Topic 11: Exercises on Integration
Level 3, Part 1

1. Find $\int \sec^3 x \tan x dx$.

$\frac{\sec^3 x}{3} + c$

2. Find $\int \frac{\sin 2x}{2 + \sin^2 x} dx$.

$\ln(\sin^2 x + 2) + c$

3. Find $\int \frac{1}{x}\sec^2(\ln x)dx$.

$\tan(\ln x)+c$

4. Find $\int \frac{e^x}{\sqrt{1-e^{2x}}}dx$.

$\sin^{-1}(e^x)+c$

5. Find $\int \frac{1}{e^x + e^{-x}} dx$.

$\tan^{-1} e^x + c$

6. Evaluate $\int_0^{\frac{1}{2}} \frac{1}{\sqrt{(1-4x^2)}} dx$.

$\frac{\pi}{4}$

7. Evaluate $\int_0^{\frac{\pi}{6}} \tan 2x \sec 2x dx$.

$\frac{1}{2}$

8. Find $\int \frac{x^2}{(x+1)(x+2)} dx$.

$$x + \ln\left(\frac{|x+1|}{(x+2)^4}\right) + c$$

9. Find $\int \frac{4x - x^2}{(x+1)(x^2+4)} dx$.

$$-\ln|x+1| + 2\tan^{-1}\left(\frac{x}{2}\right) + c$$

10. Find $\int\sqrt{\frac{1+x}{1-x}}dx$. Hint: use the substitution $x=\cos 2\theta$.

$$-\cos^{-1}x-\sqrt{1-x^2}+c$$

11. Find $\int \frac{x^3}{(x^2+1)^3} dx$.

(a) using the substitution $u = x^2 + 1$ (b) using the substitution $x = \tan\theta$.
Show that the answers agree.

$$-\frac{1}{2(x^2+1)} + \frac{1}{4(x^2+1)^2} + c.$$