

Topic 11: Exercises on Integration
Level 1

1. Find $\int \frac{\ln x}{x} dx$.

$\frac{1}{2}(\ln x)^2 + c$

2. Find $\int \frac{\cos x}{2 + \sin x} dx$.

$\ln(2 + \sin x) + c$

3. Find $\int \frac{x}{1+x^2} dx$.

$\frac{1}{2} \ln(1+x^2) + c$

4. Find $\int \frac{x}{(1+x^2)^2} dx$.

$$\boxed{-\frac{1}{2(1+x^2)} + c}$$

5. Find $\int e^{\sin x} \cos x dx$.

$$\boxed{e^{\sin x} + c}$$

6. Find $\int x\sqrt{1+x^2}dx$.

$$\frac{1}{3}(1+x^2)^{3/2} + c$$

7. Find $\int x\sec^2(x^2)dx$.

$$\frac{1}{2}\tan(x^2) + c$$

8. Evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{6}} \cot x dx$.

$$\ln\left(\sin \frac{\pi}{6}\right) - \ln\left(\sin \frac{\pi}{4}\right)$$

9. Evaluate $\int_0^2 \frac{1}{4+x^2} dx$.

$$\frac{\pi}{8}$$

10. Evaluate $\int_2^4 \frac{(x^2 - 1)^2}{x} dx$.

$$48 + \ln 2$$

11. Find $\int \frac{1}{x^2 + 2x + 2} dx$ {Show first that $x^2 + 2x + 2 = (x + 1)^2 + 1$ }.

$$\tan^{-1}(x+1) + c$$

12. Find $\int \frac{1}{\sqrt{2x-x^2}} dx$. {Show first that $2x-x^2 = 1-(x-1)^2$ }.

$$\sin^{-1}(x-1)+c$$

13. Find $\int \frac{x-1}{x^2+1} dx$.

$$\frac{1}{2} \ln(x^2+1) - \tan^{-1} x + c$$

14. Find $\int \frac{6x - 10}{(x + 1)(x - 3)} dx$.

$$4 \ln|x + 1| + 2 \ln|x - 3| + c$$

15. Evaluate $\int_{-1}^3 \frac{1}{x^2 - 2x + 5} dx$ {Show first that $5 - 2x + x^2 = 4 + (x - 1)^2$ }.

$$\frac{\pi}{4}$$

16. Evaluate $\int_{-1}^0 \frac{1}{\sqrt{3-2x-x^2}} dx$ {Show first that $3-2x-x^2 = 4-(x+1)^2$ }.

$$\frac{\pi}{6}$$

17. Find $\int \frac{x}{\sqrt{x+1}} dx$, using the substitution $u^2 = x+1$.

$$\frac{2}{3}(x+1)^{3/2} - 2\sqrt{x+1} + c$$