SYDNEY GRAMMAR SCHOOL

2011 Annual Examination

FORM I
MATHEMATICS

Thursday 3rd November 2011

General Instructions

- Writing time — 1 hour 30 minutes
- Write using black or blue pen.
- Calculators are not to be used.
- All necessary working should be shown in every question.
- Start each question on a new page.

Structure of the paper

- Total marks — 120
- All ten questions may be attempted.
- All ten questions are of equal value.

Collection

- Write your name, class and master clearly on each page of your answers.
- Staple your answers in a single bundle.
- Write your name and master on this question paper and submit it with your answers.

1PKR/ADS:	MLS	1RMF/CJW:	SJE	1DBD/MW:	LJF
1LAH/CDS:	SJG	1RBCH/LDR:	SO	1SD/BR:	(BR)
1PGM/AHWD:	LYL	1WTR/AGY:	JMR		

Checklist

- Writing paper required.
- Candidature — 189 boys

Examiner
MLS

QUESTION ONE (12 marks) Start a new page.

(a) Find:

(i) $-7+3$

(ii) $2\times(-8)$

(b) Evaluate:

(i) $10{\cdot}43-0{\cdot}17$

(ii) $\frac{2}{5}\times\frac{3}{7}$

(iii) $\frac{1}{8}+\frac{1}{2}$

(iv) $\frac{5}{7}\div\frac{1}{7}$

(c) Simplify:

(i) $10x-3x$

(ii) $a^9\div a^3$

(d) Solve:

(i) $x-3=5$

(ii) $7x=-28$

(iii) $\frac{x}{5}=10$

(e) Find the perimeter of an equilateral triangle with side length 2·1 cm.

Exam continues next page ...

QUESTION TWO (12 marks) Start a new page.

(a) Solve:

(i) $2x + 9 = 3$

(ii) $\dfrac{4x}{3} = 12$

(b) Express as a fraction:

(i) 17%

(ii) 0·23

(c) Express as a percentage:

(i) 2·3

(ii) $\dfrac{3}{4}$

(d) Find in simplest form:

(i) $\dfrac{8}{15} - \dfrac{3}{4}$

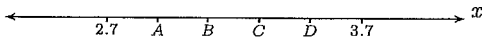
(ii) $1\dfrac{1}{8} \div \dfrac{5}{6}$

(iii) $42 \div 0{\cdot}6$

Exam continues overleaf ...

QUESTION THREE (12 marks) Start a new page.

(a)



The points on the above line are all equally spaced. Which point is closest to 3·15?

(b) Copy and complete the following table for $y = 2x - 1$.

x	-1	0	1
y			

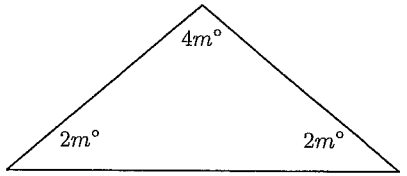
(c) Given $a = 0{\cdot}48$ and $b = 0{\cdot}12$, find:

(i) $a \times 2b$

(ii) $\dfrac{a}{b}$

(d) Find 3% of 19 metres. Give your answer in centimetres.

(e)



The triangle above has angles $4m°$, $2m°$ and $2m°$ as shown. Find the value of m, giving a reason.

(f) Using a pencil, ruler and compasses only, complete the following constructions. Do not erase any construction markings.

(i) Draw an interval 10 cm long near the bottom of a new page. Label it AB.

(ii) Construct an equilateral triangle ABC, with C lying above the interval AB.

(iii) Bisect angle CAB.

Exam continues next page ...

QUESTION FOUR (12 marks) Start a new page.

(a) Find the value of $\dfrac{18+18}{18-9}$.

(b) Expand and simplify $3x(2-4x)+5(x+1)$.

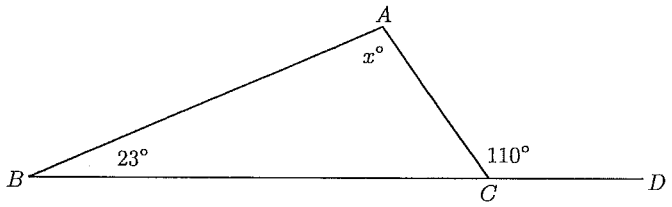
(c) I have five numbers written on cards: 486, 888, 639, 762 and 574.
Ben takes out a number and tells Sam that it is:
smaller than 870
and greater than 540
and even
and divisible by three.
What is the number?

(d) Tides alternate between low and high. The time between low and high tide at Bondi is 6 hours and 10 minutes. There is a low tide at 7:14 am. When will the next low tide occur?

(e) If 8 oranges cost \$5.60, how much do 3 oranges cost?

(f) A coffee mug holds 350 mL. How many mugs can be filled from 2·8 L of coffee?

(g)

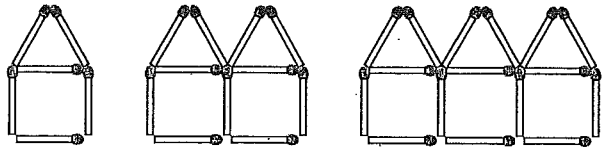


Find the value of x in the figure above, giving reasons.

Exam continues overleaf ...

QUESTION FIVE (12 marks) Start a new page.

(a)



H_1 H_2 H_3

The three matchstick diagrams above are called H_1, H_2 and H_3. The first diagram H_1 consists of one house, the second diagram H_2 consists of two houses, and so on.

n	1	2	3	4
m				

(i) Copy and complete the table above, showing the number of matchsticks m required to make n houses.

(ii) How many matchsticks are needed to construct H_6?

(iii) Write an algebraic rule for m in terms of n. That is, write a rule in terms of n for the number of matchsticks needed to construct H_n.

(b) A train travels at 80 kilometres per hour.

(i) How far does it travel in one minute?

(ii) How far does it travel in 36 minutes?

(iii) It slows down so that it covers 50 km in 100 minutes. How fast is it now travelling in kilometres per hour?

(c) Each of the numbers 11, 21, 31, 41, 51, 61, 71, 81 and 91 is written on a card. A card is selected at random. Find, in simplest form, the probability that the number on the card is:

(i) an odd number,

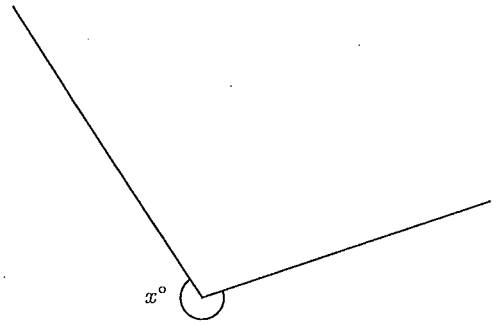
(ii) divisible by 3,

(iii) not divisible by 9,

(iv) a prime number.

Exam continues next page ...

(d)



Use a protractor to find the size of the angle marked x°. Give your answer to the nearest degree.

Exam continues overleaf ...

QUESTION SIX (12 marks) Start a new page.

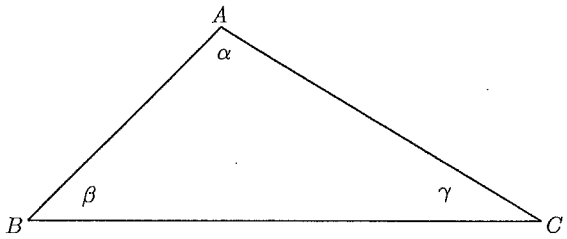
(a) Given that $2025 = 5^2 \times 3^4$, find the square root of 2025.

(b) How many small cubes with side length 2 cm can be stacked inside a box with dimensions 8 cm by 10 cm by 12 cm?

(c) (i) How many cubic centimetres of water are equivalent to six and a half litres of water? You may like to be reminded that $1\,\text{cm}^3 = 1\,\text{mL}$.

(ii) Six and a half litres of water just fill an open rectangular prism with base area $50\,\text{cm}^2$. Find the height in metres of the prism.

(d)



Copy the diagram above on your answer sheet and complete the following proof.

Theorem: The sum of the interior angles of a triangle is 180°.
Given: Let ABC be a triangle. Let $\angle A = \alpha$, $\angle B = \beta$ and $\angle C = \gamma$.
Aim: To prove that $\alpha + \beta + \gamma = 180^\circ$.
Construction:
Proof:

(e)

x	1	2	3	4
y	7	4	1	-2

Write the rule connecting x and y in the table above.

Exam continues next page ...

QUESTION SEVEN (12 marks) Start a new page.

(a) Evaluate $\dfrac{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}}{2+4+8}$.

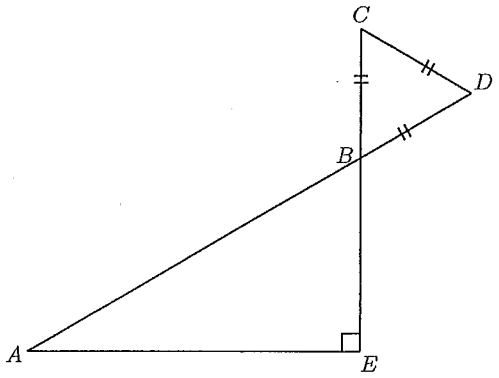
(b) Solve for x:

(i) $5x+1=2x+10$

(ii) $\dfrac{1}{2}(6-2x)=x-7$

(iii) $\dfrac{x}{7}-\dfrac{x}{8}=1$

(c)



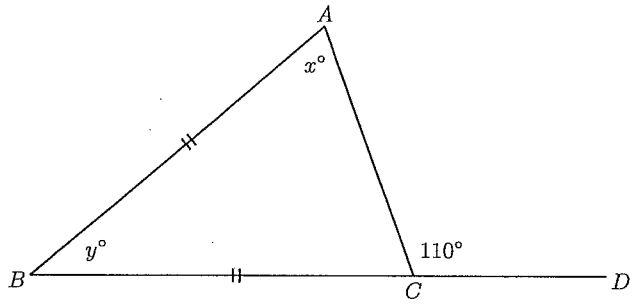
In the diagram above, the lengths BC, CD and BD are equal and $\angle BEA$ is a right-angle. Find the size of $\angle BAE$. Give reasons for your answer.

Exam continues overleaf ...

QUESTION EIGHT (12 marks) Start a new page.

(a) Evaluate $1\frac{3}{4}+2\frac{4}{5}\times 3\frac{1}{3}-2\frac{1}{2}$.

(b)



In the diagram above $AB=BC$ and $\angle ACD$ is 110°.
Find the values of x and y. Give reasons for your answers.

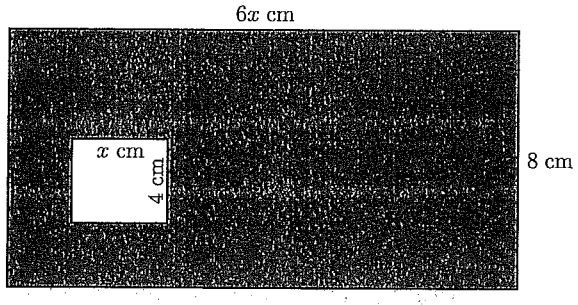
(c) (i) Using half a page, plot the points $A(-2,-1), B(4,-1)$ and $C(1,4)$ on a well labelled number plane. Use the scale 1 cm = 1 unit.

(ii) Join the points to form AB, BC and CA. What is the name of the shape formed?

(iii) Find the area of the shape formed.

Exam continues next page ...

QUESTION NINE (12 marks) Start a new page.



In the diagram above there is a small rectangle inside a large rectangle. The area of the shaded region is 121 cm^2. Find the value of x.

(b) A man has enough timber to build 24 metres of fencing around a rectangular garden. The length and width of his garden must be whole numbers. What is the maximum possible area of his garden? Show all necessary working.

(c) The third and fourth numbers in this sequence are formed by adding the two previous numbers together. Find the two missing numbers in this sequence. Show your working.

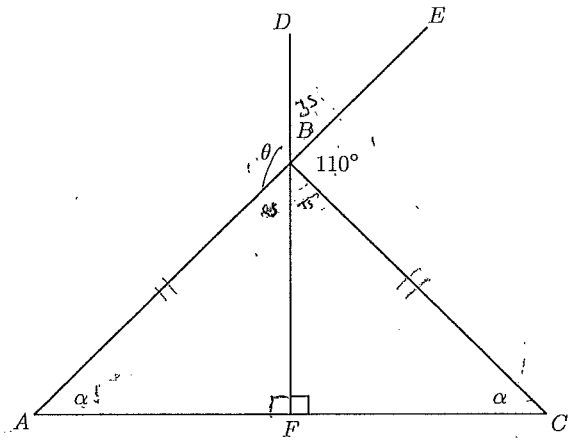
4, ..., ..., 22

(d) The length of a rectangle is increased by 15% and the width decreased by 10%. If the area of the original rectangle is 100 cm^2, find the area of the new rectangle.

Exam continues overleaf ...

QUESTION TEN (12 marks) Start a new page.

(a)



Consider the triangle ABC drawn above. The line DF passes through B and is perpendicular to AC, and the line AB is extended to E.
It is known that $\angle EBC = 110°$. Let $\angle BAC = \angle BCA = \alpha$, and let $\angle ABD = \theta$.
Find, giving reasons, the values of α and θ.

(b) In a hall, 1776 chairs are arranged so that each row contains the same number of chairs: There are also two aisles.
In each row, the number of chairs between the aisles is 39 and the rest of the chairs are equally divided between the two side sections.

(i) Suppose that there are m chairs in each side section of each row. Write an expression, using m, for the total number of chairs in a row.

(ii) Let the number of rows be n. Write an expression, using m and n, for the number of chairs in the hall.

(iii) Find the number of rows in the hall.

(c) Arrange the numbers from 1 to 15 so that any pair of adjacent numbers add to make a perfect square.

END OF EXAMINATION

SGS Form 1 November 2011. Total 120

Q1.

(a) (i) $-7+3=-4$ ✓

(ii) $2\times(-8)=-16$ ✓

(b) (i) $10.43-0.17=10.26$ ✓

(ii) $\frac{2}{5}\times\frac{3}{7}=\frac{6}{35}$ ✓

(iii) $\frac{1}{8}+\frac{1}{2}=\frac{1}{8}+\frac{4}{8}$

$=\frac{5}{8}$ ✓

(iv) $\frac{5}{7}\div\frac{1}{7}=5$ ✓

(c) (i) $10x-3x=7x$ ✓

(ii) $a^9\div a^3=a^6$ ✓

(d) (i) $x-3=5$

$x=8$ ✓

(ii) $7x=-28$

$x=-4$ ✓

(iii) $\frac{x}{5}=10$

$x=50$ ✓

(e) perimeter $=3\times2.1$ cm

$=6.3$ cm. ✓

Q2

(a) (i) $2x+9=3$

$2x=-6$

$x=-3$ ✓

(ii) $\frac{4x}{3}=12$

$4x=36$

$x=9$ ✓

(b) (i) $17\%=\frac{17}{100}$ ✓

(ii) $0.23=\frac{23}{100}$ ✓

(c) (i) $2.3=2.3\times100\%$ ✓

$=230\%$ ✓

(ii) $\frac{3}{4}=75\%$ ✓

(d) (i) $\frac{8}{15}-\frac{3}{4}=\frac{32}{60}-\frac{45}{60}$ ✓

$=\frac{-13}{60}$ ✓

(ii) $1\frac{1}{8}\div\frac{5}{6}=\frac{9}{8_4}\times\frac{6^3}{5}$ ✓

$=\frac{27}{20}$ or $1\frac{7}{20}$ ✓

(iii) $42\div0.6=420\div6$ ✓

$=70.$ ✓

Q3

(a) B ✓

(b)

x	-1	0	1
y	-3	-1	1

✓

(c) (i) $a \times 2b = 0.48 \times 2 \times 0.12$

$= 0.96 \times 0.12$

$= 0.1152$ ✓ ✓

(ii) $\frac{0.48}{0.12} = \frac{48}{12}$

$= 4$ ✓

(d) 3% of $1900\text{ cm} = \frac{3}{100} \times 1900\text{ cm}$ ✓

$= 57\text{ cm}$ ✓

(e) $4m + 2m + 2m = 180°$,

$8m = 180°$

$m = 22\frac{1}{2}°$ ✓

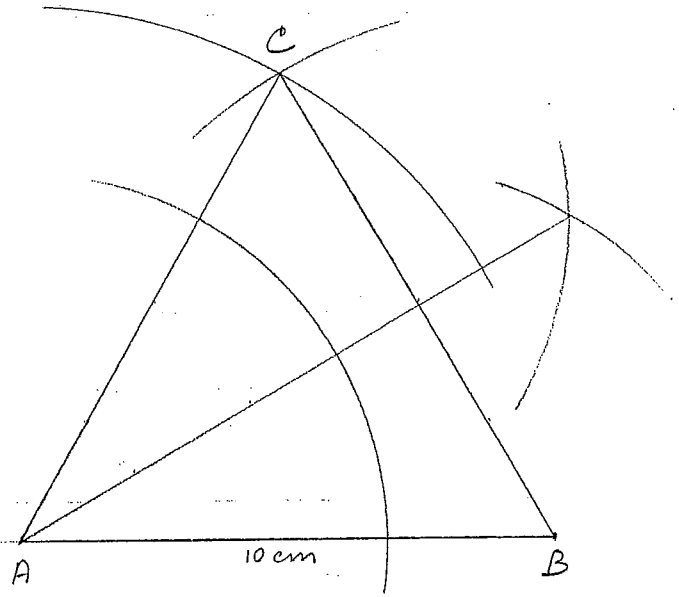
The angle sum of a triangle is $180°$ ✓

Q3 (c)

(i) ✓ for AB correct length

(ii) ✓ for C on correct perp bisect, showing construct lines

(iii) ✓ for correct bisection, showing all construction lines.



Q4.

(a) $\frac{18+18}{18-9} = \frac{36}{9} = 4$ ✓

(b) $3x(2-4x) + 5(x+1)$
$= 6x - 12x^2 + 5x + 5$ ✓
$= 11x - 12x^2 + 5$ ✓

(c) 762 ✓

(d)

Low	High	Low
7:14 am	1:24pm ✓	7:34pm ✓

(e) 8 oranges cost $5.60
1 orange costs $\$\frac{5.60}{8}$ ✓
3 oranges cost $\$\frac{5.60 \times 3}{8} = \$2.10$ ✓

(f) $2.8\,L = 2800\,mL$ ✓
Number of mugs $= \frac{2800}{350}$
$= 8$ ✓

(g) The exterior angle of a triangle equals the sum of the interior opposite angles ✓
So $x + 23 = 110$
$x = 87°$ ✓
(Can do other ways as well).

Q5

(a)(i)

n	1	2	3	4
m	6	11	16	21

✓ ✓

(ii) We need $21 + 5 + 5 = 31$ matchsticks ✓

(iii) $m = 5n + 1$ (Must need $5n+1$) ✓

(b)(i) It travels 80 km in 60 min
It travels $\frac{80}{60} = \frac{4}{3}$ km in 1 min ✓

(ii) It travels $\frac{4}{3} \times 36$ km in 36 min
$= 48$ km in 36 min ✓

(iii) It travels 50 km in 100 minutes
It travels $\frac{1}{2}$ km in 1 minute
It travels $\frac{1}{2} \times 60 = 30$ km in 60 minutes or 1 hour ✓

(c)(i) P(odd) $= 1$ ✓
(ii) P(div by 3) $= \frac{3}{9} = \frac{1}{3}$ ✓
(iii) P(not div by 9) $= \frac{8}{9}$ ✓
(iv) P(prime) $= \frac{5}{9}$ ✓

(d) $360° - 106° = 254°$ ($\pm 2°$) ✓

Q6.

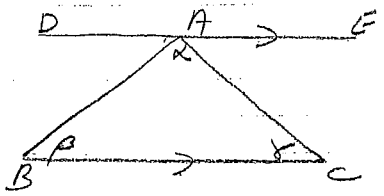
(a) $\sqrt{5^2 \times 3^4} = 5 \times 3^2$ ✓ (accept either).
$= 45$

(b) $4 \times 5 \times 6 = 120$ cubes ✓

(c) (i) $6\frac{1}{2}$ L = 6500 mL gives 6500 cm^3 ✓

(ii) Volume = 50 × h.
$6520 = 50 \times h$. ✓
$h = \frac{6520}{50}$
$= 130$ cm ✓ (need units here)
$= 1.3$ m. ✓

(d)



Construction: construct DE through A parallel to BC ✓ (or just draw it).

Proof: $\angle DAB = \beta$, alternate angles are equal, DE // BC ✓

$\angle EAC = \gamma$, alternate angles are equal, DE // BC ✓

Now adjacent angles on straight angle DAE sum to 180° ✓

So $\beta + \alpha + \gamma = 180°$ as required.

(e) $y = 10 - 3x$ ✓ ✓

Q7.

(a) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{4}{8} + \frac{2}{8} + \frac{1}{8} =$
$= \frac{7}{8}$ ✓.

$\frac{\frac{1}{2} + \frac{1}{4} + \frac{1}{8}}{2+4+8} = \frac{7}{8} \div 14$ ✓

$= \frac{7}{8} \times \frac{1}{14_2}$

$= \frac{1}{16}$. ✓

(b)

(i) $5x + 1 = 2x + 10$
$3x = 9$ ✓
$x = 3$ ✓

(ii) $\frac{1}{2}(6 - 2x) = x - 7$
$3 - x = x - 7$ ✓
$2x = 10$
$x = 5$ ✓

(iii) $\frac{x}{7} - \frac{x}{8} = 1$

$\frac{8x - 7x}{56} = 1$ ✓

$x = 56$ ✓

(c) $\angle CBD = 60°$, all angles of equilateral $\triangle CBD$ are 60° ✓

$\angle ABE = 60°$, vertically opposite angles are equal. ✓

$\angle BAE + 60° + 90° = 180°$, angle sum of $\triangle ABE$ is 180° ✓

$\angle BAE = 30°$

Q8.

(a) $1\frac{3}{4} + 2\frac{4}{5} \times 3\frac{1}{3} - 2\frac{1}{2}$.

$= \frac{7}{4} + \frac{14}{5} \times \frac{10^2}{3} - \frac{5}{2}$ ✓

$= \frac{7}{4} + \frac{28}{3} - \frac{5}{2}$ ✓

$= \frac{21}{12} + \frac{112}{12} - \frac{30}{12}$ ✓

$= \frac{103}{12}$ or $8\frac{7}{12}$ ✓

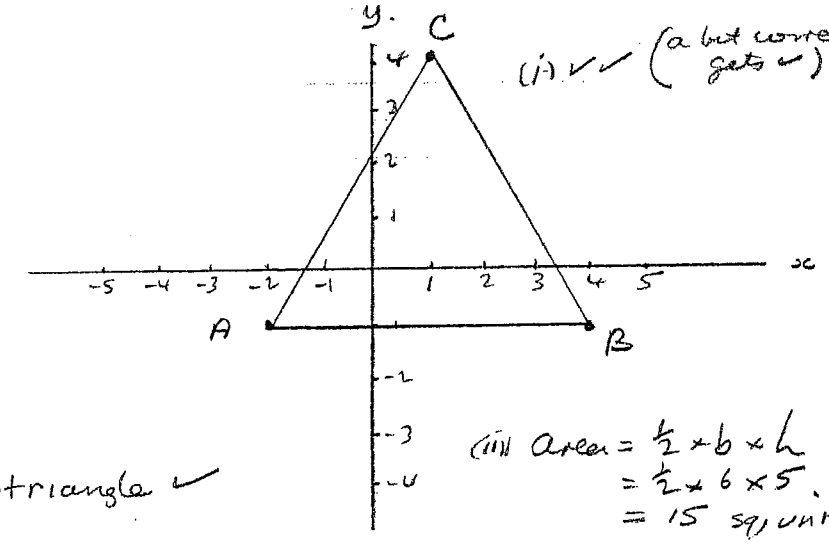
(b) $\angle ACB = 180° - 110°$, adjacent angles on straight angle BCD. ✓
$= 70°$

$\angle BAC = 70°$, angles opposite equal sides are equal. ✓

$y + 70° + 70° = 180°$, angle sum of $\triangle ABC$ is $180°$ ✓

$y = 40°$

(c)



(i) ✓✓ (a but correct gets ✓).

(ii) triangle ✓

(iii) Area $= \frac{1}{2} \times b \times h$
$= \frac{1}{2} \times 6 \times 5$. ✓
$= 15$ sq units ✓

Q9.

(a) Large rectangle area $= 6x \times 8$ cm^2
$= 48x$ cm^2 ✓

Small rectangle area $= 4 \times x$ cm^2
$= 4x$ cm^2 ✓

Shaded area $= 48x - 4x = 121$ cm^2 ✓

$44x = 121$

$x = \frac{11}{4}$ or $2\frac{3}{4}$ cm. ✓

(b)

we want $2l + 2b = 24$ or $l + b = 12$.

l	b	area
1	11	11
2	10	20
3	9	27 ✓
4	8	32
5	7	35
6	6	(36)

and we repeat.

So the largest area is 36 m^2. ✓

(c) $4, x, 4+x, 22$.

Let x be the second number

Then $x + (4+x) = 22$

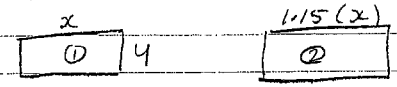
$2x + 4 = 22$

$x + 2 = 11$

$x = 9$

So missing numbers are 9 and 13. ✓✓

(d)



Area ① $= xy = 100$ cm^2 ✓

area ② $= 1.15 \times 0.9 \times x \times y$. ✓
$= 1.15 \times 0.9 \times 100$
$= 103.5$ cm^2 ✓

Q10.

(a)

The exterior angle of a triangle equals the sum of the interior opposite angles

So $2\alpha = 110°$

$\alpha = 55°$ ✓

The angle sum of $\triangle BAF$ is $180°$

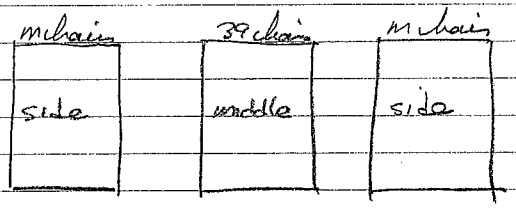
So $55° + \angle ABF + 90° = 180°$

$\angle ABF = 35°$ ✓

$\theta + 35° = 180°$, adjacent angles on a straight angle

So $\theta = 145°$ ✓

(b)



(i) $2m + 39$ ✓ is number of chairs in each row.

(ii) $(39 + 2m)n$ ✓ is number of chairs

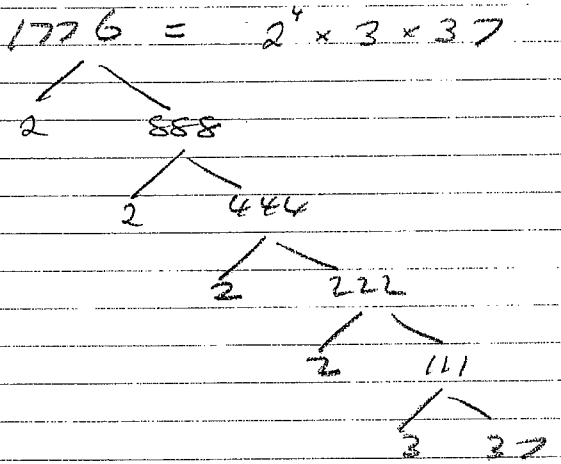
(iii) We need to find n.

Now $(39 + 2m)n = 1776$ ✓

and $39 + 2m$ is odd and it is a multiple of 1776.

So find the factors of 1776 ✓

$1776 = 2^4 \times 3 \times 37$



the odd factors are 1, 3, 37, 111

So 111 is the one we need.

$39 + 2m = 111$ ✓

But $1776 = 111 \times 16$

So n is 16 ✓

(d)

1	3	8 15
2	2	14
3	1	6 13
4	5	12
5	4	11
6	3	10
7	2	9
8	1	← must be on an end
9	7	← must be on an end.
10	6	15
11	5	14
12	4	13
13	3	12
14	2	11
15	1	10

Jews

9 7 2 14 11 5 4 12 13 3 6 10 15 1 $

✓ ✓ ✓