Topic 11: Exercises on Integration
Level 3, Part 2

1. Evaluate $\int_0^2 \sqrt{\frac{x}{4-x}}\,dx$.

Hint: use the substitution $x = 4\sin^2\theta$.

$\boxed{\pi - 2}$

2. Evaluate $\int_{0}^{\pi/2} \frac{1}{3+5\cos x}\,dx$.

$\frac{1}{4}\ln 3$

3. Evaluate $\int_0^{\pi/2} \frac{1}{3\cos x + 4\sin x + 5} dx$.

$\frac{1}{6}$

4. Find $\int \sin 6x \sin 2x dx$.

$$\frac{1}{8}\sin 4x - \frac{1}{16}\sin 8x + c$$

5. Evaluate $\int_0^{\pi/2} \sqrt{\cos x}\sin^3 x dx$.

$\frac{8}{21}$

6. Evaluate $\int_0^{\pi/3} \frac{1}{9-10\sin^2 x} dx$.

Hint: Show that $\frac{1}{9-10\sin^2 x} = \frac{\sec^2 x}{9-\tan^2 x}$ and use the substitution $u = \tan x$.

$\frac{1}{6}\ln(2+\sqrt{3})$

7. Evaluate $\int_2^4 \sqrt{\frac{5-x}{x-1}}$.

Hint: use the substitution $x = 5\sin^2\theta + \cos^2\theta$.

$\frac{2\pi}{3}$

8. If $I_n = \int \sin^n x dx$ for $n \geq 0$, show that $I_n = -\frac{1}{n}\cos x \sin^{n-1} x + \frac{(n-1)}{n} I_{n-2}$ for $n \geq 2$.

Hence show that if $I_n = \int_0^{\pi/2} \sin^n x dx$ for $n \geq 0$, then $I_n = \frac{(n-1)}{n} I_{n-2}$ for $n \geq 2$ and deduce that $I_5 \cdot I_6 = \frac{\pi}{12}$.