why $AB||CD$ in the diagram above.

(c) Tear off the last sheet of this examination and do the constructions described there. Bundle the sheet with the rest of Question 4.

Exam continues next page ...

QUESTION FIVE (11 marks) Start a new page.

(a)



Richard is building a fence out of matches. The three diagrams above show a fence with one, two and three panels respectively.

(i)

p	1	2	3	4
m				

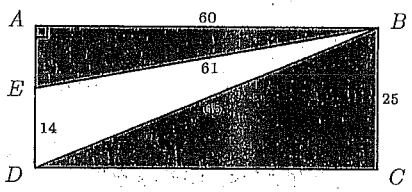
Copy and complete the table above, showing the number of matches m required to build a fence with p panels.

(ii) How many matches are required to add a panel to an existing fence?

(iii) Write down a formula for the number of matches required to make a fence with p panels.

(iv) Richard's finished fence contains 133 matches, how many panels long is Richard's fence?

(b)



The diagram above shows a rectangle $ABCD$. All measurements are in metres. Find:

(i) the perimeter of triangle DEB,

(ii) the area of the triangle DEB,

(iii) the area of the shaded compound region.

(c) Henry thought of a number. After he doubled the number, he added 7 and the result was 53.

(i) Let the number be n and write an equation to express the above statement.

(ii) Solve your equation to find the number.

(d) Given that $3439 \times 112 = 385168$, evaluate:

(i) $34{\cdot}39 \times 11{\cdot}2$

(ii) $0{\cdot}385168 \div 1{\cdot}12$

Exam continues overleaf ...

QUESTION SIX (11 marks) Start a new page.

(a) Solve $5(x-2) - 3(x+5) = -19$.

(b) Calculate:

(i) $6\frac{3}{7} - 4\frac{11}{14}$

(ii) $\frac{7}{2} - \frac{4}{9} \times (1\frac{1}{2})^3$

(c) Suppose $P = \{\text{positive square numbers less than } 15\}$, $Q = \{1, 2, 3, ...9, 10\}$, $R = \{\text{positive odd numbers less than } 20\}$ and $S = \{\text{primes less than } 25\}$.

(i) State TRUE or FALSE for each of the following:

(α) $1 \in S$

(β) $P \subset Q$

(γ) $|R| = |Q|$

(ii) List all the subsets of P.

Exam continues next page ...

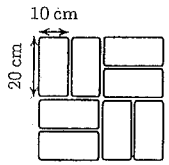
QUESTION SEVEN (11 marks) Start a new page.

(a) Draw a number plane with axes labelled from -5 to $+5$.

(i) Plot the following points and join them in the order given: $A(-3,-3), B(1,2)$ and $C(4,-3)$.

(ii) Find the area the of shape ABC.

(b)



Joseph is going to use the pattern above to pave his backyard.
He is going to pave an area of 12 m^2. How many 20 cm×10 cm pavers will he need?

(c) Charlton rode 30 km to the top of a hill and then back down again the same way. It took him 90 minutes to reach the top and 1 hour to return downhill.

(i) What was his average speed for the entire journey in km/h?

(ii) The next day he rides up and down the hill again. He rides up at the same speed as yesterday but rides down more quickly and so completes the entire journey in 2 hours and 15 minutes.

At what speed did he ride down the hill?

(d) (i) Find the prime factorisation of 450.

(ii) A rectangular block of cooking chocolate with base 15 cm×6 cm and height 5 cm is melted and poured into a square cake tin and allowed to cool, forming a new block of chocolate. This new block has a height less than the original block and the dimensions are integers.

Find the height of the new block of chocolate.

Exam continues overleaf ...

QUESTION EIGHT (11 marks) Start a new page.

(a) What is the value of $(1+\frac{1}{5})\times(1+\frac{1}{6})\times(1+\frac{1}{7})\times(1+\frac{1}{8})\times(1+\frac{1}{9})$?

(b) Simplify:

(i) $-4a^3b^5 \times 3a^2b$

(ii) $320m^5n^2 \div 80m^3n^2$

(iii) $(5m^4n)^2$

(c) A rectangle has dimensions 20 mm×10 mm. If each dimension is increased by 10%, what is the area of the new rectangle?

(d)

$$1+4\times 2=(1+2)^2$$
$$1+4\times 2+8\times 2=(1+2+2)^2$$
$$1+4\times 2+8\times 2+12\times 2=(1+2+2+2)^2$$

If the same pattern always holds, calculate the value of

$$\sqrt{1+4\times 2+8\times 2+12\times 2+\cdots+36\times 2}$$

QUESTION NINE (11 marks) Start a new page.

(a) A team is made by selecting three boys from a group of five boys: Anthony, Ben, Charlie, David and Eric. Assuming the boys are chosen at random, what is the probability that Anthony and Ben will not be in the team?

(b) Chris had been saving his pocket money and decided to go shopping. He spent \$24 on a new T-shirt and 20% of the remainder on music. He still had $\frac{2}{3}$ of his money left. How much money did he have before he began spending?

(c) Calculate $\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1-\cfrac{1}{1+2}}}}$

(d) Determine the number of digits in the product $25^7 \times 2^{19}$.

(e) Blake built a fence using n equally spaced posts, each 1 metre apart, to enclose a square field.

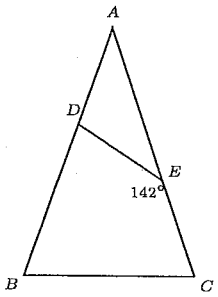
(i) Write an expression in terms of n for the area bounded by the fence.

(ii) If the area of the field is 20·25 m^2, calculate the number of posts that Blake used.

Exam continues next page ...

QUESTION TEN (11 marks) Start a new page.

(a)



Consider the isosceles triangle ABC, with $AB = AC$, as in the diagram above. Suppose D is on AB, and E is on AC, so that $AD = DE$. If $\angle CED = 142°$, calculate $\angle ABC$. Give reasons.

(b) For all positive numbers a and b, the operation $a * b$ is defined by $a * b = \dfrac{5a^2}{b}$.

(i) Find w if $w * 10 = 8$.

(ii) Find $(a * b) * (b * a)$.

(c) The digits 1, 2, 3, 4, 5 and 6 are each used in the 6-digit number $abcdef$. The 3-digit number abc is divisible by 4, while bcd is divisible by 5, cde is divisible by 3 and def is divisible by 11. Find the values of a, b, c, d, e and f.

(d) How many ways are there of picking out the name $ANNA$ from the following table by moving from each letter to an adjacent letter? You may move up or down, left or right, or diagonally. The same letter must not be used twice in one $ANNA$.

A A A A
A N N A
A N N A
A A A A

END OF EXAMINATION

Tear-off pages follow ...

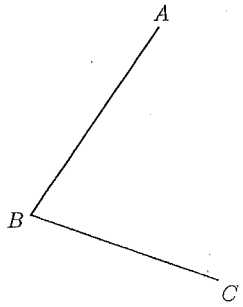
NAME: ..... CLASS: ..... MASTER: .....

DETACH THIS SHEET AND BUNDLE IT WITH THE REST OF QUESTION FOUR.

QUESTION FOUR

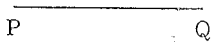
(c) Using a pencil, ruler and compasses only, complete the following constructions. Do not erase any construction markings.

(i)



Bisect the angle ABC in the diagram above.

(ii)



The interval PQ is shown above.

(α) Construct an angle of $90°$ at P.

(β) Complete the diagram by constructing the $\triangle PQR$ in which $\angle P = 90°$ and $PR = 6$cm.

FORM ONE SOLUTIONS

QUESTION ONE

1 (i) $5 - 3 \times 7$
$= 5 - 21$
$= -16$ ✓

(ii) $3 \cdot 27 \times 100$
$= 327$ ✓

(iii) $0 \cdot 3 \times 1 \cdot 5$
$= 0 \cdot 45$ ✓

2 (i) $5 - (-8)$
$= 5 + 8$
$= 13$ ✓

(ii) $-42 \div 7$
$= -6$ ✓

(iii) -3^2
$= -9$ ✓

(ii) $\frac{2}{3} + \frac{4}{9}$
$= \frac{6}{9} + \frac{4}{9}$
$= \frac{10}{9}$
$= 1\frac{1}{9}$ ✓

(ii) $\frac{2}{3} \times \frac{6}{5}$
$= \frac{12}{15}$
$= \frac{4}{5}$ ✓

(d) (i) $2b \times 3b$
$= 6b^2$ ✓

(ii) $-8y + 3y$
$= -5y$ ✓

(e) $7 - 3(-1)$
$= 7 + 3$
$= 10$ ✓

QUESTION TWO

(a) (i) $2 \cdot 367 + 0 \cdot 09 = 2 \cdot 457$ ✓

(ii) $64 \div 0 \cdot 4$
$= 640 \div 4$
$= 160$ ✓

(b) (i) $\frac{32}{100} = \frac{8}{25}$ ✓

(ii) $\frac{9}{20} = 0 \cdot 45$ ✓

(iii) $\frac{48}{100} \times \frac{600}{1} = 288$ ✓

(iv) $\frac{5}{8} \times \frac{100}{1}$ ✓
$= \frac{500}{8}$
$= 62 \cdot 5\%$ ✓

(c) 8 burgers cost $28
1 burger costs $3.50 ✓
5 burgers cost $17.50 ✓

3 (i) $2(5 - 3x) = 10 - 6x$ ✓

(ii) $-y(3 - y) = -3y + y^2$ ✓

QUESTION THREE

1 $P(\text{yellow}) = \frac{7}{12}$ ✓

b (i) $x + 19 = 32$
$x = 13$ ✓

(ii) $5x - 3 = 17$ ✓
$5x = 20$
$x = 4$ ✓

c (i) $7x - 2 + x + 9$
$= 8x + 7$ ✓

(ii) $\frac{x}{5} + \frac{x}{4}$
$= \frac{4x + 5x}{20}$
$= \frac{9x}{20}$ ✓

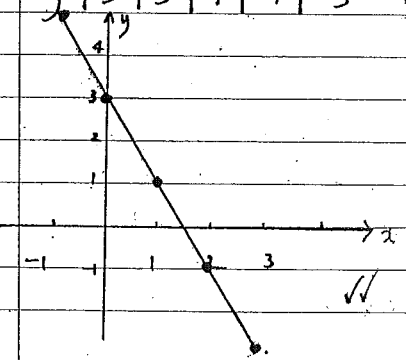
(iii) $48a^5 \div 16a^3$
$= 3a^2$ ✓

d (i)

x	-1	0	1	2	3
y	5	3	1	-1	-3

✓✓

(ii)



✓✓

QUESTION FOUR

(a)
(i) $x + 58 = 180°$
(cointerior angles, $AB \parallel CD$) ✓
$x = 122°$ ✓

(ii) $\frac{x}{2} + 64 + 128 + 108 = 360$
(angle sum of a quadrilateral) ✓
$x = 60°$ ✓

(iii) $x + 70 = 142°$
(exterior angle of a triangle) ✓
$\therefore x = 72°$ ✓

(b) $\angle PQB = \angle QRD$
corresponding angles are equal.
$\therefore AB \parallel CD$. ✓
Solutions for constructions at the end.

QUESTION FIVE

(a) (i)

p	1	2	3	4
m	5	9	13	17

✓

(ii) 4 ✓

(iii) $m = 4p + 1$ ✓

(iv) $133 = 4p + 1$
$4p = 132$
$p = 33$
$\therefore$ Richards fence contains 33 panels. ✓

(b)(i) $P = 61 + 65 + 14$
$= 140\text{m}$ ✓

(ii) $A = \frac{1}{2} \times 60 \times 14$
$= 420\text{m}^2$ ✓

(iii) Total area $= 25 \times 60$
$= 1500\text{m}^2$
Shaded area $= 1500 - 420$
$= 1080\text{m}^2$ ✓

(c)(i) $2n + 7 = 53$ ✓

(ii) $2n = 46$
$n = 23$ ✓

(d)(i) 34.39×11.2
$= 385.168$ ✓

(ii) $0.385168 \div 1.12$
$= 0.3439$ ✓

QUESTION SIX

a) $5(x-2) - 3(x+5) = 19$
$5x - 10 - 3x - 15 = -19$ ✓
$2x = 6$
$x = 3$ ✓

b)(i) $6\frac{3}{7} - 4\frac{11}{14}$
$= 6\frac{6}{14} - 4\frac{11}{14}$ ✓
$= 5\frac{20}{14} - 4\frac{11}{14}$
$= \frac{9}{14}$ ✓

(ii) $\frac{7}{2} - \frac{4}{9} \times \left(\frac{1}{2}\right)^3$
$= \frac{7}{2} - \frac{4}{9} \times \left(\frac{3}{2}\right)^3$
$= \frac{7}{2} - \frac{4}{9} \times \frac{27}{8}$ ✓
$= \frac{7}{2} - \frac{3}{2}$
$= \frac{4}{2}$
$= 2$ ✓

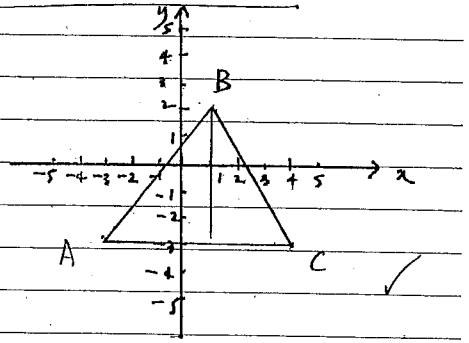
(c)(i) (α) FALSE ✓
(β) TRUE ✓
(γ) TRUE ✓

(ii) ϕ $\{1\}$ $\{4\}$ $\{9\}$ $\{1,4\}$ $\{1,9\}$ $\{4,9\}$ $\{1,4,9\}$ ✓✓

* 1 mark for listing at least 4 subsets
* A complete list is require to achieve 2 marks

QUESTION SEVEN

(i)



✓

(ii) AC = 7 units
height = 5 units ✓

Area $= \frac{1}{2}bh$
$= \frac{1}{2} \times 7 \times 5$
$= \frac{35}{2}\text{ units}^2$ ✓

(c) 1 paver $= 200\text{cm}^2$
Total area $= 12\text{m}^2$
$= 120\,000\text{cm}^2$ ✓
No. of pavers required
$= 120000 \div 200$ ✓

(d)(i) $S = \frac{30+30}{1.5+1}$
$= \frac{60}{2.5}$
$= 24\text{ km/h}$ ✓

(ii) Downhill ride = 45 minutes
Speed $= \frac{30}{3/4}$
$= 30 \div \frac{3}{4}$ ✓
$= 30 \times \frac{4}{3}$
$= 40\text{ km/h}$ ✓

e)(i) 450

$\therefore 450 = 2 \times 3^2 \times 5^2$ ✓

(ii) Possible squares bases:
$2 \times 15 \times 15$
$18 \times 5 \times 5$
$50 \times 3 \times 3$
$450 \times 1 \times 1$ ✓
The new height must be less than 5, therefore the new height is 2cm. ✓

QUESTION EIGHT

a) $\left(1+\frac{1}{5}\right) \times \left(1+\frac{1}{6}\right) \times \left(1+\frac{1}{7}\right) \times \left(1+\frac{1}{8}\right) \times \left(1+\frac{1}{9}\right)$
$= \frac{6}{5} \times \frac{7}{6} \times \frac{8}{7} \times \frac{9}{8} \times \frac{10}{9}$
$= \frac{10}{5}$
$= 2$ ✓

b(i) $-4a^3b^5 \times 3a^2b$
$= -12a^5b^6$ ✓✓

(ii) $320m^5n^2 \div 80m^3n^2$
$= 4m^2$ ✓✓

(iii) $(5m^4n)^2$
$= 25m^8n^2$ ✓✓

(c) New dimensions $= 22\text{mm} \times 11\text{mm}$ ✓
Area $= 242\text{mm}^2$ ✓

d) $\sqrt{1 + (4\times2) + (8\times2) + (12\times2) + (16\times2) + (20\times2) + (24\times2) + (28\times2) + (32\times2) + 36\times2}$
$= \sqrt{(1+2+2+2+2+2+2+2+2+2)^2}$ ✓
$= \sqrt{19^2}$

QUESTION 9

Let the 5 boys be A, B, C, D & E.
List the possibilities:

ABC	ACD	BCD	CDE
ABD	ACE	BCE	
ABE	ADE	BDE	

✓

There are nine teams that include Anthony or Ben

- $P(\text{not } A+B) = \frac{1}{10}$ ✓

b) Let x be the amount of money that Chris had saved.

Chris spent $\$24 + \frac{20}{100}(x-24)$

$24 + \frac{20}{100}(x-24) = \frac{x}{3}$ ✓

$72 + \frac{6}{10}(x-24) = x$

$720 + 6x - 144 = 10x$ ✓

$4x = 576$

$x = \$144$

∴ Chris had \$144 before he started shopping. ✓

c)
$$\frac{1}{1+\frac{1}{1+\frac{1}{1-\frac{1}{1+2}}}}$$

$$= \frac{1}{1+\frac{1}{1+\frac{1}{1-\frac{1}{3}}}}$$

$$= \frac{1}{1+\frac{1}{1+\frac{1}{2/3}}}$$ ✓

$$= \frac{1}{1+\frac{1}{1+3/2}}$$

$$= \frac{1}{1+\frac{1}{5/2}}$$

$$= \frac{1}{1+\frac{2}{5}}$$

$$= \frac{1}{7/5}$$

$$= \frac{5}{7}$$ ✓

(d) $25^7 \times 2^{19}$

$= (5^2)^7 \times 2^{19}$

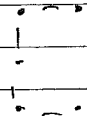
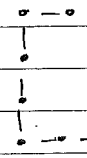
$= 5^{14} \times 2^{19}$

$= (10^{14}) \times 2^5$ ✓

$= 32 \times 10^{14}$

∴ there are 16 digits ✓

9(e)

		POSTS	SIDE LENGTH	AREA
(i)		4	1	$1 \times 1 = 1m^2$
		8	2	$2 \times 2 = 4m^2$
		12	3	$3 \times 3 = 9m^2$
		n posts	$\frac{n}{4}$	$\frac{n}{4} \times \frac{n}{4} = \left(\frac{n^2}{16}\right)m^2$ ✓

(ii) $\frac{n^2}{16} = \frac{81}{4}$

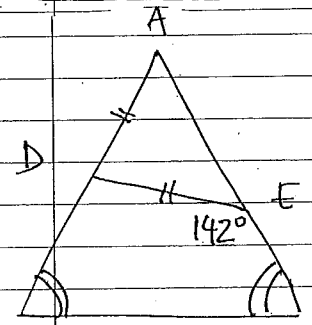
$n^2 = 324$

$n = 18$

∴ Blake used 18 posts
But this is impossible as this would mean he used $\frac{18}{4} = 4.5$ posts on each side. ✓

QUESTION TEN

(a)



$\angle ABC = \angle AEB$ (isosceles Δ)
$\angle AED = 38°$ (straight angle)
$\angle DAE = 38°$ (triangle ADE is isosceles, AD = DE) ✓

Let $\angle ABC = \angle ACB = x$

$2x + 38 = 180°$ (angle sum of a Δ) ✓

$2x = 142°$

$x = 71°$.

(b) (i) $\frac{5w^2}{10} = 8$

$5w^2 = 80$ ✓

$w^2 = 16$

$w = 4$ ✓

(ii) $\frac{5a^2}{b} * \frac{5b^2}{a}$

$= 5\left(\frac{5a^2}{b}\right)^2 \div \frac{5b^2}{a}$

$= 5\left(\frac{25a^4}{b^2}\right) \times \frac{a}{5b^2}$ ✓

$= \frac{25a^5}{b^4}$ ✓

(c) bcd must end in 5, therefore $d = 5$.

abc is divisible by 4;

possible endings: 12, 24, 16, 32

i.e. $c = 2, 4$ or 6

cde is divisible by 3

possibilities : ~~156~~, ~~153~~, ~~453~~, ~~651~~, (456), ~~654~~

def is divisible by 11; possibilities = 561

$\therefore e = 6$ and $f = 1$

$cde = 456$ ✓

$c = 4$

$\therefore$ abc ends in 24 ; $b = 2$, $a = 3$

$\therefore a = 3$

$b = 2$

$c = 4$

$d = 5$ ✓

$e = 6$

$f = 1$

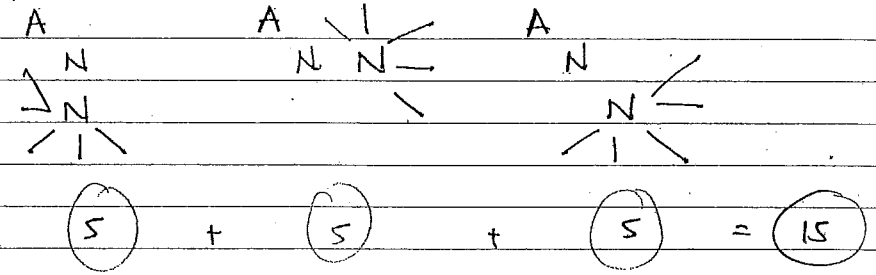
A A A A

A N N A

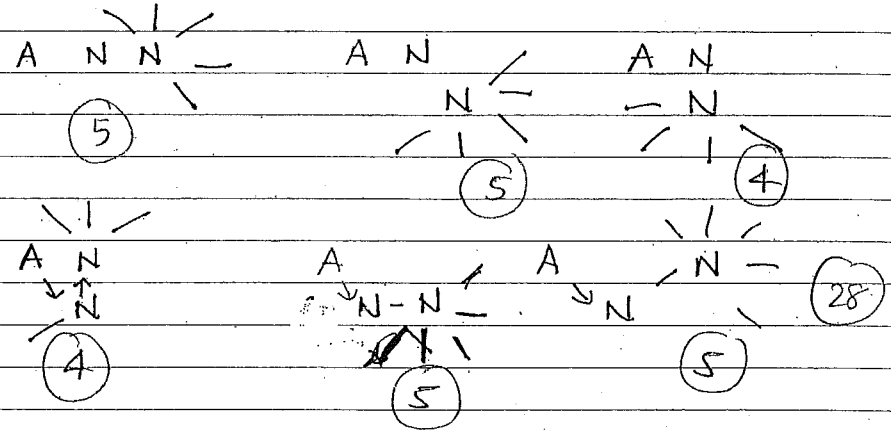
A N N A

A A A A

Starting at a Corner



Starting at an Edge



4 Corners = 4 × 15

8 Edges = 8 × 28

= 4 × 56

= 2 × 112

= 224

224

60

∴ 284 ways ✓

1 mark for a systematic attempt at counting

1 mark for final correct answer

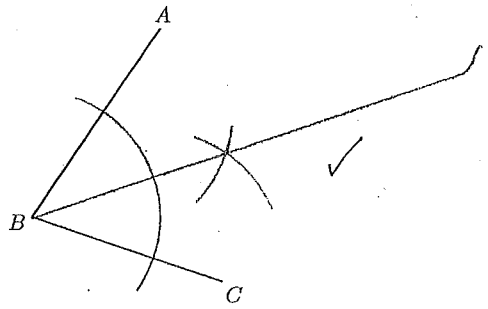
NAME: ..... CLASS: ..... MASTER: .....

DETACH THIS SHEET AND BUNDLE IT WITH THE REST OF QUESTION FOUR.

QUESTION FOUR

(c) Using a pencil, ruler and compasses only, complete the following constructions. Do not erase any construction markings.

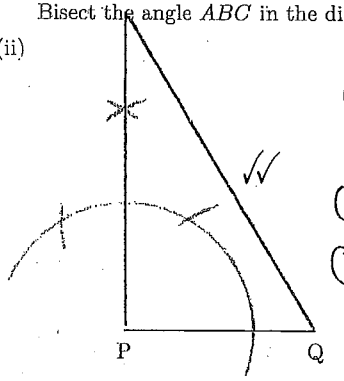
(i)



R ✓

Bisect the angle ABC in the diagram above.

(ii)



① for drawing arcs to indicate 60° and 120°
① correctly constructing 90° at P.
① completing ΔPQR such that PR=6cm
(*) Deduct ① mark for inaccurate construction.

The interval PQ is shown above.

(α) Construct an angle of 90° at P.

(β) Complete the diagram by constructing the $\triangle PQR$ in which $\angle P = 90°$ and $PR = 6$cm.