Topic 11: Exercises on Integration
Level 3, Part 3

1. If $I_n = \int \sec^n x dx$ for $n \geq 0$, show that $I_n = \frac{1}{n-1}\tan x \sec^{n-2} x + \frac{n-2}{n-1} I_{n-2}$ for $n \geq 2$.

Hence show that if $I_n = \int_0^{\pi/4} \sec^n x dx$ for $n \geq 0$, then $I_n = \frac{(\sqrt{2})^{n-2}}{n-1} + \frac{n-2}{n-1} I_{n-2}$ for $n \geq 2$ and deduce that $I_6 = \frac{28}{15}$.

2. If $I_n = \int_0^1 x(1-x^3)^n\,dx$ for $n \geq 0$, show that $I_n = \dfrac{3n}{2+3n} I_{n-1}$ for $n \geq 1$. Hence find an expression for I_n in terms of n for $n \geq 0$.

$$I_n = \frac{3^n n!}{(3n+2)(3n-1)\ldots 8\cdot 5\cdot 2}$$

3. Show that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$. Hence show that

$\int_0^{\pi/2} \frac{\cos^3 x}{\cos^3 x + \sin^3 x}dx = \int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x}dx$ and hence evaluate the given integral.

$\boxed{\frac{\pi}{4}}$

4. Show that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$. Hence show that $\int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x} dx = \frac{\pi}{4}$.