ST SPYRIDON COLLEGE

2011
Year 11
Preliminary Assessment Task 2
Half Yearly Examination
Wednesday 1st June

# Mathematics

**Weighting:** **35 %**

**Reading time:** 5 minutes

**Working time:** 100 minutes

**Total marks:** **48**

**Topics examined:** Real functions, Linear functions and Tangent to a curve and derivative of a function.

**Outcomes assessed:** P4, P5, P6 and P7

**General instructions:**

- Candidates are advised to read the **whole** paper during the allocated reading time
- Write using blue or black pen on the writing paper provided
- Board-approved calculators and templates may be used
- All necessary working should be shown in every question
- Questions are of equal value
- Full marks may not be awarded for careless or badly arranged work
- Questions are not necessarily arranged in order of difficulty
- Begin Questions 1 – 4 on a new page
- Use the answer sheet provided on page 6 to answer question 4 (d).



Marks

QUESTION 1 - (12 Marks) (Begin on a NEW page)

(a) Differentiate with respect to $x$:

i) $7x^3+5x^2-3x$ 1

ii) $\frac{x^5+3x^3}{x^2}$ 2

(b) Find from first principles the derivative of $f(x)=4x^2+2$ with respect to $x$. 2

(c) Given that $f(x)=x^3+3x+2$ Find:

(i) the equation of the tangent at the point $P(0,2)$. 2

(ii) the equation of the normal at the point $P(0,2)$. 2

(iii) the area of the triangle formed with the $x$-axis, the tangent and the normal. (Hint: A sketch would help here.) 3

## Question 2:

(a) $m(AC) = \frac{3-0}{6-2} = \frac{3}{4}$ ✓

(b) eqn AC: $y - 0 = \frac{3}{4}(x-2)$ ✓

$4y = 3x - 6$.

$3x - 4y - 6 = 0$. ✓

(-one mark for general form)

(c) $\tan\theta = m$

$\tan\theta = \frac{3}{4}$ ✓

$\theta = 36^\circ 52'$ ✓ (-1 mark for nearest minute only once).

(d) $m(BA) = -3$ ✓

$\tan\theta = -3$

$\theta = 108^\circ 26'$ ✓ (obtuse angle)

$\therefore \angle BAC = 108^\circ 26' - 36^\circ 52'$ ✓

$= 71^\circ 34'$

(e) $3x - 4y - 6 = 0$ (AC)

B (0, 6)

perpendicular distance $= \frac{|3(0) - 4(6) - 6|}{\sqrt{3^2 + (-4)^2}}$ ✓

$= \frac{30}{5} = 6$ ✓ units

(f) $AC = \sqrt{(6-2)^2 + (3-0)^2}$

$= \sqrt{25}$

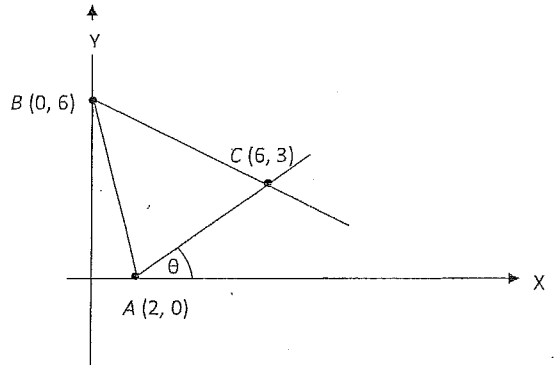
$= 5$ ✓

Area $\triangle ABC = \frac{1}{2}bh$

$= \frac{1}{2} \times 5 \times 6$

$= 15$ units$^2$ ✓

QUESTION 2 - (12 Marks) (Begin on a NEW page) Marks




Not to Scale

The points *A, B, C* have the coordinates (2, 0), (0, 6) and (6, 3) as shown in the diagram.
The angle between the line *AC* and the *x*-axis is $\theta$.

(a) Find the gradient of the line *AC*. 1

(b) Find the equation of the line *AC* in general form. 2

(c) Calculate the size of the angle $\vartheta$ to the nearest minute. 2

(d) Find the size of angle *BAC* to the nearest minute. 3

(e) Find the perpendicular distance of the point *B* to the line *AC*. 2

(f) Find the length of *AC* and hence calculate the area of *ABC*. 2



QUESTION 3 - (12 Marks) (Begin on a NEW page) Marks

(a) The line $2kx - (k + 1)y = 5$ is perpendicular to the line $3x + y - 4 = 0$. Find the value of k. 2

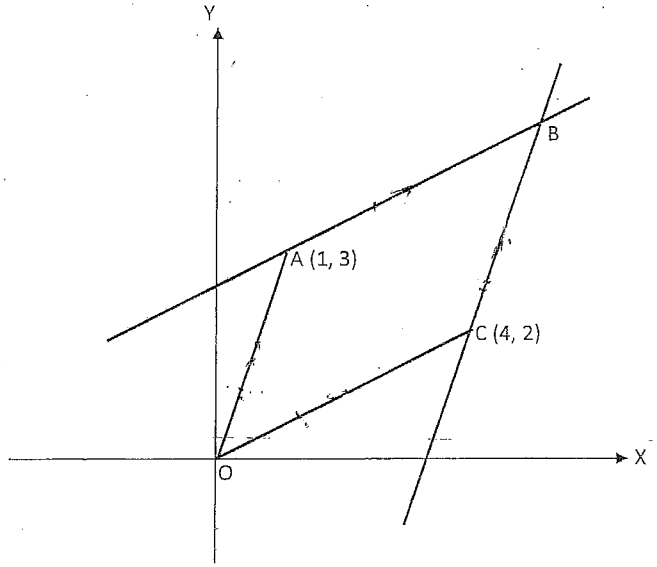
(b) A, B and C are the vertices of a right-angled triangle with the right angle at B. The coordinates of A are (2, -2) and B(-4, 1).

(i) Show that the gradient of AB is $-\frac{1}{2}$. 1

(ii) Show that the equation of AB is $x + 2y + 2 = 0$. 2

(iii) Derive the equation of the line BC 2

(c)




The equation of *AB* is $x-2y+5=0$, the equation of *BC* is $3x-y-10=0$, the point *A* has coordinates (1, 3) and the point *C* has coordinates (4, 2).

(i) Prove that the coordinates of *B* are (5, 5). 2

(ii) Prove that *OABC* is a parallelogram. 3

QUESTION 4 - (12 Marks) (Begin on a NEW page) Marks

(a) If $f(x) = 3kx + 1$ and $f(-2) = 7$, find the value of $k$ 2

(b) For the function $y = x^2 - 2x$

(i) Find $f(-x)$ expressed in its simplest form. 1

(ii) Find $f(x+2)$ expressed in its simplest form. 2

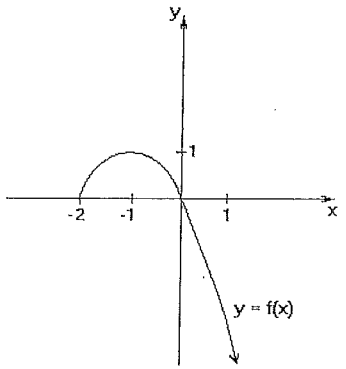
(iii) Is the function $y = x^2 - 2x$, odd, even or neither? Give reasons for your answer. 2

(c) (i) Sketch the following graph, clearly showing x and y intercepts. 1

(ii) State the domain and range. 2

$y = |x - 2| + 3$

(d) On your answer sheet provided, draw a sketch of the function $y = 1 - f(x)$. 2




**END OF EXAMINATION**

2011

## Question 1:

(a) (i) $y = 7x^3 + 5x^2 - 3x$

$\frac{dy}{dx} = 21x^2 + 10x - 3$ ✓

(ii) $y = \frac{x^5}{x^2} + \frac{3x^3}{x^2}$

$y = x^3 + 3x$ ✓

$\frac{dy}{dx} = 3x^2 + 3$ ✓

(b) $f(x) = 4x^2 + 2$

$f(x+h) = 4(x+h)^2 + 2$

$= 4(x^2 + 2xh + h^2) + 2$

$= 4x^2 + 8xh + 4h^2 + 2$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{4x^2 + 8xh + 4h^2 + 2 - 4x^2 - 2}{h}$ ✓

$= \lim_{h \to 0} \frac{h(8x + 4h)}{h}$

$= 8x + 0$

$= 8x$ ✓

$\therefore f'(x) = 8x$

if left off $\lim_{h \to 0}$ take off marks in future!

(c) $f(x) = x^3 + 3x + 2$

$f'(x) = 3x^2 + 3$ ✓

(i) At $P(0,2)$, $m = 3(0)^2 + 3$

$= 3$ ✓

eqn of tangent: $y - 2 = 3x$

$y = 3x + 2$ ✓

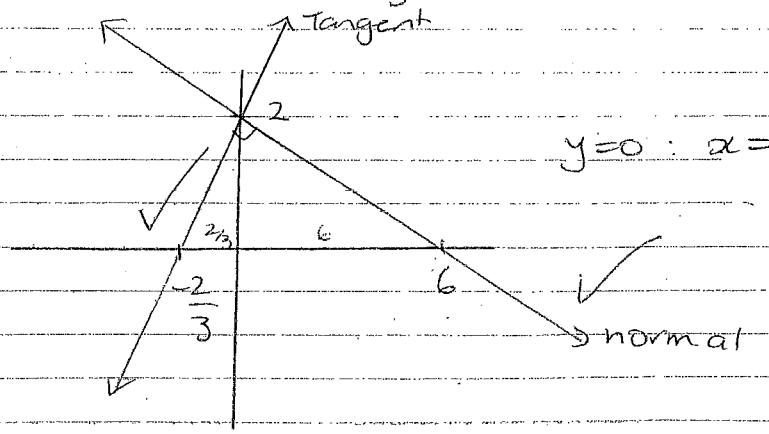
(ii) $m(\text{normal}) = -\frac{1}{3}$ ✓

eqn of normal: $y - 2 = -\frac{1}{3}(x - 0)$

$y = -\frac{1}{3}x + 2$ ✓

OR: $x + 3y - 6 = 0$

(iii)




$y = 0$: $3x = -2$

$x = -\frac{2}{3}$

$y = 0$: $x = 6$

$A = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 6\frac{2}{3} \times 2$

$= 6\frac{2}{3}\ u^2$ ✓

## Question 3:

(a) $2kx-(k+1)y=5$

$(k+1)y=2kx-5$

$y=\frac{2k}{k+1}x-\frac{5}{k+1}$

$m(l_1)=\frac{2k}{k+1}$ ✓

$3x+y-4=0$

$y=-3x+4$ OR ✓

$m(l_2)=-3$ ✓ $m(l_1)=\frac{1}{3}$

Perpendicular $\Rightarrow m(l_1)\times m(l_2)=-1$

ie. $\frac{2k}{k+1}x-3=-1$ ✓

$\frac{-6k}{k+1}=-1$

$6k=k+1$

$5k=1$

$k=\frac{1}{5}$ ✓

(b) (i) $m(AB)=\frac{1+2}{-4-2}$ ✓

$=\frac{3}{-6}$

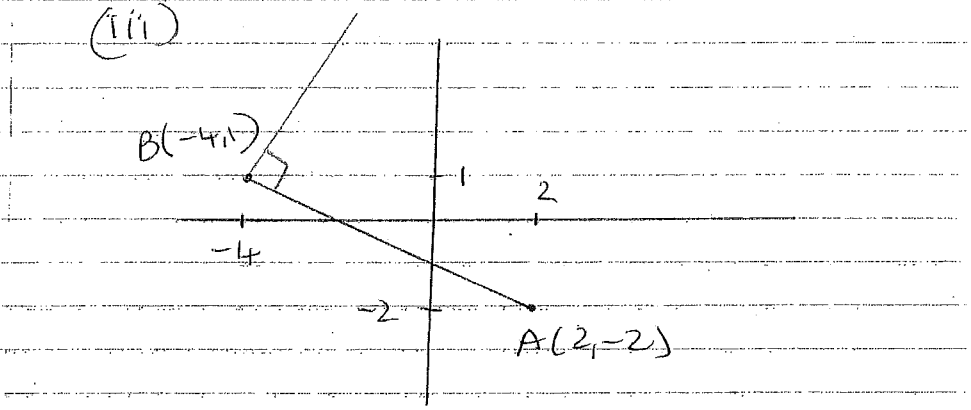
$=-\frac{1}{2}$ ✓

(ii) eqn AB: $y+2=-\frac{1}{2}(x-2)$ ✓

$2y+4=-x+2$

$x+2y+2=0$ ✓

(iii)




right $\angle$ at B $\Rightarrow$ m of BC $= 2$ ✓

eqn of BC: $y-1=2(x+4)$

$y-1=2x+8$

$2x-y+9=0$ ✓

(c) (i) AB: $x - 2y + 5 = 0$ — (1)
BC: $3x - y - 10 = 0$ — (2)

From (2): $y = 3x - 10$

sub in (1): $x - 2(3x - 10) + 5 = 0$ ✓

$x - 6x + 25 = 0$

$5x = 25$

$x = 5$ ✓

$y = 15 - 10 = 5$

∴ lines AB and BC intersect at B

∴ B(5, 5).

OR AB: sub in (5, 5).

$x - 2y + 5$

$= 5 - 2(5) + 5$

$= 0$.

∴ lies on AB. ✓

BC: $3x - y - 10$

$= 3(5) - 5 - 10$

$= 0$.

∴ lies on BC. ✓

∴ B (5, 5).

Note: Not sufficient to substitute B(5, 5) into AB OR BC – Must be both AB and BC.

(ii) $m(OA) = 3$.

$m(BC) = \frac{5-2}{5-4} = 3$ ✓

∴ OA || BC.

$m(OC) = \frac{2}{4} = \frac{1}{2}$

$m(AB) = \frac{5-3}{5-1} = \frac{1}{2}$ ✓

∴ OC || AB.

∴ OABC is a parallelogram ✓ as the opposite sides are parallel.

Other appropriate answers accepted

## Question 4:

(a) $f(x) = 3kx + 1$

$f(-2) = 7 \Rightarrow 3k(-2) + 1 = 7$ ✓

$-6k + 1 = 7$

$-6k = 6$

$k = -1$. ✓

(b) $y = x^2 - 2x$

$f(x) = x^2 - 2x$

(i) $f(-x) = (-x)^2 - 2(-x)$

$= x^2 + 2x$. ✓

(ii) $f(x+2) = (x+2)^2 - 2(x+2)$ ✓

$= x^2 + 4x + 4 - 2x - 4$ ✓

$= x^2 + 2x$. ✓

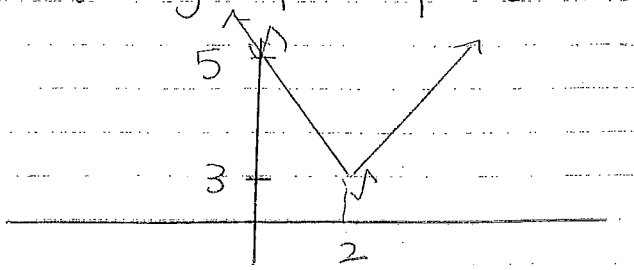
(iii) $-f(x) = -x^2 + 2x$. ✓

$f(x) \neq f(-x)$

$f(-x) \neq -f(x)$ ✓

$\therefore$ $f(x)$ is neither odd or even.

(c) (i) $y = |x - 2| + 3$




$x = 0, y = 5$

$y = 0$, no soln

✓

(ii) D: all real $x$. ✓

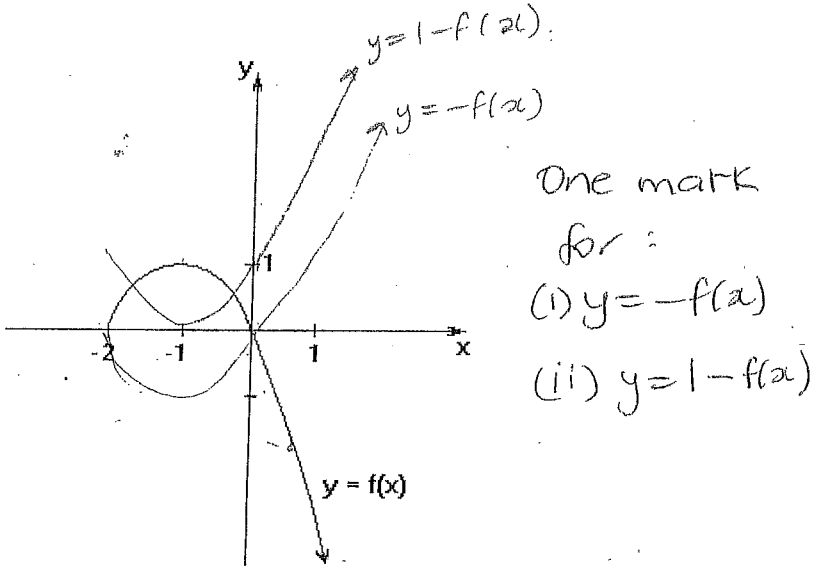
R: $y \geq 3$. ✓

(d) See sheet.

Name: _____

**Use this sheet to answer Question 4:**

(d) You must detach this sheet and hand it in with your answers for question 4.




One mark for:

(i) $y = -f(x)$

(ii) $y = 1 - f(x)$