2011

Year 11

Preliminary Assessment Task 2

Half Yearly Examination

Monday, 30th May

ST SPYRIDON COLLEGE

Mathematics Extension 1

Weighting: 35%

Reading time: 5 minutes

Working time: 100 minutes

Marks: 48 marks

	MARK	TOTAL
Q1		12
Q2		12
Q3		12
Q4		12
		48

Topics examined:
Linear functions and lines
Internal and external division of lines into given ratios
Tangent to a curve and the derivative of a function

Outcomes assessed: P5, P6, P7, P8, PE5 and PE6

General instructions:
- Write using blue or black pen
- Board-approved calculators may be used
- All necessary working should be shown in every question
- Questions are of equal value
- Full marks may not be awarded for careless or badly arranged work
- Questions are not necessarily arranged in order of difficulty
- Begin each question on a new page.

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Attempt Questions 1-4
All questions are of equal value
Answer each question on a SEPARATE page.

Question 1 (12 marks) **Marks**

(a) The distance between $A(1,-2)$ and $B(-2,k)$ is $\sqrt{18}$. Find the value(s) of k. **3**

(b) Find the equation of the line passing through the point of intersection of $3x+y+6=0$ and $5x-2y-1=0$ which is perpendicular to $y=2x+6$. **3**

(c) The interval AB has end points $A(-1,3)$ and $B(7,-8)$.

(i) Find the coordinates of the point P which divides the interval AB **externally** in the ratio 5:3. **2**

(ii) In what ratio does the point B divide the interval PA? **1**

(d) The line $3x+ky-5=0$ makes an angle of 135° with the positive x-axis.

(i) Show that the gradient of the line is $\frac{-3}{k}$. **1**

(ii) Find the value of k. **2**

Question 2 (12 marks) Begin writing on a separate page. **Marks**

(a) Differentiate the following:

(i) $y = 3x^6 + 5x - 1 + \frac{3}{x}$ **2**

(ii) $f(x) = x(1 - 3x)^4$ **2**

(iii) $y = \frac{2x+1}{3x-2}$ **2**

(b) If the gradient of the curve $y = ax^3 - b$ at the point $(1,4)$ is equal to 2, find the values of a and b. **2**

(c) Find the gradient function of $y = \frac{x^5-7}{x^3}$ **2**

(d) Find $\lim_{x\to 3} \frac{x^2+x-12}{x^2-5x+6}$ **2**

Question 3 (12 marks) Begin writing on a separate page. **Marks**

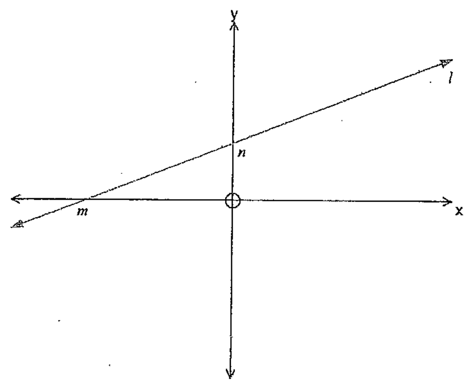
(a) Differentiate $f(x) = \frac{1}{x}$ from first principles. **3**

(b) (i) On the same set of axes, sketch the lines $4x - y + 1 = 0$ and $12x + 6y - 5 = 0$, clearly showing x and y intercepts. **2**

(ii) On your diagram, shade the region which satisfies both inequalities $4x - y + 1 \geq 0$ and $12x + 6y - 5 \leq 0$. **1**

(iii) Find the acute angle between the lines drawn in part (i), to the nearest degree. **3**

(c) **3**



In the diagram above, the line l, intersects with the axes at m and n.
Given that the area of the triangle formed by the line and the axes is 12 square units.
Find the perpendicular distance from the origin to the line, l, in terms of m and n.

Question 4 (12 marks) Begin writing on a separate page. **Marks**

(a) If $y = x\sqrt{x}$.

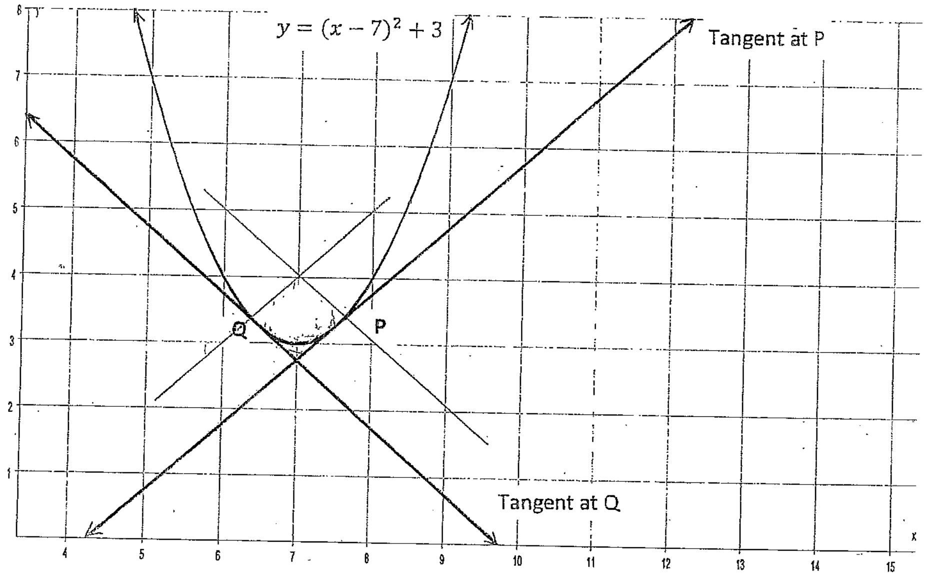
(i) Find $\frac{dy}{dx}$, leaving your answer in surd form. **1**

(ii) Find the coordinates of the point on the graph of $y = x\sqrt{x}$ at which the normal has a slope of $\frac{-1}{4}$. **2**

(b) (i) Show that the equation of the tangent to the curve $y = \frac{k}{x}$ at the point where $x = t$ is $yt^2 = -kx + 2kt$, where k is a constant. **2**

(ii) Find A and B, the x and y-intercepts of the tangent respectively. **2**

(iii) Hence, show that the area of the triangle AOB, where O is the origin, is equal to $2k$. **1**



The diagram above shows the graph of $y = (x - 7)^2 + 3$ and the tangents P and Q have gradients 1 and -1 respectively.

(i) Find the x-coordinates of the points P and Q. **2**

(ii) Show that the square formed by the tangents and normals at P and Q has area $\frac{1}{2}$ square units. **2**

End of exam.

YEAR 11 HALF-YEARLY EXAMINATION MATHS EXT 1

QUESTION 1

(a) $d = \sqrt{(-2-1)^2 + (k--2)^2}$

$d = \sqrt{9+(k+2)^2}$ ✓

$d^2 = 9+(k+2)^2$

(since $d = \sqrt{18}$

$\sqrt{18}^2 = 9+(k+2)^2$

$18 = 9+(k+2)^2$

$9 = (k+2)^2$

$\therefore k+2 = \pm 3$

$\therefore k = 1$ ✓ or $k = -5$ ✓

(b). $3x+y+6=0 \quad \ldots(1)$

$5x-2y-1=0 \quad \ldots(2)$

$6x+2y+12=0 \quad \ldots(3)$

$5x-2y-1=0 \quad \ldots(4)$

$11x \quad +11 = 0$

$11x = -11$

$x = -1$

$\therefore$ sub $x=-1$ into (1)

$3x-1+y+6=0$

$y+3=0$

$\therefore y=-3$

$\therefore$ point of intersection is $(-1,-3)$ ✓

$y = 2x+6 \quad \therefore m_1 = 2$

$m_2 = -\frac{1}{2}$ ✓ since $m_1 \times m_2 = -1$

$y--3 = \frac{-1}{2}(x--1)$

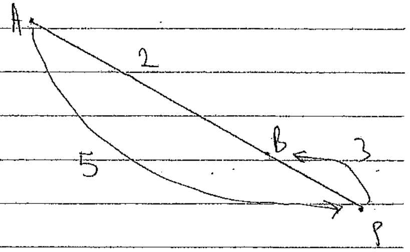
$y+3 = \frac{-1}{2}(x+1)$

$2y+6 = -x-1$

$\therefore x+2y+7=0$ ✓

(c) $A(-1,3)$ $B(7,-8)$ $P(x,y)$ $m:n$

x_1, y_1 $x_2\ y_2$ a, b $5:-3$



$a = \frac{mx_2+nx_1}{m+n}$, $b = \frac{my_2+ny_1}{m+n}$

$a = \frac{5\times7+-3\times-1}{5+-3}$, $b = \frac{5\times-8+-3\times3}{5+-3}$

$a = \frac{35+3}{2}$, $b = \frac{-40-9}{2}$

$a = \frac{38}{2}$, $b = \frac{-49}{2}$

$\therefore (a = 19$ ✓, $b = -24\frac{1}{2})$ ✓

(ii) $PB : BA$

$3 : 2$ ✓

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QUESTION 1 continued ...

(d)(i) $3x+ky-5=0$

$ky = -3x+5$

$y = \frac{-3}{k}x + \frac{5}{k}$ ✓

$\therefore m = \frac{-3}{k}$ shown

(ii) Since $\tan 135 = -\frac{3}{k}$ ✓

$-1 = -\frac{3}{k}$

$\therefore k=3$ ✓

12

QUESTION 2

(a)(i) $y = 3x^6+5x-1+3x^{-1}$

$\frac{dy}{dx} = 18x^5+5-3x^{-2}$ ✓ 2 terms

OR

$\frac{dy}{dx} = 18x^5+5-\frac{3}{x^2}$ ✓✓ all term

(ii) $u = x$ $\quad v = (1-3x)^4$

$u' = 1$ $\quad v' = 4(1-3x)^3 \times -3$

$v' = -12(1-3x)^3$

$\therefore f'(x) = (1-3x)^4 - 12x(1-3x)^3$ ✓✓

$f'(x) = (1-3x)^3(1-3x-12x)$ OR

$\therefore f'(x) = (1-3x)^3(1-15x)$

(iii) $y = \frac{2x+1}{3x-2}$ $\quad \frac{u'v-v'u}{v^2}$

$u = 2x+1$ $\quad v = 3x-2$

$u' = 2$ $\quad v' = 3$

$y' = \frac{2(3x-2)-3(2x+1)}{(3x-2)^2}$ ✓

$y' = \frac{6x-4-6x-3}{(3x-2)^2}$

$y' = \frac{-7}{(3x-2)^2}$ ✓

QUESTION 2 continued...

(b) $y = ax^3 - b$

$y' = 3ax^2$

since $m = 2$

$3ax^2 = 2$

at $x = 1$

$3a = 2$

$a = \frac{2}{3}$

$\therefore y = \frac{2}{3}x^3 - b$

sub $x=1$, $y = 4$

$4 = \frac{2}{3} - b$

$12 = 2 - 3b$

$3b = -10$

$b = \frac{-10}{3}$

$\therefore a = \frac{2}{3}$, $b = \frac{-10}{3}$ ✓ ✓

(c) $y = \dfrac{x^5 - 7}{x^3}$

$y = x^2 - 7x^{-3}$

$\therefore y' = 2x + 21x^{-4}$ ✓ ✓

OR $y' = 2x + \dfrac{21}{x^4}$

(d) $\lim\limits_{x \to 3} \dfrac{x^2 + x - 12}{x^2 - 5x + 6}$

$= \lim\limits_{x \to 3} \dfrac{(x+4)\cancel{(x-3)}}{\cancel{(x-3)}(x-2)}$ ✓

$= \dfrac{3+4}{3-2}$

$= \dfrac{7}{1}$

$= 7$ ✓

QUESTION 3

(a) $f(x) = \frac{1}{x}$

Using $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$f'(x) = \lim\limits_{h \to 0} \dfrac{\frac{1}{x+h} - \frac{1}{x}}{h}$ ✓

$f'(x) = \lim\limits_{h \to 0} \dfrac{\frac{x - (x+h)}{x(x+h)}}{h}$

$f'(x) = \lim\limits_{x \to 0} \dfrac{\frac{-h}{x(x+h)}}{h}$

$f'(x) = \lim\limits_{x \to 0} \dfrac{-h}{x(x+h)} \times \dfrac{1}{h}$

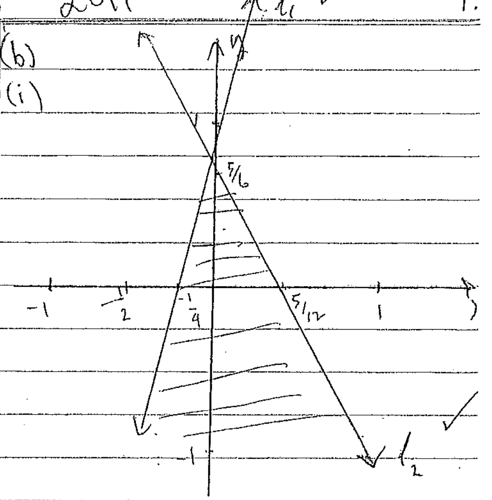
$f'(x) = \lim\limits_{x \to 0} \dfrac{-1}{x(x+h)}$ ✓

as $h \to 0$

$\therefore f'(x) = -\dfrac{1}{x^2}$ ✓

(b)

(i)



$4x - y + 1 = 0$: l_1

at $x = 0$: $-y + 1 = 0$, $y = 1$

at $y = 0$: $4x + 1 = 0$, $4x =$, $x = -\frac{1}{4}$

$12x + 6y - 5 = 0$: l_2

at $x = 0$: $6y = 5$, $y = \frac{5}{6}$

at $y = 0$: $12x = 5$, $x = \frac{5}{12}$

(ii) $4x - y + 1 \geq 0$

test $(0,0)$

$0 - 0 + \quad \geq 0$

$1 \geq 0$ Yes. ✓ correct region.

$12x + 6y - 5 \leq 0$

test $(0,0)$

$-5 \leq 0$

Yes.

QUESTION 3 continued...

(b) (iii)

$4x - y + 1 = 0$ $\quad$ $12x + 6y - 5 = 0$

$m = 4$ $\quad$ $m = -2$

Using $\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$, $m_1 = -2$, $m_2 = 4$

$\tan\theta = \frac{-2-4}{1+(-2\times 4)}$

$\tan\theta = \frac{-6}{-7}$ $\quad \therefore \theta = 41°$

(c)

Equation of l:

$y = \frac{n}{m}x + n$

$my = nx + mn$

$my - nx - mn = 0$ (in general form)

Also, Area of triangle

$A = \frac{1}{2}mn$

$12 = \frac{1}{2}mn$

$\therefore mn = 24$

$\therefore$ Equation of l: $\underline{my - nx - 24 = 0} \longrightarrow \underline{nx - my + 24 = 0}$

$a = n$, $b = m$, $c = 24$

$x_1 = 0$, $y_1 = 0$

Perpendicular distance:

$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$

$d = \frac{|0 + 0 + 24|}{\sqrt{n^2+m^2}}$

$d = \frac{24}{\sqrt{n^2+m^2}}$

QUESTION 4

(a) $y = x\sqrt{x}$

$\therefore y = x^{3/2}$

(i) $\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}}$

OR

$\therefore \frac{dy}{dx} = \frac{3\sqrt{x}}{2}$

(ii) Given normal's gradient $= -\frac{1}{4}$

$\therefore$ tangents' gradient $= 4$

$\therefore \frac{3\sqrt{x}}{2} = 4$

$3\sqrt{x} = 8$

$\sqrt{x} = \frac{8}{3}$

$\therefore x = \frac{64}{9}$

sub $x = \frac{64}{9}$

$y = \frac{64}{9} \times \sqrt{\frac{64}{9}}$

$y = \frac{64}{9} \times \frac{8}{3} = \frac{512}{27}$

$\therefore$ Coordinates $\left(\frac{64}{9}, \frac{512}{27}\right)$

$\left(7\frac{1}{9}, 18\frac{26}{27}\right)$

(b) $y = kx^{-1}$

(i)

$y' = -kx^{-2}$

at $x = t$

$y' = \frac{-k}{t^2}$

also, at $x = t$, $y = \frac{k}{t}$

$\therefore$ EQUATION OF THE TANGENT:

$y - \frac{k}{t} = \frac{-k}{t^2}(x - t)$

$t^2y - kt = -kx + kt$

$t^2y = -kx + 2kt$

SHOWN.

(ii) For A (sub $y = 0$)

$0 = -kx + 2kt$

$2kt = kx$

$2t = x$

$\therefore A(2t, 0)$

For B (sub $x = 0$)

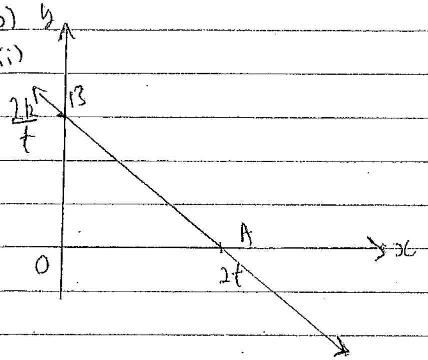
$yt^2 = 2kt$

$y = \frac{2kt}{t^2}$ $\quad \therefore y = \frac{2k}{t}$

$\therefore B\left(0, \frac{2k}{t}\right)$

QUESTION 4 continued...

(b) (iii)



Area of $\triangle AOB = \frac{1}{2} \times 2t \times \frac{2k}{t}$

$= 2k$ units^2

$\therefore$ SHOWN.

(c) $y = (x-7)^2 + 3$

$\therefore y' = 2(x-7)$

FOR P $(m = 1)$

$2(x-7) = 1$

$x - 7 = \frac{1}{2}$

$\therefore x = 7\frac{1}{2}$

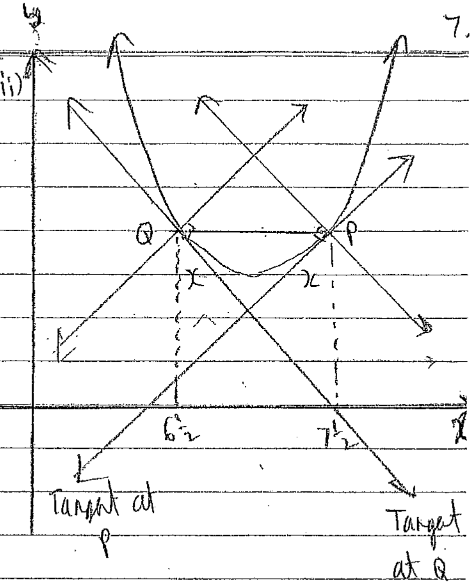
FOR Q $(m = -1)$

$-1 = 2(x-7)$

$x - 7 = -\frac{1}{2}$

$\therefore x = 6\frac{1}{2}$

(ii)



$QP^2 = x^2 + x^2$

$QP^2 = 2x^2$

$x^2 = \frac{1}{2} QP^2$

(since $QP = 1$)

$\therefore x^2 = \frac{1}{2}$, which is the area of the square.

END OF EXAM