ACE Examinations

Student Name: ____________________

2014

YEAR 12

YEARLY EXAMINATION

Mathematics

General Instructions

- Reading time - 5 minutes
- Working time - 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Show all necessary working in Questions 11-16

Total marks - 100

Section I
10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II
90 marks

- Attempt Questions 11-16
- Allow about 2 hour 45 minutes for this section

STUDENT NUMBER/NAME: ....

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Section I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

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1 What is the primitive of $\frac{2}{x}-\cos x$?

(A) $\frac{-2}{x^2}+\sin x+C$

(B) $\frac{-2}{x^2}-\sin x+C$

(C) $2\ln x+\sin x+C$

(D) $2\ln x-\sin x+C$

2 What are the values of x for which $|4-3x|<13$?

(A) $x<-3$ or $x<\frac{17}{3}$

(B) $x>-3$ or $x>\frac{17}{3}$

(C) $x>-3$ or $x<\frac{17}{3}$

(D) $x<-3$ or $x>\frac{17}{3}$

3 What is the simultaneous solution to the equations $2x+y=7$ and $x-2y=1$?

(A) $x=3$ and $y=1$

(B) $x=-1$ and $y=9$

(C) $x=2$ and $y=3$

(D) $x=5$ and $y=1$

HSC Mathematics

4 Factorise $2x^2-7x-15$.

(A) $(2x-3)(x-5)$

(B) $(2x+3)(x-5)$

(C) $(2x-5)(x-3)$

(D) $(2x+5)(x-3)$

5 The value of $\frac{5.79+0.55}{\sqrt{4.32-3.28}}$ is closest to:

(A) 4

(B) 6

(C) 9

(D) 10

6 What are the values of p and q given $\left(3\sqrt{12}+\sqrt{75}\right)\left(2+\sqrt{48}\right)=p+q\sqrt{3}$?

(A) $p=132$ and $q=15$

(B) $p=396$ and $q=15$

(C) $p=132$ and $q=22$

(D) $p=396$ and $q=22$

7 The line $6x-ky=8$ passes through the point (3,2). What is the value of k?

(A) −13

(B) −5

(C) 5

(D) 15

8 The semi-circle $y=\sqrt{4-x^2}$ is rotated about the x-axis. Which of the following expressions is correct for the volume of the solid of revolution?

(A) $V=\pi\int_0^2(4-x^2)dx$

(B) $V=2\pi\int_0^2(4-x^2)dx$

(C) $V=\pi\int_0^2(4-y^2)dy$

(D) $V=2\pi\int_0^2(4-y^2)dy$

9 A circle has the equation $4x^2 - 4x + 4y^2 + 24y + 21 = 0$. What is the radius and centre?

(A) Centre $(\frac{1}{2}, -3)$ and radius of 2.

(B) Centre $(\frac{1}{2}, 3)$ and radius of 2.

(C) Centre $(\frac{1}{2}, -3)$ and radius of 4.

(D) Centre $(\frac{1}{2}, 3)$ and radius of 4.

10 An infinite geometric series has a first term of 12 and a limiting sum of 15. What is the common ratio?

(A) $\frac{1}{5}$

(B) $\frac{1}{4}$

(C) $\frac{1}{3}$

(D) $\frac{1}{2}$

Section II

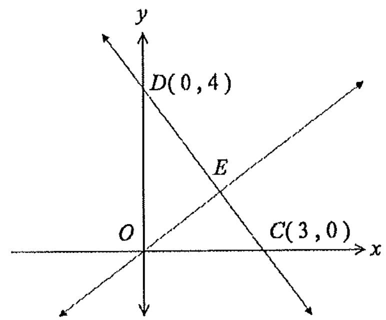
90 marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet.

All necessary working should be shown in every question.

Question 11 (15 marks) **Marks**

(a)



The coordinates of *O*, *D* and *C* are (0,0), (0,4) and (3,0) respectively. Point *E* lies on *CD*. Copy the diagram onto your workbook.

(i) Show that the equation of *CD* is $4x + 3y - 12 = 0$ **1**

(ii) Equation *OE* is $3x - 4y = 0$. Explain why *OE* is perpendicular to *CD*. **2**

(iii) Prove that ΔDOE is similar to ΔOCE. **2**

(iv) Show that $\frac{OE}{DE} = \frac{CE}{OE} = \frac{3}{4}$. **1**

(v) Find the ratio of the areas of triangles *DOE* and *OCE*. **1**

(b) Find the equation of the tangent to the curve $y = \log_e x - 1$ at the point $(e, 0)$. **2**

(c) The equation of a parabola is given by $y = x^2 - 2x + 5$.

(i) Find the coordinates of its vertex. **2**

(ii) What is its focal length? **1**

(iii) Find the equation of the normal at the point $P(2,5)$. **2**

(iv) For what values of x is the parabola concave upwards? **1**

Question 12 (15marks) **Marks**

(a) There are 200 tickets sold in a raffle with only two prizes. These tickets are placed in a bag and two are drawn, one at a time. Once a ticket is drawn it is not placed back in the bag. One boy bought 3 tickets.

(i) What is the probability he wins first prize? **1**

(ii) What is the probability he wins both prizes? **1**

(iii) What is the probability he wins second prize but not first prize? **1**

(iv) What is the probability he does not win a prize? **1**

(b) Differentiate with respect to x.

(i) $e^{3x}\tan x$ **2**

(ii) $\dfrac{\sin x}{5-x}$ **2**

(c) Find

(i) $\displaystyle\int\frac{dx}{e^{4x}}$ **2**

(ii) $\displaystyle\int_0^{\pi}\sec^2\frac{x}{3}\,dx$ **2**

(d) The roots of the equation $2x^2-x-15=0$ are α and β. Find the value of:

(i) $\alpha+\beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\dfrac{1}{\alpha^2}+\dfrac{1}{\beta^2}$ **1**

Question 13 (15 marks) **Marks**

(a) The sum of the firsts n terms of a certain arithmetic series is given by:

$$S_n=\frac{3n^2+n}{2}$$

(i) Calculate S_1 and S_2. **1**

(ii) Find the first three terms of the series. **2**

(iii) Find an expression for the nth term. **1**

(b) Let $f(x)=x^3-3x^2-9x+22$

(i) Find the coordinates of the stationary points and determine their nature. **3**

(ii) Find the coordinates of the point of inflexion. **2**

(iii) Sketch the graph of $y=f(x)$, indicating where the curve meets the y-axis, stationary points and points of inflexion. **2**

(iv) For what values of x is the graph of $y=f(x)$ concave down? **1**

(c) Alex and Bella leave from point O at the same time. Alex travels at 20 km/h along a straight road in the direction 085° T. Bella travels at 25 km/h along another straight road in the direction 340° T.

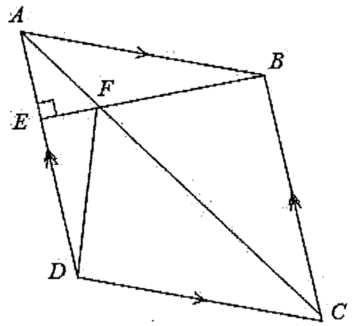
Draw a diagram to represent this information.

(i) Show that $\angle AOB$ is 105° where $\angle AOB$ is the angle between the directions taken by Alex and Bella. **1**

(ii) Find the distance Alex and Bella are apart to the nearest kilometre after two hours. **2**

Question 14 (15 marks) **Marks**

(a)



ABCD is a rhombus, BE is perpendicular to AD and intersects AC at F.
Copy the diagram onto your workbook.

(i) Explain why $\angle BCA = \angle DCA$. **1**

(ii) Prove that the triangles BFC and DFC are congruent. **3**

(iii) Show that $\angle FBC$ is a right angle. **1**

(iv) Hence or otherwise find the size of $\angle FDC$. **1**

(b) A scientist grows the number of bacteria according to the equation

$$N(t) = Ae^{0.15t}$$

where t is measured in days and A is a constant.

(i) Show that the number of bacteria increases at a rate proportional to the number present. **2**

(ii) When $t = 3$ the number of bacteria was estimated at 1.5×10^8. Evaluate A. Answer correct to 2 significant figures. **1**

(iii) The number of bacteria doubles every x days. Find x. Answer correct to 1 decimal place. **2**

(c) The speed of a car at intervals of two minutes is shown below.

t (h)	0	$\frac{1}{30}$	$\frac{1}{15}$	$\frac{1}{10}$	$\frac{2}{15}$
v (km/h)	0	35	45	50	60

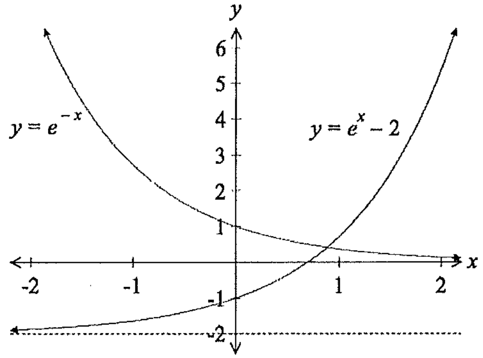
Use Simpson's rule with these five function values to estimate $\int_0^{\frac{2}{15}} v\,dt$. **2**

Answer correct to 3 significant figures.

(d) Solve the equation $(\cos x + 2)(2\cos x + 1) = 0$ in the domain $0 \le x \le 2\pi$. **2**

Question 15 (15 marks) **Marks**

(a)



The diagram shows the graphs of $y = e^x - 2$ and $y = e^{-x}$.

(i) Find the area between the curves from $x = 1$ and $x = 2$. Leave your answer in terms of e. **3**

(ii) Show that the curves intersect when $e^{2x} - 2e^x - 1 = 0$. **1**

(iii) Show that the x-coordinate of the point of intersection of the curves is approximately 0.881. **3**

(b) The velocity of a object moving along the x-axis is given by

$$v = 2\sin t + 1 \text{ for } 0 \le t \le 2\pi$$

where v is measured in metres per second and t in seconds.

(i) When is the object at rest? **2**

(ii) Sketch the graph of v as a function of t for $0 \le t \le 2\pi$ **2**

(iii) Find the maximum velocity of the object for this period. **1**

(iv) When is the object travelling in the negative direction during this period? **1**

(v) Calculate the total distance travelled by the object in the period $\pi \le t \le 2\pi$. **2**

Question 16 (15 marks) **Marks**

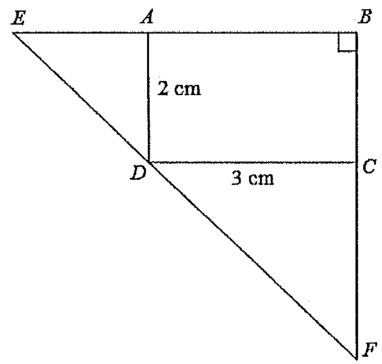
(a) George is saving for a holiday. He opens a savings account with an interest rate of 0.4% per month compounded monthly at the end of each month. George decides to deposit $450 into the account on the first of each month. He makes his first deposit on the 1st December 2011 and his last on the 1st June 2014. George withdraws the entire amount, plus interest, immediately after his final interest payment on the 30th June 2014.

(i) How much did George deposit into his saving account? Answer correct to the nearest dollar. **1**

(ii) How much did George withdraw from his account on the 30th June 2014? Answer correct to the nearest dollar. **3**

(iii) George's holiday is postponed due to family illness. He decides to deposit $12 000 into a different account with an interest rate of 5% p.a. compounded quarterly for 2 years. How much will George receive at the end of the investment period? Answer correct to the nearest dollar. **2**

(b)



$ABCD$ is a rectangle with $CD = 3$ cm and $AD = 2$ cm. F and E lie on the lines BC and BA, so that F, D and E are collinear. Let $CF = x$ cm and $AE = y$ cm.

(i) Show that ΔFCD and ΔDAE are similar. **3**

(ii) Show that $xy = 6$. **1**

(iii) Show that the area (A) of ΔFBE is given by $A = 6 + \frac{3}{2}x + \frac{6}{x}$. **2**

(iv) Find the height and base of ΔFBE with minimum area. Justify your answer. **3**

End of paper

ACE Examination 2014

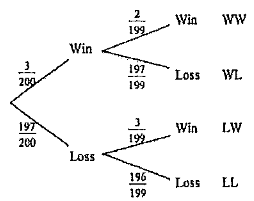
HSC Mathematics Yearly Examination

Worked solutions and marking guidelines

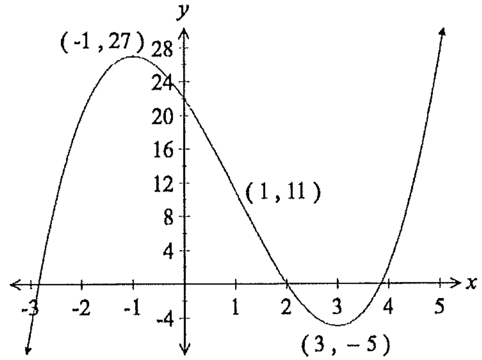
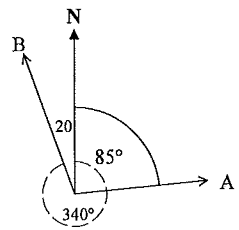
Section I		
	Solution	**Criteria**
1	$\int \frac{2}{x} - \cos x dx = 2\ln x - \sin x + C$	1 Mark: D
2	$\|4-3x\| < 13$ $4-3x<13$ and $-4+3x<13$ $-3x<9$ $3x<17$ $x>-3$ $x<\frac{17}{3}$	1 Mark: C
3	$2x+y=7$ (1) $x-2y=1$ (2) Multiply eqn (1) by 2 $4x+2y=14$ (3) Eqn (2)+(3) $5x=15$ or $x=3$ Substitute $x=3$ into eqn (1) $6+y=7$ or $y=1$ Solution is $x=3$ and $y=1$.	1 Mark: A
4	$2x^2-7x-15=(2x+3)(x-5)$	1 Mark: B
5	$\frac{5.79+0.55}{\sqrt{4.32-3.28}}=6.216881484$ ≈ 6	1 Mark: B
6	$\left(3\sqrt{12}+\sqrt{75}\right)\left(2+\sqrt{48}\right)=\left(6\sqrt{3}+5\sqrt{3}\right)\left(2+4\sqrt{3}\right)$ $=12\sqrt{3}+72+10\sqrt{3}+60$ $=132+22\sqrt{3}$ Therefore $p=132$ and $q=22$	1 Mark: C
7	The point (3,2) satisfies the equation $6x-ky=8$. $6\times 3-k\times 2=8$ $18-2k=8$ $-2k=-10$ $k=5$	1 Mark: C

8	Now $y=\sqrt{4-x^2}$ or $y^2=4-x^2$ $V=\pi\int_{-2}^{2} y^2 dx$ $=2\pi\int_0^2 (4-x^2)dx$	1 Mark: B
9	$4x^2-4x+4y^2+24y+21=0$ $x^2-x+y^2+6y=-\frac{21}{4}$ $(x-\frac{1}{2})^2-\frac{1}{4}+(y+3)^2-9=-\frac{21}{4}$ $(x-\frac{1}{2})^2+(y+3)^2=4$ Centre $(\frac{1}{2},-3)$ and radius of 2.	1 Mark: A
10	$a=12$ and $S=15$ $S=\frac{a}{1-r}$ $15=\frac{12}{1-r}$ $15-15r=12$ $15r=3$ $r=\frac{1}{5}$	1 Mark: A
Section II		
11(a) (i)	Equation of CD $\frac{x}{a}+\frac{y}{b}=1$ $\frac{x}{3}+\frac{y}{4}=1$ $4x+3y=12$ $4x+3y-12=0$	1 Mark: Correct answer.
11(a) (ii)	$4x+3y-12=0$, $y=-\frac{4}{3}x+4$ Gradient is $-\frac{4}{3}$ $3x-4y=0$, $y=\frac{3}{4}x$ Gradient is $\frac{3}{4}$ Perpendicular lines then $m_1m_2=-1$ $-\frac{4}{3}\times\frac{3}{4}=-1$ True	2 Marks: Correct answer. 1 Mark: Finds the gradient of OE or recognises $m_1m_2=-1$.

11(a) (iii)	In ΔDOE and ΔOCE Let $x = \angle ECO$ $\angle ECO + \angle CDO + \angle DOC = 180^\circ$ (angle sum of triangle is 180) $\angle CDO = 180^\circ - 90^\circ - x = 90^\circ - x$ $\angle DOE + \angle EDO + \angle DEO = 180^\circ$ (angle sum of triangle is 180°) $\angle DOE = 180^\circ - (90^\circ - x) - 90^\circ$ $\angle DOE = x$ $\angle DEO = \angle CEO = 90^\circ$ (OE is perpendicular to CD) $\angle DOE = \angle ECO$ (Both equal to x) $\angle EOC = \angle EDO$ (Both equal to 90 - x) ΔDOE is similar to ΔOCE (equiangular)	2 Marks: Correct answer. 1 Mark: Shows some understanding
11(a) (iv)	$\frac{OE}{DE} = \frac{CE}{OE} = \frac{OC}{OD} = \frac{3}{4}$ (corresponding sides in similar triangles)	1 Mark: Correct answer.
11(a) (v)	$\frac{\Delta DOE}{\Delta OCE} = \frac{\frac{1}{2}DE \times OE}{\frac{1}{2}CE \times OE}$ $= \frac{DE}{OE} \times \frac{OE}{CE}$ $= \frac{4}{3} \times \frac{4}{3} = \frac{16}{9}$	1 Mark: Correct answer.
11(b)	$y = \log_e x - 1$ At the point $(e, 0)$ $\frac{dy}{dx} = \frac{1}{e}$ $\frac{dy}{dx} = \frac{1}{x}$ Point slope formula $y - y_1 = m(x - x_1)$ $y - 0 = \frac{1}{e}(x - e)$ $y = \frac{1}{e}x - 1$ or $x - ey - e = 0$	2 Marks: Correct answer. 1 Mark: Finds the gradient of the tangent
11(c) (i)	$y = x^2 - 2x + 5$ $y = (x-1)^2 + 4$ $y - 4 = (x-1)^2$ Vertex is (1, 4)	2 Marks: Correct answer. 1 Mark: Completes the square
11(c) (ii)	$y - k = 4a(x-h)^2$ Focal length is $^1/_4$ $y - 4 = 4 \times \frac{1}{4}(x-1)^2$	1 Mark: Correct answer.

11(c) (iii)	$\frac{dy}{dx} = 2x - 2$ At the point $(2,5)$ $\frac{dy}{dx} = 2 \times 2 - 2 = 2$ $m_1 m_2 = -1$ Equation of the normal $y - y_1 = m(x - x_1)$ $m_1 \times 2 = -1$ $y - 5 = -\frac{1}{2}(x-2)$ $m = -\frac{1}{2}$ $x + 2y - 12 = 0$	2 Marks: Correct answer. 1 Mark: Finds gradient of the tangent
11(c) (iv)	$\frac{d^2y}{dx^2} = 2 > 0$ Parabola is concave up for all real x	1 Mark: Correct answer.
12(a) (i)	 $P(W) = \frac{3}{200}$	1 Mark: Correct answer.
12(a) (ii)	$P(WW) = \frac{3}{200} \times \frac{2}{199}$ $= \frac{3}{19900}$	1 Mark: Correct answer.
12(a) (iii)	$P(LW) = \frac{197}{200} \times \frac{3}{199}$ $= \frac{591}{39800}$	1 Mark: Correct answer.
12(a) (iv)	$P(LL) = \frac{197}{200} \times \frac{196}{199}$ $= \frac{9653}{9950}$	1 Mark: Correct answer.
12(b) (i)	$\frac{d}{dx}\left(e^{3x} \tan x\right) = e^{3x}(\sec^2 x) + \tan x 3e^{3x}$ $= e^{3x}(\sec^2 x + 3\tan x)$	2 Marks: Correct answer. 1 Mark: Applies the product rule
12(b) (ii)	$\frac{d}{dx}\left(\frac{\sin x}{5-x}\right) = \frac{(5-x)\cos x - \sin x \times -1}{(5-x)^2}$ $= \frac{(5-x)\cos x + \sin x}{(5-x)^2}$	2 Marks: Correct answer. 1 Mark: Applies the quotient rule

12(c) (i)	$\int \frac{dx}{e^{4x}} = \int e^{-4x}dx$ $= -\frac{1}{4}e^{-4x} + C$	2 Marks: Correct answer. 1 Mark: Shows some understanding.
12(c) (ii)	$\int_0^\pi \sec^2 \frac{x}{3}dx = 3\left[\tan\frac{x}{3}\right]_0^\pi$ $= 3\left[\tan\frac{\pi}{3} - \tan\frac{0}{3}\right]$ $= 3\sqrt{3}$	2 Marks: Correct answer. 1 Mark: Finds the integral.
12(d) (i)	$\alpha + \beta = -\frac{b}{a}$ $= -\frac{-1}{2} = \frac{1}{2}$	1 Mark: Correct answer.
12(d) (ii)	$\alpha\beta = \frac{c}{a}$ $= \frac{-15}{2}$	1 Mark: Correct answer.
12(d) (iii)	$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha^2\beta^2}$ $= \frac{\left(\frac{1}{2}\right)^2 - 2\times\left(\frac{-15}{2}\right)}{\left(\frac{-15}{2}\right)^2} = \frac{61}{225}$	1 Mark: Correct answer.
13(a) (i)	$S_n = \frac{3n^2 + n}{2}$ $\quad S_n = \frac{3n^2+n}{2}$ $S_1 = \frac{3\times1^2+1}{2} = 2$ $\quad S_2 = \frac{3\times2^2+2}{2} = 7$	1 Mark: Correct answer.
13(a) (ii)	$a = T_1 = S_1 = 2$ $T_2 = S_2 - S_1 = 7 - 2 = 5$ $d = T_2 - T_1 = 5 - 2 = 3$ Sequence is {2, 5, 8, …}	2 Marks: Correct answer. 1 Mark: Finds the first term or the common difference.
13(a) (iii)	$T_n = a + (n-1)d$ $= 2 + (n-1)3$ $= 2 + 3n - 3$ $= 3n - 1$	1 Mark: Correct answer.

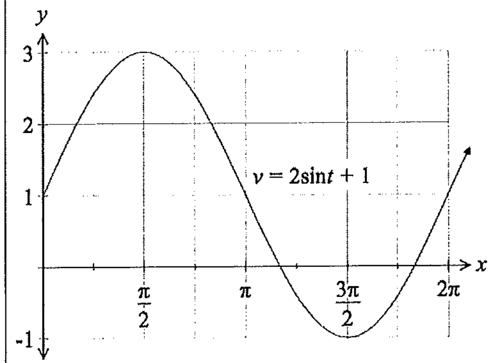
13(b) (i)	$f(x) = x^3 - 3x^2 - 9x + 22$ Stationary points $f'(x) = 0$ $f'(x) = 3x^2 - 6x - 9$ $\quad 3(x^2 - 2x - 3) = 0$ $= 3(x^2 - 2x - 3)$ $\quad 3(x-3)(x+1) = 0$ $f''(x) = 6x - 6$ $\quad x = -1,\ x = 3$ When $x = -1, y = 27$ then $f''(x) = -12 < 0$ Maxima. When $x = 3, y = -5$ then $f''(x) = 12 > 0$ Minima. Maximum turning point at $(-1, 27)$. Minimum turning point at $(3, -5)$.	3 Marks: Correct answer. 2 Marks: Finds the stationary points. 1 Mark: Correct differentiation to determine the stationary points.
13(b) (ii)	Possible points of inflexion $f''(x) = 0$ $\quad 6x - 6 = 0$ $6(x-1) = 0$ $x = 1$ When $x = 1, y = 11$ Check for change in concavity When $x = 0.9$ then $f''(x) = 6\times0.9 - 6 < 0$ When $x = 1.1$ then $f''(x) = 6\times1.1 - 6 > 0$ Hence (1,11) is a point of inflexion.	2 Marks: Correct answer. 1 Mark: Finds the point of inflexion.
13(b) (iii)	 	2 Marks: Correct answer. 1 Mark: Correct shape or shows some understanding.
13(b) (iv)	Function is concave down when $x < 1$ (from the graph)	1 Mark: Correct answer.
13(c) (i)	 $\angle AOB = 85° + 20°$ $= 105°$	1 Mark: Correct answer.

13(c) (ii)	After 2 hours Alex travels 40 km and Bella travels 50 km. $AB^2 = 40^2 + 50^2 - 2\times 40\times 50\times \cos 105^\circ$ $AB^2 = 5135.27618...$ $AB = 71.66084133...$ $AB \approx 72$ km Alex and Bella are 72 km apart after 2 hours.	2 Marks: Correct answer. 1 Mark: Uses the cosine rule with some correct values

14(a) (i)	$\angle BCA = \angle DCA$ (diagonals of a rhombus bisect the angles through which they pass)	1 Mark: Correct answer.
14(a) (ii)	In ΔBFC and ΔDFC $CF = CF$ (common side) $\angle BCF = \angle DCF$ (proven from part (i)) $BC = DC$ (adjacent sides of a rhombus are equal) $\therefore \Delta BFC \equiv \Delta DFC$ (SAS)	3 Marks: Correct answer. 2 Marks:Makes significant progress. 1 Mark: One relevant statement and reason.
14(a) (iii)	$\angle AEB = \angle EBC$ (alternate angles are equal, $AD//BC$) Now $\angle AEB = 90^\circ$ $\therefore \angle FBC = 90^\circ$	1 Mark: Correct answer.
14(a) (iv)	$\angle FBC = \angle FDC$ (matching sides in congruent triangles) $\therefore \angle FBC = 90^\circ$	1 Mark: Correct answer.
14(b) (i)	$N(t) = Ae^{0.15t}$ $\frac{dN}{dt} = A\times 0.15e^{0.15t}$ $= 0.15N$ The number of bacteria increases at a rate proportional to the number present.	2 Marks: Correct answer. 1 Mark: Finds $\frac{dN}{dt}$.
14(b) (ii)	We need to find A when $t=3$ and $N = 1.5\times 10^8$ $N(t) = Ae^{0.15t}$ $1.5\times 10^8 = Ae^{0.15\times 3}$ $A = \frac{1.5\times 10^8}{e^{0.45}}$ $= 95\ 644\ 222.74$ $\approx 9.6\times 10^7$	1 Mark: Correct answer.

14(b) (iii)	When $t = 3+x$ the number has doubled or $N = 2\times(1.5\times 10^8)$. $N(t) = Ae^{0.15t}$ $3.0\times 10^8 = 95\ 644\ 222.74\times e^{0.15(3+x)}$ $e^{0.15(3+x)} = \frac{3.0\times 10^8}{95\ 644\ 222.74}$ $0.15(3+x) = \log_e\left(\frac{3.0\times 10^8}{95\ 644\ 222.74}\right)$ $3+x = \log_e\left(\frac{3.0\times 10^8}{95\ 644\ 222.74}\right)\div 0.15$ $x = \log_e\left(\frac{3.0\times 10^8}{95\ 644\ 222.74}\right)\div 0.15 - 3$ $= 4.620981204$ ≈ 4.6 days	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.
14(c)	$\int_0^{\frac{2}{15}} v\,dt = \frac{h}{3}\left[y_0 + y_4 + 4(y_1 + y_3) + 2y_2\right]$ $= \frac{\frac{1}{30}}{3}\left[0 + 60 + 4\times(35+50) + 2\times 45\right]$ $= 5.44444444....$ ≈ 5.44	2 Marks: Correct answer. 1 Mark: Uses Simpson's rule with one correct value.
14(d)	$2\cos x + 1 = 0$ $\cos x = -\frac{1}{2}$ or $x = \frac{\pi}{3}$ $\cos x + 2 = 0$ $\cos x = -2$ No solution In domain $0 \le x \le 2\pi$ the solution is $x = \frac{2\pi}{3}, \frac{4\pi}{3}$	2 Marks: Correct answer. 1 Mark: Finds one solution or shows some understanding.

15(a) (i)	$A=\int_1^2 (e^x-2)dx-\int_1^2 e^{-x}dx$ $=\left[e^x-2x+e^{-x}\right]_1^2$ $=\left(e^2-4+e^{-2}\right)-\left(e-2+e^{-1}\right)$ $=e^2+e^{-2}-e-e^{-1}-2$ square units	3 Marks: Correct answer. 2 Marks: Makes significant progress. 1 Mark: Correctly sets up one integral
15(a) (ii)	Solve the equations simultaneously $e^x-2=e^{-x}$ $e^x-2=\frac{1}{e^x}$ $e^{2x}-2e^x-1=0$	1 Mark: Correct answer.
15(a) (ii)	The x coordinate is the solution of the equation $e^{2x}-2e^x-1=0$ Let $m=e^x$ then $m^2-2m-1=0$ $m=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $=\frac{-(-2)\pm\sqrt{(-2)^2-4\times1\times-1}}{2\times1}$ $=\frac{2\pm\sqrt{8}}{2}$ $=1\pm\sqrt{2}$ $\therefore e^x=1+\sqrt{2}$ or $\therefore e^x=1-\sqrt{2}$ $x=\log_e(1+\sqrt{2})$ $x=\log_e(1-\sqrt{2})$ $=0.881373587$ No solution ≈ 0.881	3 Marks: Correct answer. 2 Marks: Solves the quadratic equation. 1 Mark: Recognises the quadratic equation.
15(b) (i)	Particle at rest when $v=0$ $v=2\sin t+1$ $0=2\sin t+1$ $\sin t=-\frac{1}{2}$ $t=\frac{7\pi}{6}$ or $\frac{11\pi}{6}$	2 Marks: Correct answer. 1 Mark: Finds $2\sin t+1=0$ or calculates one answer.

15(b) (ii)		2 Marks: Correct answer. 1 Mark: Correct shape of the curve.
15(b) (iii)	Maximum velocity is 3 metres per second (from the graph)	1 Mark: Correct answer.
15(b) (iv)	Negative direction occurs when $v<0$ $\frac{7\pi}{6}\le t\le\frac{11\pi}{6}$ (from the graph and 15(b)(i))	1 Mark: Correct answer.
15(b) (v)	Distance travelled is the area under the curve between $\pi\le t\le 2\pi$. $d=2\int_\pi^{\frac{7\pi}{6}}(2\sin t+1)dt+\left\|\int_{\frac{7\pi}{6}}^{\frac{11\pi}{6}}(2\sin t+1)dt\right\|$ $=2\left[-2\cos t+t\right]_\pi^{\frac{7\pi}{6}}+\left\|\left[-2\cos t+t\right]_{\frac{7\pi}{6}}^{\frac{11\pi}{6}}\right\|$ $=2\left(\frac{\pi}{6}+\sqrt{3}-2\right)+\left\|\left(\frac{4\pi}{6}-2\sqrt{3}\right)\right\|$ $=4\sqrt{3}-4-\frac{\pi}{3}$	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.

16(a) (i)	31 deposits between 1st December 2011 and 1st June 2014. Total deposited $= \$450\times 31$ $= \$13950$	1 Mark: Correct answer.
16(a) (ii)	1st deposit - $A=P(1+r)^n$ $=450(1+0.004)^{31}$ $S=450(1.004)+450(1.004)^2+450(1.004)^3+\ldots+450(1.004)^{31}$ G.P. with $a=450(1.004)$, $r=1.004$ and $n=31$ $s=\dfrac{450(1.004)\left[1.004^{31}-1\right]}{1.004-1}$ $=\$14879.57127$ $\approx \$14880$ George withdraws \$14880 from his account.	3 Marks: Correct answer. 2 Marks: Identifies G.P. with 31 terms. 1 Mark: Uses compound interest formula with one correct value.
16(a) (iii)	$P=\$12000$, $r=\dfrac{0.05}{4}=0.0125$ and $n=2\times 4=8$ $A=P(1+r)^n$ $=12000(1+0.0125)^8$ $=\$13253.83321$ $\approx \$13254$ George will receive \$13254 after 2 years.	2 Marks: Correct answer. 1 Mark: Uses compound interest formula with one correct value.
16(b) (i)	$BC \parallel AD$ (opposite sides of a rectangle are parallel) $\angle BCD=\angle DAB=90^\circ$ (angles of a rectangle equal 90°) $\angle BCD+\angle FCD=180^\circ$ (straight angle is 180°) $90^\circ+\angle FCD=180^\circ$ $\angle FCD=90^\circ$ Similarly $\angle DAE=90^\circ$ In ΔFCD and ΔDAE $\angle FCD=\angle DAE=90^\circ$ (from above) $\angle BFD=\angle ADE$ (corresponding angles are equal, $BC\parallel AD$) $\therefore \Delta FCD$ is similar to ΔDAE (equiangular)	3 Marks: Correct answer. 2 Marks: Makes significant progress. 1 Mark: One relevant statement and reason.
16(b) (ii)	$\dfrac{CF}{AD}=\dfrac{CD}{AE}$ (matching sides in similar triangles) $\dfrac{x}{2}=\dfrac{3}{y}$ $xy=6$	1 Mark: Correct answer.

16(b) (iii)	$A=\dfrac{1}{2}bh$ $=\dfrac{1}{2}(2+x)(3+y)$ $=\dfrac{1}{2}(6+2y+3x+xy)$ Now $xy=6$ and $y=\dfrac{6}{x}$ $A=\dfrac{1}{2}(6+2\times\dfrac{6}{x}+3x+6)$ $=6+\dfrac{3}{2}x+\dfrac{6}{x}$	2 Marks: Correct answer. 1 Mark: Finds the correct expression for area containing both x and y.
16(b) (iv)	$A=6+\dfrac{3}{2}x+6x^{-1}$ $\dfrac{dA}{dx}=\dfrac{3}{2}-6x^{-2}$ $=\dfrac{3}{2}-\dfrac{6}{x^2}$ Minimum area occurs when $\dfrac{dA}{dx}=0$ $\dfrac{3}{2}-\dfrac{6}{x^2}=0$ $\dfrac{6}{x^2}=\dfrac{3}{2}$ $3x^2=12$ $x^2=4$ $x=\pm 2$ Since x is a length the $x>0$ $\therefore x=2$ and $y=3$ Test if a minimum $\dfrac{d^2A}{dx^2}=12x^{-3}=\dfrac{12}{x^3}>0$ for all x ($x>0$) Therefore minimum value when $x=2$ $\therefore BE=6$ cm and $BF=4$ cm	3 Marks: Correct answer. 2 Marks: Finds $x=2$ and tests for minimum value. 1 Mark: Calculates the first derivative or has some understanding of the problem.