6. Quadratic Functions

1. For the graph with equation $y = 2x^2 - 3$, which of the following points lie on the graph?

A. (1, -1) B. (-1, -1) C. (-1, 1) D. (0, 0) E. (0, -3)

Ans: A, B, E

2. The number and type of the factors of a quadratic equation $ax^2 + bx + c = 0$ can give information about the graph with equation $y = ax^2 + bx + c$. Which of the following statements are true?

A. If the equation has two real factors then the graph crosses the X-axis twice.
B. If the equation has two real factors then the graph crosses the Y-axis twice.
C. If the equation has no real factors then it does not cross the X-axis.
D. If the equation has no real factors then it does not cross the Y-axis.
E. If the equation has rational factors which are the same then the graph touches the X-axis.
F. If the equation has rational factors which are the same then the graph touches the Y-axis.
G. If the expression $ax^2 + 6x + c$ is a difference of perfect squares then the graph is symmetrical about the X-axis.

Ans: A, C, E, G

3. For solving the quadratic inequation $x^2 + 3x > 4$ which of the following can be correct parts of the method?

A. Factorise the LHS of the equation.
B. Use the null factor law on the LHS of the equation.
C. Draw a graph.
D. Set up a table of values of $x^2 + 3x$.
E. Use the quadratic formula on $x^2 + 3x$.

Ans: C, D

4. For the graphs with the given equations, state which properties are true when compared to the graph with equation $y = x^2$.

A. $y = 2x^2$ is stretched by a factor of 2 parallel to the Y-axis.
B. $y = -x^2$ is upside down.
C. $y = \frac{1}{2}x^2$ is stretched by a factor of 2 parallel to the X-axis.
D. $y = \frac{1}{2}(x - 2)^2$ is translated 1 unit to the right.
E. $y = 2(x + 1)^2$ is translated 1 unit to the left and stretched by a factor of 2 parallel to the Y-axis.
F. $y = (x + 1)^2 - 3$ is translated 1 unit to the right and 3 units down.
G. $y = (x - 3)^2 + 4$ is translated a total of 5 units.

Ans: A, B, E, G

5. The turning point of the graph with equation $y = x^2 + 8x + 1$ is:

A. (8, 1) B. (4, 15) C. (-4, -15) D. (15, 4) E. (-15, -4)

Ans: C

6. The turning point of the graph with equation $y = 2x^2 + 4x + 8$ is:

A. (4, 8) B. (-2, -6) C. (2, 6) D. (1, 3) E. (-1, 6)

Ans: E

7. A solution of the equation $x^2 + x - 3 = 0$ is approximately:

A. $x = 2.61$ B. $x = 1.3$ C. $x = 2.3$ D. $x = -1.3$ E. $x = -2.3$

Ans: B

8. Solve for x: $(x - 4)(x + 3) = 0$

Ans: 4, -3

9. Solve for x: $x(x + 1) = 0$

Ans: 0, -1

10. Solve for a: $(a - 2)^2 = 0$

Ans: 2

11. Solve for b: $(b + 3 - \sqrt{2})(b + 3 + \sqrt{2}) = 0$

Ans: $-3 \pm \sqrt{2}$

12. Solve for x: $(x - 2 + \sqrt{3})(x - 2 - \sqrt{3}) = 0$

Ans: $2 \pm \sqrt{3}$

13. Solve for x: $(x - a + b)(x - 2a - b) = 0$

Ans: a-b, 2a + b

14. Solve $x^2 + 3x - 18 = 0$

Ans: -6, 3

15. Solve $x^2 - 6x = 0$

Ans: 0, 6

16. Solve $x^2 = 16$

Ans: ± 4

17. Solve $a^2 + 14a = -40$

Ans: -4, -10

18. Solve $g^2 - g = 72$

Ans: 9, -8

19. Solve $3b^2 + 5b - 2 = 0$

Ans: $\frac{1}{3}$, -2

20. Solve $2y^2 + 5y = 3$

Ans: $\frac{1}{2}$, -3

21. Solve $6a^2 + 43a + 7 = 0$

Ans: $\frac{1}{6}$, -7

22. Solve $4x^2 + 4x = -1$

Ans: -½

23. Solve $a^2 - 2a = 0$

Ans: 0, 2

24. Solve $6x^2 = 24$

Ans: ±2

25. Solve $3a^2 - 18 = 9$

Ans: ±3

26. Solve $(x - 1)^2 - 3(x - 1) - 18 = 0$

Ans: 7, -2

27. Solve $(a + 2)^2 + 14(a + 2) + 40 = 0$

Ans: -6, -12

28. Solve $6x^2 - 72 = 0$

Ans: $\pm 2\sqrt{3}$

29. Solve $(x - 1)^2 - 5 = 0$

Ans: $1 \pm \sqrt{5}$

30. Solve $x^2 + 6x - 1 = 0$

Ans: $-3 \pm \sqrt{10}$

31. Solve $a^2 + 8a - 1 = 0$

Ans: $-4 \pm \sqrt{17}$

32. Solve $c^2 + 4c + 7 = 0$

Ans: $-2 \pm \sqrt{11}$

33. Solve $y^2 - 14y + 22 = 0$

Ans: $7 \pm 3\sqrt{3}$

34. Solve $x^2 + 16x + 37 = 0$

Ans: $-8 \pm 3\sqrt{3}$

35. Solve $3a^3 - 4a^2 + 3a - 4 = 0$

Ans: $\frac{4}{3}$

36. Find the x and y-intercepts of the graph with equation $y = x^2 - 5x + 4$

Ans: $x = 1, 4$; $y = 4$

37. Find the x and y-intercepts of the graph with equation $y = x^2 - 5x + 7$

Ans: no x-intercepts; $y = 7$

38. Find the x and y-intercepts of the graph with equation $y = (x + 2)^2 - 1$

Ans: $x = -1, -3$; $y = 3$

39. Find the x and y-intercepts of the graph with equation $y = x^2 - 2$

Ans: $x = \pm\sqrt{2}$; $y = -2$

40. Find the x and y-intercepts of the graph with equation $y = x^2 + 2$

Ans: no x-intercepts; $y = 2$

41. Solve $a^2 - 6a + 8 \leq 0$

Ans: $2 \leq a \leq 4$

42. Solve $-b^2 + b + 12 > 0$

Ans: $-3 < b < 4$

43. Solve $c^2 + 10c + 16 > 0$

Ans: $c < -8$ or $c > -2$

44. Solve $y^2 + 6y + 10 \geq 0$

Ans: all real values of y

45. Solve $a^2 \leq 4a$

Ans: $0 \leq a \leq 4$

46. Solve $b^2 \geq b$

Ans: $b < 0$ or $b > 1$

47. Find the point of intersection of the graphs with equations:
$y = x^2 - 4x + 5$ and $y = 4x - 7$

Ans: (2, 1) (6, 17)

48. Find the point of intersection of the graphs with equations:
$y = x^2 - 4x + 4$ and $y = 2x - 4$

Ans: (2, 0)

49. Find the point of intersection of the graphs with equations:
$y = x^2 - 4x + 5$ and $y = x + 1$

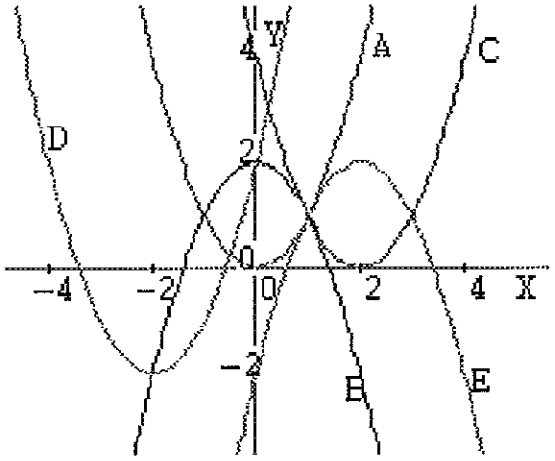
Ans: (4,5), (1,2)

50. Find the point of intersection of the graphs with equations
$y = 2x + 3$ and $y = 3x^2 + 6x - 4$

Ans: $(\frac{-7}{3}, \frac{-5}{3})$, (1,5)

51. Match each graph A to E with its equation (a) to (e):

(a) $y = (x - 2)^2$
(b) $y = -x^2 + 2$
(c) $y = -(x - 2)^2 + 2$
(d) $y = (x + 2)^2 - 2$
(e) $y = x^2$



Ans: (a) C; (b) B, (c) E, (d) D, (e) A

52. For the graph with equation $y = -\frac{1}{3}(x + 9)^2 + 2$, which of the following are true when it is compared with the graph with equation $y = x^2$?

A. it is inverted, moved 9 units to the right and 2 units up
B. it is inverted, moved 2 units to the right and 9 units up
C. it is inverted, moved 9 units to the left and 2 units up
D. it is wider
E. it is narrower

Ans: C, D

53. For the graph with equation $y = 2(x - 1)^2 - 3$ the turning point is:

A. (-2, -3) B. (2, -3) C. (-1, 3) D. (-1, -3) E. (1, -3)

Ans: E

54. A rectangle of area 20 cm^2 has its length 8 cm longer than its breadth. Which of the following is feasible?

A. The length of the rectangle is 4 cm.
B. The width of the rectangle is 2 cm.
C. The length of the rectangle is 2 cm.
D. The length of the rectangle is 10 cm.
E. The width of the rectangle is 8 cm.

Ans: B, D

55. Which of the following points lie above the graph with equation $y = x^2 - 4x + 5$?

A. (0, 0) B. (2, 1) C. (6, 18) D. (-1, 4) E. (2, 2)

Ans: C, E

56. Which of the following points lie below the graph with the equation $y = 1 - x^2$?

A. (0, 0) B. (1, 0) C. (0, 1) D. (0,-1) E. (1, 1)

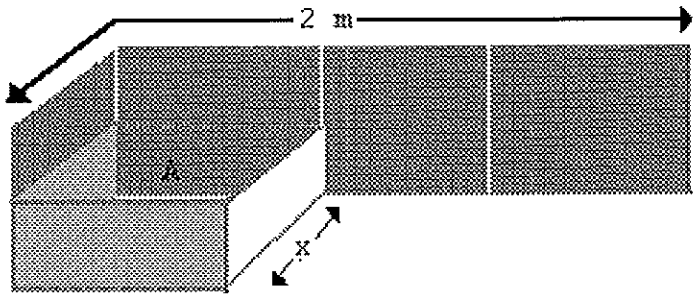
Ans: A, D

57. Which of the following points lie above or on the graph with equation $y = x - 1$ and below the graph with equation $y = 1 - x^2$?

A. (0, 0) B. (-1, -1) C. (½, -½) D. (-½, ½) E. (1, 1)

Ans: A, B, C, D

58. A box in the form of a rectangular prism has base width x cm. It is to be wrapped in brown paper 2 m long as shown:



The brown paper just reaches around the box.

(a) The area, A, of the base of the box is:

A. $x(2 - 2x)$ B. $x(100 - x)$ C. $x(2 - x)$

D. $x(200 - x)$ E. $x^2 - 2x$

(b) The area, A, of the base of the box is zero when:

A. $x = 0$ or 100 B. $x = 0$ or 1 C. $x = 0$ only

D. $x = 0$ or 200 E. $x = 0$ or 2

(c) The area, A, of the base of the box is a maximum when:

A. $x = 0$ B. $x = 25$ C. $x = 100$

D. $x = 1$ E. $x = 50$

(d) The maximum area of the base of the box is:

A. 1 m^2 B. 2500 cm^2 C. 10000 cm^2

D. 3750 cm^2 E. 50 m^2

Ans: (a) B (b) A (c) E (d) B

59. A ball thrown up in the air has a path described by the equation:

$$h = 2x - \frac{1}{5}x^2 + 2$$

where h is the height above the ground in metres and x is the horizontal distance of the ball from the thrower.

(a) When the ball was thrown, how high above the ground was it?

A. $\frac{1}{5}$ m B. 10 m C. 2 m D. 0 m E. 20 cm

(b) The equation of the path of the ball in turning point form is:

A. $h = -\frac{1}{5}(x - 5)^2 + 7$ B. $h = -\frac{1}{5}(x - 5)^2 - 2$

C. $h = \frac{1}{5}(x - 5)^2 + 7$ D. $h = (x + 5)^2 - 2$ E. none of these

(c) The maximum height reached by the ball was:

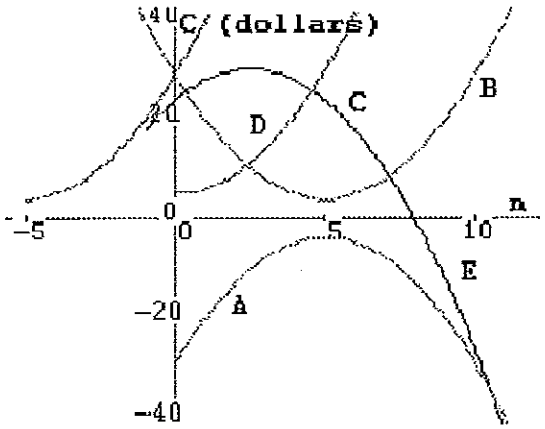
A. 5 m B. 2 m C. 7 m D. -5m E. none of these

Ans: (a) C (b) A (c) C

60. The cost of making a jar of coffee, C dollars, depends on the number made, n, in thousands, according to the quadratic model:

$$C = 4 + (n - 5)^2$$

(a) Which of the following graphs represents the cost plotted against number made?



(b) How many jars should be made so that each has minimum cost?

A. 4000 B. 4 C. 0 D. 5 E. 5000

(c) What is the minimum cost for each jar?

A. \$4 B. \$5 C. 40¢ D. 50¢ E. \$0

Ans: (a) B (b) E (c) A

61. On his daily walk, Denis usually takes t hours at an average speed of (t - 1) kph. Yesterday he walked more than 20 km. What was his average speed.?

A. 4 kph B. more than 4 kph C. 5 kph

D. more than 5 kph E. more than 3 kph

Ans: D

62. The profit on a business venture \$P depends on the number of extra people, x, employed according to the rule $P = -x^2 - x + 20$. Which of the following are true?

A. A (positive) profit is made for $-5 < x < 4$
B. A loss is made for $x > 4$
C. A (positive) profit is made if no extra people are employed
D. A loss is made when $x < 0$
E. A (positive) profit is always made.

Ans: B, C

63. The cost \$C of making n ring binders is given by:

$$C = n^2 + 7n + 6$$

The selling price \$5 of n ring binders is given by:

$$S = 3n + 2$$

There is no profit made on the ring binders when:

A. $n = 2$ B. $n = -2$ C. $n = 4$ D. $n = -4$ E. none of these

Ans: E

64. If a total of 560 children are to be lined up in a rectangular array for assembly in such a way as there are 19 more rows than there are columns, how many rows of children is this?

A. 10 B. 56 C. 35 D. 16 E. 8

Ans: D

65. Find the equation of the parabola of the form $y = (x - a)^2 + b$ if the turning point is $(-1, 3)$.

Ans: $y = x^2 + 2x + 4$

66. Find the equation of the parabola of the form $y = a(x - h)^2 + k$ if the turning point is $(2, 1)$ and the y-intercept is 9.

Ans: $y = 2x^2 - 8x + 9$

67. To solve the equation $3p^2 - 5p + 1 = 0$ the formula which can be used is:

A. $p = \frac{5 \pm \sqrt{25-12}}{6}$ B. $p = \frac{-5 \pm \sqrt{25-12}}{6}$ C. $p = \frac{-5 \pm \sqrt{25+12}}{6}$

D. $p = \frac{5 \pm \sqrt{25+12}}{6}$ E. $p = \frac{5 \pm \sqrt{25-12}}{3}$

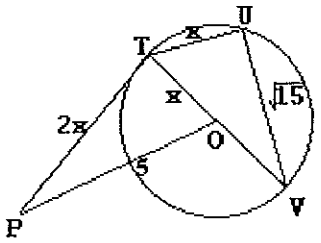
Ans: A

68. The null factor law states that:

A. if $x \times y = 0$ then $x = 0$ and $y = 0$;
B. if $x \times y = 0$ then $x \neq 0$ or $y \neq 0$;
C. if $x \times y = 0$ then $x = 0$ or $y = 0$;
D. if $x \times y = 0$ then $x = 0$ and/or $y = 0$;
E. if $x \times y = 0$ then $x \neq 0$

Ans: D

69. To find the value of x in the following diagram where O is the centre of the circle and $\overline{PT}$ is a tangent:



which of the following methods could be used?

A. Pythagoras' Theorem on ΔTUV;
B. Pythagoras 'Theorem on ΔTOP;
C. Pythagoras' Theorem on ΔTOP then on ΔTUV;
D. Angle sum of ΔTUV;
E. Angle sum of ΔTOP;

Ans: A, B

70. Which of the following is true for the graph of a parabola?

A. Substituting x = 0 and y = 0 in the equation gives the turning point
B. Substituting x = 0 in the equation gives the y-intercept
C. Substituting x = 0 in the turning point form of the equation gives the x-intercept
D. Substituting y = 0 in the equation gives the x-intercept/s
E. The turning point can be found by completing the square.

Ans: B, D, E

71. If $x \times y \times z = 0$ which of the following are true?

A. x, y and z must all be zero
B. At least one of x, y and z must be zero
D. x can be zero
E. x and y can both be zero

Ans: B, D, E

72. For the equation in x, $x^2 + 7x + b = 0$, which of the following are true?

A. It is quadratic
B. It has rational solutions if $b = 17$
C. It has real solutions if $b = 17$
D. It has no rational solutions if $b = 6$
E. It has rational solutions if $b = 6$

Ans: A, E

73. If the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ is used to solve the equation $(x+3)^2 - 18 = 0$, which of the following is true?

A. $a = 1,\ b = 3,\ c = -18$
B. $a = 1,\ b = 6,\ c = 9$
C. $a = 1,\ b = 6,\ c = -9$
D. $a = 1,\ b = 3,\ c = 9$
E. $a = 1,\ b = 3,\ c = -9$

Ans: C

74. Which of the following statements are true?

A. a parabola and a straight line cut in two places.
B. A parabola and a straight line may not intersect.
C. A parabola and a straight line cannot be parallel where they touch.
D. Simultaneous solution of a quadratic equation and a linear equation will result in no more than two solutions.
E. Simultaneous solution of a quadratic equation and a linear equation will result in at least one solution.

Ans: B, D

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