



## 5. Factorisation

1. Which of the following are true?

- A. The factors of 6 are 3 and 12.
- B. 3 and 12 are factors of 36.
- C. 8 is a factor of 36.
- D. 1 is a factor of 36.
- E. There are 9 factors of 36.

Ans: B, D, E

2. Which of the following lists contain only prime numbers?

- A. 3, 11, 17, 53
- B. 1, 2, 3, 5, 7
- C. 13, 17, 23, 71
- D. 13, 17, 23, 51
- E. 7, 19, 63, 71

Ans: A, C

3. Complete the following with a number:

A difference of perfect squares method may be used to factorise an expression with ... terms.

Ans: Two

4. Complete the following with a number:

A quadratic trinomial has ... terms.

Ans: Three

5. Complete the following with numbers:

An expression with four terms can be factorised by grouping together ... and ... or ... and ...

Ans: two, two, three, one

6. List the factors of 96.

Ans: 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96

7. List the factors of 126.

Ans: 1, 2, 3, 6, 7, 9, 14, 18, 21, 42, 63, 126

8. List the prime factors of 96.

Ans: 2, 3

9. List the prime factors of 126.

Ans: 2, 3, 7

10. Write 24 as a product of prime factors.

Ans:  $2 \times 2 \times 2 \times 3$

11. Write 56 as a product of prime factors.

Ans:  $2 \times 2 \times 2 \times 7$

12. Write 100 as a product of prime factors.

Ans:  $2 \times 2 \times 5 \times 5$

13. Which of the following are common factors of 30 and 40?

- A. 2 and 5
- B. 3 and 4
- C. 5 and 10
- D. 10 and 20
- E. 30 and 40

Ans: A, C

14. Which of the following are common factors of 180 and 240?

- A. 18 and 24
- B. 40 and 60
- C. 20 and 30
- D. 9 and 12
- E. 36 and 48

Ans: C

15. What is the HCF of 12 and 18?

Ans: 6

16. What is the HCF of 45 and 54?

Ans: 9

17. What is the HCF of  $2a^3b^2$  and  $6ab^3$ ?

Ans:  $2ab^2$

18. What is the HCF of  $75abc$  and  $30ab^3$ ?

Ans:  $15ab$

19. What is the HCF of  $9(x - 1)^2y^3$  and  $15(x - 1)yz$ .

Ans:  $3(x - 1)y$

20. Factorise  $3a + 15b$ .

Ans:  $3(a + 5b)$

21. Factorise  $24x^2 - 3yz$ .

Ans:  $3(8x^2 - yz)$

22. Factorise  $24x^2 - 3xyz$

Ans:  $3x(8x - yz)$

23. Factorise  $a^3 - a^2b$

Ans:  $a^2(a - b)$

24. Factorise  $-2ab^2c - 3a^3b + 6a^2bc$

Ans:  $-ab(2bc + 3a^2 - 6ac)$

25. Factorise  $x^2(x + 1) - x(x + 1)^2$

Ans:  $-x(x + 1)$

26. Factorise  $a^2(b - c)^2 - a^2(b - c)$

Ans:  $a^2(b - c)(ab - ac - 1)$

27. Factorise  $(x + 1)^2 + x + 1$

Ans:  $(x + 1)(x + 2)$

28. Factorise  $(x + 1)^2 - (x + 1)$

Ans:  $x(x + 1)$

29. Factorise  $(x + 1)^2 - x - 1$

Ans:  $x(x + 1)$

30. Factorise  $(x + y)p - (x + y)(q + 1)$

Ans:  $(x + y)(p - q - 1)$

31. Factorise  $(x + y)p - (x + y)q$

Ans:  $(x + y)(p - q)$

32. Factorise  $x^2 - 64$

Ans:  $(x - 8)(x + 8)$

33. Factorise  $2x^2 - 128$

Ans:  $2(x - 8)(x + 8)$

34. Factorise  $9a^2 - b^2$

Ans:  $(3a - b)(3a + b)$

35. Factorise  $9a^2 - 36b^2$

Ans:  $9(a - 2b)(a + 2b)$

36. Factorise  $(x + y)^2 - 36$

Ans:  $(x + y - 6)(x + y + 6)$

37. Factorise  $(x + y)^2 - (x + 1)^2$

Ans:  $(2x + y + 1)(y - 1)$

38. Factorise  $(x + y)^2 - (x + 3y)^2$

Ans:  $-4y(x + 2y)$

39. Factorise  $2(x + y)^2 - 72$

Ans:  $2(x + y - 6)(x + y + 6)$

40. Factorise  $4(x + y)^2 - 16(x + 1)^2$

Ans:  $4(3x + y + 1)(y - x - 2)$

41. Factorise  $32(x + y)^2 - 8(x + 3y)^2$

Ans:  $8(x - y)(3x + 4y)$

42. Factorise  $x^2 + 11x + 30$

Ans:  $(x + 5)(x + 6)$

43. Factorise  $x^2 + 16x + 63$

Ans:  $(x + 7)(x + 9)$

44. Factorise  $x^2 - 2x - 63$

Ans:  $(x + 7)(x - 9)$

45. Factorise  $x^2 - x - 30$

Ans:  $(x + 5)(x - 6)$

46. Factorise  $x^2 - 5x + 6$

Ans:  $(x - 3)(x - 2)$

47. Factorise  $x^2 - 10x + 24$

Ans:  $(x - 4)(x - 6)$

48. Factorise  $x^2 - 10xy + 21y^2$

Ans:  $(x - 3y)(x - 7y)$

49. Factorise  $x^2 - 8xy + 15y^2$

Ans:  $(x - 3y)(x - 5y)$

50. Factorise  $x^2 + 5xy - 24y^2$

Ans:  $(x + 8y)(x - 3y)$

51. Factorise  $3x^2 + 16x + 5$

Ans:  $(3x + 1)(x + 5)$

52. Factorise  $8x^2 + 26x + 21$

Ans:  $(2x + 3)(4x + 7)$

53. Factorise  $10x^2 + 18x - 4$

Ans:  $(5x - 1)(2x + 4)$

54. Factorise  $12x^2 + x - 6$

Ans:  $(3x - 2)(4x + 3)$

55. Factorise  $14x^2 - 38x + 20$

Ans:  $2(7x - 5)(x - 2)$

56. Factorise  $24x^2 - 32x + 10$

Ans:  $2(6x - 5)(2x - 1)$

57. Factorise  $6x^2 - 23x + 15$

Ans:  $(6x - 5)(x - 3)$

58. Factorise  $6x^2 + 13xy + 6y^2$

Ans:  $(2x + 3y)(3x + 2y)$

59. Factorise  $6x^2 + 7xy + 6y^2$

Ans:  $(6x + y)(x + 6y)$

60. Factorise  $6x^2 - 10xy + 3y^2$

Ans:  $(3x - y)(2x - 3y)$

61. Factorise  $x^3 - 5x^2 + 6x$

Ans:  $x(x - 3)(x - 2)$

62. Factorise  $3x^3 - 6x^2 + 3x$

Ans:  $3x(x - 1)^2$

63. Factorise  $x^4 + 2x^2 + x^2$

Ans:  $x^2(x + 1)^2$

64. Factorise  $(x - 2)^2 + 4(x - 2) + 4$

Ans:  $x^2$

65. Factorise  $(x + 4)^2 - (x + 4) - 30$

Ans:  $(x - 2)(x + 9)$

66. Factorise over R:  $x^2 - 5$

Ans:  $(x - \sqrt{5})(x + \sqrt{5})$

67. Factorise over R:  $x^2 - 12$

Ans:  $(x - 2\sqrt{3})(x + 2\sqrt{3})$

68. Factorise over R:  $5y^2 - 25$

Ans:  $5(y - \sqrt{5})(y + \sqrt{5})$

69. Factorise over R:  $6x^2 - 72$

Ans:  $6(x - 2\sqrt{3})(x + 2\sqrt{3})$

70. Factorise over R:  $3a^2 - 24$

Ans:  $3(a - 2\sqrt{2})(a + 2\sqrt{2})$

71. Factorise over R:  $(x - 1)^2 - 5$

Ans:  $(x - 1 - \sqrt{5})(x - 1 + \sqrt{5})$

72. Factorise over R:  $(x - 4)^2 - 6$

Ans:  $(x - 4 - \sqrt{6})(x - 4 + \sqrt{6})$

73. Factorise over R:  $(x + 3)^2 - 8$

Ans:  $(x + 3 - 2\sqrt{2})(x + 3 + 2\sqrt{2})$

74. Factorise over R:  $(x + 1)^2 - 24$

Ans:  $(x + 1 - 2\sqrt{6})(x + 1 + 2\sqrt{6})$

75. Factorise over R:  $x^2 + 6x - 1$

Ans:  $(x + 3 - \sqrt{10})(x + 3 + \sqrt{10})$

76. Factorise over R:  $x^2 - 4x - 7$

Ans:  $(x - 2 - \sqrt{11})(x - 2 + \sqrt{11})$

77. Factorise over R:  $a^2 + 8a - 1$

Ans:  $(a + 4 - \sqrt{17})(a + 4 + \sqrt{17})$

78. Factorise over R:  $b^2 - 10b + 3$

Ans:  $(b - 5 - \sqrt{22})(b - 5 + \sqrt{22})$

79. Factorise over R:  $c^2 + 4c + 1$

Ans:  $(c + 2 - \sqrt{3})(c + 2 + \sqrt{3})$

80. Factorise over R:  $x^2 + 6x + 1$

Ans:  $(x + 3 - 2\sqrt{2})(x + 3 + 2\sqrt{2})$

81. Factorise over R:  $y^2 - 14y + 22$

Ans:  $(y - 7 - 3\sqrt{3})(y - 7 + 3\sqrt{3})$

82. Factorise over R:  $x^2 + 16x + 37$

Ans:  $(x + 8 - 3\sqrt{3})(x + 8 + 3\sqrt{3})$

83. Factorise  $2ab - bc + 2a - c$

Ans:  $(2a - c)(b + 1)$

84. Factorise  $a^3 + 9 - 9a - a^2$

Ans:  $(a - 1)(a - 3)(a + 3)$

85. Factorise  $pq - r + pq^2 - rq$

Ans:  $(pq - r)(1 + q)$

86. Factorise  $a^2 - b^2 - 5a + 5b$

Ans:  $(a - b)(a + b - 5)$

87. Factorise  $-6a + bc + ab - 6c$

Ans:  $(a + c)(b - 6)$

88. Factorise  $a^2 - b^2 + 3a - 3b$

Ans:  $(a - b)(a + b + 3)$

89. Factorise  $3x^3 - 4x^2 + 3x - 4$

Ans:  $(3x - 4)(x^2 + 1)$

90. Factorise  $3x^3 - 4x^2 - 3x + 4$

Ans:  $(3x - 4)(x - 1)(x + 1)$

91. Factorise  $x^2 - 4x + 4 - 9y^2$

Ans:  $(x - 2 - 3y)(x - 2 + 3y)$

92. Factorise  $x^2 + 8xy + 16y^2 - z^2$

Ans:  $(x + 4y - z)(x + 4y + z)$

93. Factorise  $x^2 - y^2 - 12y - 36$

Ans:  $(x + y + 6)(x - y - 6)$

94. Factorise and simplify  $\frac{6a + 3}{4a + 2}$

Ans:  $\frac{3}{2}$

95. Factorise and simplify  $\frac{x^2 + 4x + 4}{x^2 - 4}$

Ans:  $\frac{x + 2}{x - 2}$

96. Factorise and simplify  $\frac{x^2 - 2x - 15}{x^2 + 5x + 6}$

Ans:  $\frac{x - 5}{x + 2}$

97. Factorise and simplify  $\frac{3x + 18}{x - 5} \times \frac{2x - 10}{6}$

Ans:  $x + 6$

98. Factorise and simplify  $\frac{x^2 + 5x + 6}{x^2 + x - 2} \times \frac{x^2 - 4}{x^2 + x - 6}$

Ans:  $\frac{x + 2}{x - 1}$

99. Factorise and simplify  $\frac{5x+15}{2} + \frac{x+3}{2x+6}$

Ans:  $5(x+3)$

100. Factorise and simplify  $\frac{x^2-x-2}{x^2-x-12} + \frac{x^2-4}{x^2-2x-8}$

Ans:  $\frac{x+1}{x+3}$

101. Which of the following is the HCF of 360 and 630?

- A. 15 B. 90 C. 18 D. 105 E. 45

Ans: B

102. Which of the following is the HCF of  $6a^2b^3$  and  $27ab^2$ ?

- A.  $3ab$  B.  $3a^2b^3$  C.  $6a^2b^3$  D.  $3ab^2$  E.  $6ab^2$

Ans: D

103. Which of the following is the HCF of  $12(x-y)^3(z-x)$  and  $18(x-y)^2(z-x)^2$ ?

- A.  $12(x-y)^3(z-x)^2$  B.  $12(x-y)^2(z-x)$  C.  $6(x-y)^2(z-x)^2$   
 D.  $18(x-y)^3(z-x)^2$  E.  $6(x-y)^2(z-x)$

Ans: E

104. Which of the following gives the factors of  $4(x-y)^2 - 4(x+3y)^2$ ?

- A.  $-32y(x+y)$  B.  $4y(x+y)$  C.  $4(x-2y)(x+y)$   
 D.  $(2x-2y)(2x+6y)$  E.  $-6y^2(x+y)$

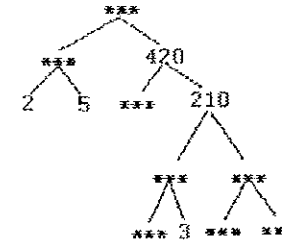
Ans: A

105. Complete the list of factors of 96:

1, 2, 3, 4, 6, 8, 12, ..., ..., 32, 48, 96

Ans: 16, 24

106. The following factor tree (or waterfall) is incomplete. Complete it by filling in the gaps:



Ans: 4200, 10, 2, 6, 35, 2, 5, 7 (not unique)

107. Find the smallest number which is divisible by all of 2, 3, 4, 5, 8, 9, 10 and 11.

Ans: 3960

108. Which of the following numbers has the greatest number of different prime factors?

- A. 425425 B. 595595 C. 442442 D. 510510 E. 578578

Ans: D

109. Find the expression which has the following properties:

- it is divisible throughout by 3
- it has three terms
- its linear factors are  $x-2$  and  $x+1$
- it is a quadratic with one term  $(x+1)^2$

Ans:  $3(x+1)^2 - 3x - 3$

110. A rectangular prism has the following properties:

- height  $x$
- volume  $8x^3y^2 - 2x^2y - 15x$

What are the length and width?

Ans:  $(4xy + 5)$ ,  $(2xy - 3)$

111. A cone has the following properties:

- base radius  $(x - y)$
- volume  $\pi(x^3 - x^2y - xy^2 + y^3)$

What is the height?

Ans:  $3(x + y)$

112. I think of a number, square it, multiply the result by 5, subtract 8. If the number I thought of was  $x$ , what do I need to add so that the resulting quadratic trinomial has factors  $5x - 1$  and  $x + 8$ ?

Ans:  $39x$

113. I think of an expression which is a perfect square. It has two quadratic factors, both of which are perfect squares. If I multiply out the whole expression I get

$$x^4 - 10x^3 + 37x^2 - 60x + 36.$$

If I multiply out one of the quadratic factors I get

$$x^2 - 6x + 9.$$

What was the original expression?

Ans:  $(x^2 - 5x + 6)^2$

114. Which of the following is the value of  $2001^2 - 1999^2$ ?

- A. 2      B. 3999      C. 7999      D. 4      E. 8000

Ans: E

115. Why is 71 a prime number?

Ans: Only two different factors, 1, 71

116. Why is 240 divisible by 5?

Ans: Ends in 0;  $240 \div 5$  leaves no remainder

117. Why is 242 divisible by 11?

Ans:  $2 + 2 = 4$ ;  $242 \div 11$  leaves no remainder

118. Which of the following are factors of 3663?

- A. 3 and 11      B. 3 and 6      C. 3, 6 and 11  
D. 3, 6 and 9      E. 3 only

Ans: A

119. Which of the following is the value of:  $\frac{2}{5} \times 1331 - \frac{2}{5} \times 331$ ?

- A. 40      B. 4000      C. 400      D. 452E. 0

Ans: C.

120. Which of the following is the value of  $-353 \times 50 - 50 \times 353$ ?

- A. 35300      B. -35300      C. 17650      D. 0      E. -8825

Ans: B

121. Which of the following statements are true?

- A. 1 is a prime number.  
B. A prime number has only two different factors.  
C. If two numbers will divide evenly into a number it is prime.  
D. One of the two different factors of a prime number must be 1.  
E. If a number has no factors it is prime.

Ans: B, D



122. Which of the following statements are false?

- A. Even numbers are divisible by 4.
- B. Numbers ending in zero are divisible by 2, 5 and 10.
- C. If the sum of the digits of a number is 81 then it is only divisible by 9.
- D. If a number is divisible by 4 it is also divisible by 2.
- E. An even number whose sum of digits is divisible by 3 is divisible by 6.

Ans: A, C

123. Which of the following statements are true?

- A. Linear factors must contain integers.
- B. Linear expressions produce straight lines when plotted on a graph.
- C. Linear expressions in  $x$  contain rational numbers.
- D. Quadratic expressions do not contain linear terms.
- E. Linear expressions in  $x$  do not contain  $x^2$  terms.

Ans: B, E

124. Which of the following statements are false?

- A. Quadratic expressions must have linear factors.
- B. Quadratic expressions can only have linear factors.
- C. Rational factors must contain fractions.
- D. All quadratic expressions have real factors.
- E. Real factors must contain surds.

Ans: A, C, D