SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

December 2010
Assessment Task 1
Year 11

Mathematics Extension

General Instructions

- Reading Time – 5 Minutes
- Working time – 90 Minutes
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators maybe used.
- All necessary working should be shown in every question if full marks are to be awarded.
- All answers to be given in simplified exact form unless otherwise stated.
- Marks may not be awarded for messy or badly arranged work

Total Marks – 74

- Attempt questions 1-4
- All questions are **NOT** of equal value.
- Start each new question in a separate answer booklet.
- Hand in your answers in 4 separate bundles:

Question 1, Question 2, Question 3 and Question 4

Examiner: *R Boros*

This is an assessment task only and does not necessarily reflect the content or format of the Higher School Certificate

Start a new booklet.

Question 1 (18 marks). **Marks**

a) Express $\frac{7\pi^c}{18}$ in degrees. 1

b) Find the value of k if $(x-2)$ is a factor of $P(x)=x^4-3x^3+kx^2-4$ 1

c) Solve for x, $\frac{x+1}{x-3}\geq 2$ 2

d) Find the acute angle, to the nearest degree, between the lines $y=2x+3$ and $x+y=0$ 2

e) Find the general solution of $2\cos\theta-1=0$, where θ is in radians. 3

f) Prove that $\tan\left(\frac{\pi}{4}+A\right)+\tan\left(\frac{\pi}{4}-A\right)=2\sec 2A$. 3

g) (i) In how many ways can 3 men and 3 women be arranged in a circle? 1

(ii) How many of these arrangements have at least 2 women sitting next to each other? 2

h) Find the co-ordinates of the point P which divides the interval joining A(-1,-3) and B(3,7) externally in the ratio 5:3. 3

End of Question 1

Start a new booklet.

Question 2 (18 Marks). **Marks**

a) The roots of a cubic polynomial equation are 0, 1 and 3; and the coefficient of x^3 is 2. Find the polynomial in full expanded form. 3

b) Solve for n : $2 \times {}^nC_4 = 5 \times {}^nC_2$. 2

c) If α, β and γ are the roots of $x^3 - 3x + 1 = 0$, find;

(i) $\alpha + \beta + \gamma$ 1

(ii) $\alpha\beta\gamma$ 1

(iii) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ 2

d) (i) Express $\sqrt{3}\cos x - \sin x$ in the form 2

$$R\cos(x+\alpha),\ R > 0 \text{ and } 0 \le \alpha \le \frac{\pi}{2}.$$

(ii) Hence, find the general solution for $\sqrt{3}\cos x - \sin x = 1$. 2

e) α and β are acute angles, such that $\cos\alpha = \frac{3}{5}$ and $\sin\beta = \frac{1}{\sqrt{5}}$. Without finding the size of either angle, show that $\alpha = 2\beta$. 2

f) Using the identity $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$, solve the equation $\sin 3\theta = 2\sin\theta$, where $0 \le \theta \le 2\pi$. 3

End of Question 2

Start a new booklet.

Question 3 (18 marks). **Marks**

a) (i) Show that the equation $\sin x^\circ - 3\cos x^\circ = \tan\left(\frac{x}{2}\right)^\circ$ may be written as 3

$t^3 - 3t^2 - t + 3 = 0$, where $t = \tan\left(\frac{x}{2}\right)^\circ$.

(ii) Hence, find all 3 solutions of the equation $\sin x^\circ - 3\cos x^\circ = \tan\left(\frac{x}{2}\right)^\circ$, for 3

$0^\circ \le x \le 360^\circ$. Give answers to the nearest minute, if necessary.

b) Six identical yellow discs and 4 identical blue discs are placed in a straight line. 1

How many arrangements are possible?

c) (i) In how many ways can a committee of 2 Australians, 2 New Zealanders and 1 Nauruan be chosen from 6 Australians, 7 New Zealanders and 3 Nauruans? 2

(ii) In how many of these ways do 2 friends, a particular Australian and a particular New Zealander, belong to the committee? 1

d) For the parabola $x^2 = 12y$:

(i) Derive the equation of the tangent at $(6t, 3t^2)$. 2

(ii) Find the equations of the two tangents that pass through the point $(5, -2)$. 3

e) Find the value of the constants p and q if $x^2 - 4x + 3$ is a factor of 3

$x^3 + px^2 - x + q$.

End of Question 3

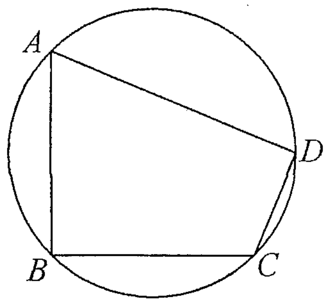
Start a new booklet.

Question 4 (20 marks). **Marks**

a) In how many ways can 8 different books be arranged in a row such that 3 particular books are always together? 2

b) $ABCD$ is a cyclic quadrilateral. One of its properties is that opposite angles are supplementary. Show that 2

$\tan A + \tan B + \tan C + \tan D = 0$



c) Eight people including James and Sarah are to be seated around a table. How many arrangements are possible if James and Sarah do not wish to sit next to each other. 2

d) The straight line $y = mx + b$ meets the parabola $x^2 = 4ay$ at the points $P\left(2ap, ap^2\right)$ and $Q\left(2aq, aq^2\right)$

(i) Find the equation of the chord PQ and, hence or otherwise, show that 2

$pq = -\dfrac{b}{a}$

(ii) Prove that $p^2 + q^2 = 4m^2 + \dfrac{2b}{a}$ 2

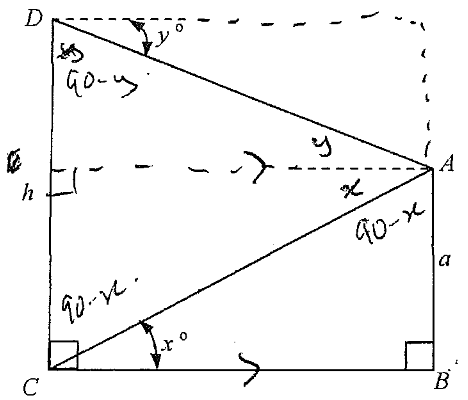
(iii) Given that the equation of the normal to the parabola at P is 2
$x + py = 2ap + ap^3$, and that, N, the point of intersection of the normals at P and Q has the co-ordinates $\left[-apq(p+q),\ a(2+p^2+pq+q^2)\right]$, express these co-ordinates in terms of a, m and b.

(iv) Suppose that the chord PQ is free to move while maintaining a fixed gradient. Find the locus of N, and show that this locus is a straight line. Verify that this line is a normal to the parabola. 2

Question 4 continues on next page

Question 4 (continued).

e)



From the foot of a tower CD, the angle of elevation of a building AB, 'a' metres high, is $x°$. From the top of the tower, D, the angle of depression to the top of the building, A is $y°$. Show that the height, 'h', of the tower is given by: 6

$$h = \frac{a\sin(x+y)}{\sin x \cos y}$$

End of Question 4

End of Examination