ACE Examinations

Student Name: ____________________

2013

YEAR 12

YEARLY EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time - 5 minutes
- Working time - 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Show all necessary working in Questions 11-14

Total marks - 70

Section I

10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II

60 marks

- Attempt Questions 11-14
- Allow about 1 hour 45 minutes for this section

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Section I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

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1 What is the exact value of the definite integral $\int_{\frac{3}{\sqrt{3}}}^{3\sqrt{3}} \frac{dx}{x^2+9}$?

(A) $\frac{\pi}{18}$ (B) $\frac{\pi}{6}$

(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

2 What are the coordinates of the point that divides the interval joining the points $A(-6,4)$ and $B(-2,-10)$ externally in the ratio 1:3?

(A) $(-8,8)$
(B) $(-8,11)$
(C) $(2,8)$
(D) $(2,11)$

3 What is the value of $f'(x)$ if $f(x)=3x^2\cos^{-1}3x$?

(A) $\frac{-6x}{\sqrt{1-3x^2}}$

(B) $\frac{-6x}{\sqrt{1-9x^2}}$

(C) $\frac{-9x^2}{\sqrt{1-3x^2}}+9x\cos^{-1}3x$

(D) $\frac{-9x^2}{\sqrt{1-9x^2}}+6x\cos^{-1}3x$

4 What is the acute angle to the nearest degree that the line $3x-2y+1=0$ makes with the y-axis?

(A) 27°
(B) 34°
(C) 56°
(D) 63°

HSC Mathematics Extension 1

5 A container contains seven identical discs except for their colour. Four are red, two are blue and one is white. Two balls are selected at random. What is the probability of getting two red balls on exactly three occasions from five selections of two balls?

(A) 0.0119 (B) 0.0298
(C) 0.1190 (D) 0.2975

6 Let α, β and γ be the roots of $y^3-5y+1=0$.

What is the value of $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$?

(A) -5
(B) -1
(C) 1
(D) 5

7 What is the indefinite integral for $\int\left(2\sin^2 x-x^2\right)dx$?

(A) $2x-\sin 2x-\frac{x^3}{3}+c$

(B) $x-\frac{1}{2}\sin 2x-\frac{x^3}{3}+c$

(C) $2x-\sin 2x-2x+c$

(D) $x-\frac{1}{2}\sin 2x-2x+c$

8 One approximate solution of the equation $\cos x-\frac{x}{3}=0$ is $x=1.1$. What is another approximation to this solution using one application of Newton's method?

(A) $x=1.052$
(B) $x=1.171$
(C) $x=1.198$
(D) $x=2.896$

9 A particle moving in simple harmonic motion starts from rest at a distance 6 metres from the centre of oscillation. The period is 4π seconds. What is the time taken to move to a point 3 metres from the origin?

(A) $t = \dfrac{\pi}{3}$

(B) $t = \dfrac{\pi}{6}$

(C) $t = \dfrac{2\pi}{3}$

(D) $t = \dfrac{5\pi}{6}$

10 An object moves in a straight line so that at time t its displacement from a fixed origin is x and its velocity is v. The acceleration is $5 - 6x$. Which of the following is the correct equation for velocity given that $v = 2$ when $x = 1$?

(A) $v = 5x - 3x^2$

(B) $v = \sqrt{5x - 3x^2}$

(C) $v = \sqrt{10x - 6x^2}$

(D) $v = \sqrt{20x - 12x^2}$

Section II

60 marks
Attempt Questions 11 □ 14
Allow about 1 hour and 45 minutes for this section

Answer each question in the appropriate writing booklet.

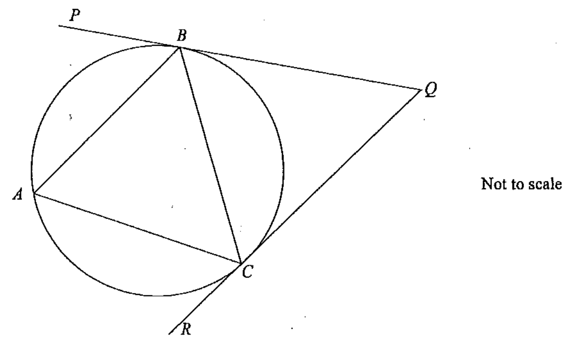
All necessary working should be shown in every question.

Question 11 (15 marks) **Marks**

(a) Use the substitution $u = 1 - x^2$ to evaluate $\int_2^3 \dfrac{2x}{(1-x^2)}\,dx$. **2**

(b) Solve $\dfrac{3}{x-2} \le 5$. **2**

(c)



In the diagram PQ and RQ are tangents to the circle touching the circle at B and C respectively. $\angle PQR = 50°$

(i) Show that $\angle QCB = 65°$ **2**

(ii) Hence find the value of $\angle CAB$ **1**

(d) Given that $\int_0^{\frac{1}{3}} \frac{dx}{1+9x^2} = n\pi$, find n. **2**

(e) (i) Express $8\cos x + 6\sin x$ in the form $R\cos(x-\alpha)$ where α is in radians. **2**

(ii) Hence or otherwise, solve the equation $8\cos x + 6\sin x = 10$ for $0 \le x \le 2\pi$. Answer correct to two decimal places. **2**

(f) The acute angle between the lines $3x - y + 7 = 0$ and $mx - y + 1 = 0$ is 45°. Find the possible values of m. **2**

Question 12 (15 marks) **Marks**

(a) A golfer hits a golf ball lying on a horizontal fairway. The ball passes over a 2.25 metre high tree after 1.5 seconds. The tree is 60 metres away from the point from which the ball was hit. Take $g = 10\ \text{ms}^{-1}$.

(i) Derive the expressions for the vertical and horizontal components of the displacement of the ball from the point of projection. **2**

(ii) Calculate the angle of projection. **2**

(iii) How far from where the golfer hits it does the ball land? **2**

(b) Consider the function $y = \frac{1}{2}\cos^{-1}(x-1)$.

(i) What is the domain and range of the function? **2**

(ii) Sketch the graph of the function showing the coordinates of the endpoints. **1**

(iii) The region in the first quadrant bounded by the curve $y = \frac{1}{2}\cos^{-1}(x-1)$ and the coordinate axes is rotated through 360° about the y-axis. Find the volume of the solid of revolution. Express your answer in simplest exact form. **3**

(c) (i) Show that $\cos 4x = 8(\cos^4 x - \cos^2 x) + 1$ **1**

(ii) Hence or otherwise solve $\cos^2 x - \cos^4 x = \frac{1}{16}$ for $0 \le x \le \frac{\pi}{2}$. **2**

Question 13 (15 marks) **Marks**

(a) Consider the function $f(x) = x\log_e x - 1$ with $x > 0$

(i) What is the stationary point and determine its nature? **2**

(ii) Let $x = 2$ be a first approximation to the root. Use Newton's method with one application to find the x-intercept. **1**

(iii) Why is the curve always concave upwards? **1**

(iv) Sketch the curve, showing all its main features. **2**

(b) A thermos of tea is cooling in a room of constant temperature $22°\text{C}$. At time t minutes its temperature T decreases according to the equation

$$\frac{dT}{dt} = -k(T - 22)$$

where t is the time in minutes and k is a positive constant.

The initial temperature of the tea is $80°\text{C}$ and it cools to $60°\text{C}$ after 10 minutes.

(i) Verify that $T = 22 + Ae^{-kt}$ is a solution of this equation, where A is a constant. **1**

(ii) Find the values of A and k. **2**

(iii) How long will it take for the temperature of the tea to cool to 30°C? Answer correct to the nearest minute. **2**

(c) Five different fair dice are thrown together.

(i) What is the probability the five scores are all different? **2**

(ii) What is the probability the five scores include at most one six? **2**

Question 14 (15 marks) **Marks**

(a) The acceleration of a particle is given by $a = 2x^3 + 2x$ where x is the displacement. The particle starts from a position 2 metres to the right of the origin with an initial velocity of 5 m/s.

(i) Show that $v = x^2 + 1$ **2**

(ii) Hence find an expression for x in terms of t. **2**

(b) $P(2p, p^2)$ and $Q(2q, q^2)$ are two points on the parabola $x^2 = 4y$. M is the midpoint of PQ.

(i) Show that $(p-q)^2 = 2(p^2+q^2) - (p+q)^2$ **1**

(ii) If P and Q move on the parabola so that $p - q = 1$, show that the locus of M is the parabola $x^2 = 4y - 1$. **2**

(iii) What is the focus of the locus of M? **1**

(c) Use mathematical induction to prove that $7^n - 1$ is divisible by 6 for positive integral values of n. **3**

(d) (i) Expand $\left(x + \frac{1}{x}\right)^5$ in descending powers of x. **1**

(iii) If $x + \frac{1}{x} = n$, express $x^5 + \frac{1}{x^5}$ in terms of n **3**

End of paper

ACE Examination 2013

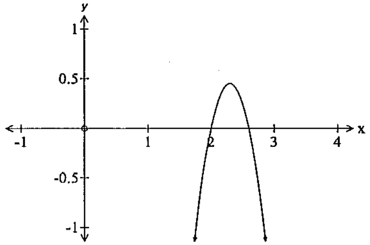
HSC Mathematics Extension 1 Yearly Examination

Worked solutions and marking guidelines

Section I

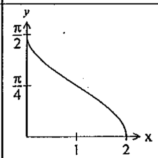
	Solution	Criteria
1	$\int_{\frac{3}{\sqrt{3}}}^{3\sqrt{3}}\frac{dx}{x^2+9}=\left[\frac{1}{3}\tan^{-1}\frac{x}{3}\right]_{\frac{3}{\sqrt{3}}}^{3\sqrt{3}}$ $=\frac{1}{3}\left(\tan^{-1}\sqrt{3}-\tan^{-1}\frac{1}{\sqrt{3}}\right)$ $=\frac{1}{3}\left(\frac{\pi}{3}-\frac{\pi}{6}\right)=\frac{\pi}{18}$	1 Mark: A
2	$x=\frac{mx_2+nx_1}{m+n}$ $\quad y=\frac{my_2+ny_1}{m+n}$ $=\frac{-1\times-2+3\times-6}{-1+3}=-8$ $\quad =\frac{-1\times-10+3\times4}{-1+3}=11$ The coordinates of point are $(-8,11)$	1 Mark: B
3	$f(x)=3x^2\cos^{-1}3x$ $=3x^2\frac{-1}{\sqrt{\frac{1}{9}-x^2}}+6x\cos^{-1}3x$ $=\frac{-9x^2}{\sqrt{1-9x^2}}+6x\cos^{-1}3x$	1 Mark: D
4	For $3x-2y+1=0$ then $m=\frac{3}{2}$ Angle the line makes with the *x*-axis $\tan\theta=\frac{3}{2}$ $\theta=56.30993247...\approx 56^\circ$ Angle the line makes with the *y*-axis $90^\circ-56^\circ=34^\circ$	1 Mark: B
5	$P(RR)=\frac{4}{7}\times\frac{3}{6}=\frac{2}{7}$ P(Two red exactly 3 times) $={}^5C_3\left(\frac{2}{7}\right)^3\left(\frac{5}{7}\right)^2$ $=0.1190$	1 Mark: C

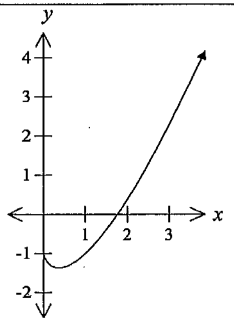
6	$\alpha\beta+\alpha\gamma+\beta\gamma=\frac{c}{a}=\frac{-5}{1}=-5$ $\alpha\beta\gamma=-\frac{d}{a}=-\frac{1}{1}=-1$ $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=\frac{\alpha\beta+\alpha\gamma+\beta\gamma}{\alpha\beta\gamma}$ $=\frac{-5}{-1}=5$	1 Mark: D
7	$\int\left(2\sin^2x-x^2\right)dx=\int 2\times\frac{1}{2}(1-\cos 2x)-x^2dx$ $=x-\frac{1}{2}\sin 2x-\frac{x^3}{3}+c$	1 Mark: B
8	$f(x)=\cos x-\frac{x}{3}$ $\quad f'(x)=-\sin x-\frac{1}{3}$ $x_1=x_0-\frac{f(x_0)}{f'(x_0)}$ $=1.1-\frac{\cos 1.1-\frac{1.1}{3}}{-\sin 1.1-\frac{1}{3}}$ $=1.170989437...=1.171$	1 Mark: B
9	$T=\frac{2\pi}{n}$ or $4\pi=\frac{2\pi}{n}$ or $n=\frac{1}{2}$ Amplitude is 6 metres $x=a\cos(nt+\alpha)$ $=6\cos(\frac{1}{2}t+\alpha)$ $=6\cos(\frac{1}{2}t)$ when $t=0$, $x=6$ implies $\alpha=0$ When $x=3$ $\quad 3=6\cos\frac{1}{2}t$ $\cos\frac{1}{2}t=\frac{1}{2}$ $\frac{1}{2}t=\frac{\pi}{3}$ or $t=\frac{2\pi}{3}$	1 Mark: C
10	$a=5-6x$ $v^2=2\int(5-6x)dx$ $=10x-6x^2+c$ When $x=1$, $v=2$ then $c=0$ $v^2=10x-6x^2$ $v=\sqrt{10x-6x^2}$ (conditions indicate positive solution)	1 Mark: C

Section II		
11(a)	$u=1-x^2$ $\frac{du}{dx}=-2x$ $-du=2xdx$ $x=3$ then $u=-8$ $x=2$ then $u=-3$ $\int_2^3 \frac{2x}{(1-x^2)}dx=\int_{-3}^{-8}-\frac{1}{u^2}du$ $=\left[u^{-1}\right]_{-3}^{-8}$ $=-\frac{1}{8}+\frac{1}{3}=\frac{5}{24}$	2 Marks: Correct answer. 1 Mark: Sets up the integration using the substitution
11(b)	$(x-2)^2\times\frac{3}{(x-2)}\le 5\times(x-2)^2$ $(x-2)3\le 5(x-2)^2 \quad x\ne 2$ $(x-2)(3-5x+10)\le 0$ $(x-2)(13-5x)\le 0$  $x<2$ and $x\ge\frac{13}{5}$	2 Marks: Correct answer. 1 Mark: Finds one correct solution. Or multiplies both sides of the inequality by $(x-2)^2$.
11(c) (i)	PQ and RQ are tangents $QB=QC$ (Tangents from an external point are equal) $\therefore \Delta CQB$ is isosceles (two equal sides) $\therefore \angle QBC=\angle QCB$ (base angles of an isosceles triangle are equal) $\angle PQR+\angle QBC+\angle QCB=180$ (angle sum of triangle is 180) $50^\circ+\angle QBC+\angle QCB=180$ $\angle QCB=\angle QBC=65^\circ$	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.
11(c) (ii)	$\angle CAB=65^\circ$ (Angle between a tangent and a chord is equal to the angle in the alternate segment)	1 Mark: Correct answer.
11(d)	$\int_0^{\frac{1}{3}}\frac{dx}{1+9x^2}=\frac{1}{3}\left[\tan^{-1}3x\right]_0^{\frac{1}{3}}$ $=\frac{1}{3}\left(\tan^{-1}1-\tan^{-1}0\right)$ $=\frac{1}{3}\left(\frac{\pi}{4}-0\right)=\frac{1\pi}{12}$ Therefore $n=\frac{1}{12}$	2 Marks: Correct answer. 1 Mark: Shows some understanding of the problem.

11(e) (i)	$R\cos(x-\alpha)=8\cos x+6\sin x$ $R\cos(x-\alpha)=R\cos x\cos\alpha+R\sin x\sin\alpha$ Hence $R\cos\alpha=8$ and $R\sin\alpha=6$ Dividing these equations $\tan\alpha=\frac{6}{8}$ $\alpha=\tan^{-1}\frac{3}{4}$ or 0.6435... Squaring and adding the equations $R^2=6^2+8^2$ $R=10$ $8\sin x+6\cos x=10\cos(x-\tan^{-1}\frac{3}{4})$	2 Marks: Correct answer. 1 Mark: Determines the value of α or R
11(e) (ii)	$10\cos(x-\tan^{-1}\frac{3}{4})=10$ from (i) $\cos(x-\tan^{-1}\frac{3}{4})=1$ $x-\tan^{-1}\frac{3}{4}=0$ or 2π $x=\tan^{-1}\frac{3}{4}$ or $2\pi+\tan^{-1}\frac{3}{4}$ $x=0.64$ or $x=6.93$	2 Marks: Correct answer. 1 Mark: Finds one of the solutions or makes significant progress towards the solution.
11(f)	For $3x-y+7=0$ then $m_1=3$ For $mx-y+1=0$ then $m_2=m$ $\tan\theta=\left\|\frac{m_1-m_2}{1+m_1m_2}\right\|$ $\tan 45=\left\|\frac{3-m}{1+3\times m}\right\|$ $1=\left\|\frac{3-m}{1+3m}\right\|$ $\frac{3-m}{1+3m}=1$ or $\frac{3-m}{1+3m}=-1$ $3-m=1+3m$ $\quad$ $3-m=-1-3m$ $4m=2$ $\quad$ $2m=-4$ $m=\frac{1}{2}$ $\quad$ $m=-2$	2 Marks: Correct answer. 1 Mark: Uses the formula for the angle between 2 lines with at least one correct value.

12(a) (i)	Horizontal Motion $\ddot{x}=0$ $\dot{x}=c_1$ (when $t=0, \dot{x}=v\cos\theta$) $\dot{x}=v\cos\theta$ $x=v\cos\theta t+c_2$ (when $t=0, x=0$) $x=v\cos\theta t$ Vertical Motion $\ddot{y}=-10$ $\dot{y}=-10t+c_1$ (when $t=0, \dot{y}=v\sin\theta$) $\dot{y}=-10t+v\sin\theta$ $y=-5t^2+v\sin\theta t+c_2$ (when $t=0, y=0$) $y=-5t^2+v\sin\theta t$	2 Marks: Correct answer. 1 Mark: Derives either the horizontal or vertical equations of motion.
12(a) (ii)	when $x=60$, $t=1.5$ $x=v\cos\theta t$ $60=v\cos\theta\times 1.5$ $v\cos\theta=40$ (1) when $y=2.25$, $t=1.5$ $y=-5t^2+v\sin\theta t$ $2.25=-5\times 1.5^2+v\sin\theta\times 1.5$ $v\sin\theta=9$ (2) Equation (2) $\div$ (1) $\frac{v\sin\theta}{v\cos\theta}=\frac{9}{40}$ $\tan\theta=\frac{9}{40}$ $\theta=12.68038349..=12°41'$ Angle of projection is 12°41'	2 Marks: Correct answer. 1 Mark: Calculates the value of $v\cos\theta$ or $v\sin\theta$.
12(a) (iii)	Ball lands when $y=0$ $-5t^2+v\sin\theta t=0$ $t(-5t+v\sin\theta)=0$ Hence $t=0$ (initially) or $-5t+v\sin\theta=0$ or $t=\frac{v\sin\theta}{5}$ Now from eqn (2) $v\sin\theta=9$ Therefore $t=\frac{9}{5}$ Distance travelled by the ball $x=v\cos\theta t$ (using eqn (1) $v\cos\theta=40$) $=40\times\frac{9}{5}$ $=72$ m The golfer hits the ball 72 metres	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.

12(b) (i)	Domain: $-1\le x-1\le 1$ or $0\le x\le 2$. Range: $0\le 2y\le\pi$ or $0\le y\le\frac{\pi}{2}$.	2 Marks: Correct answer.
12(b) (ii)	 	1 Mark: Correct answer.
12(b) (iii)	$x=1+\cos 2y$ $V=\pi\int_0^{\frac{\pi}{2}}(1+\cos 2y)^2\,dy$ $=\pi\int_0^{\frac{\pi}{2}}(1+2\cos 2y+\cos^2 2y)\,dy$ $=\pi\int_0^{\frac{\pi}{2}}(1+2\cos 2y+\frac{1}{2}(1+\cos 4y))\,dy$ $=\pi\left[\frac{3}{2}y+\sin 2y+\frac{1}{8}\sin 4y\right]_0^{\frac{\pi}{2}}$ $=\pi\left[\left(\frac{3}{2}\times\frac{\pi}{2}+\sin\pi+\frac{1}{8}\sin 2\pi\right)-\left(\frac{3}{2}\times 0+\sin 0+\frac{1}{8}\sin 0\right)\right]$ $=\frac{3}{4}\pi^2$	3 Marks: Correct answer. 2 Marks: Calculates the primitive function. 1 Mark: Sets up the integral in terms of y
12(c) (i)	$\cos 4x=2\cos^2 2x-1$ $=2\times(2\cos^2 x-1)^2-1$ $=2\times(4\cos^4 x-4\cos^2 x+1)-1$ $=8(\cos^4 x-\cos^2 x)+1$	1 Mark: Correct answer.
12(c) (ii)	$\cos 4x=8(\cos^4 x-\cos^2 x)+1$ $\cos^4 x-\cos^2 x=\frac{\cos 4x-1}{8}$ $\cos^2 x-\cos^4 x=\frac{1-\cos 4x}{8}$ Therefore $\frac{1-\cos 4x}{8}=\frac{1}{16}$ $\cos 4x=\frac{1}{2}$ $4x=\frac{\pi}{3}$ or $4x=\frac{5\pi}{3}$ $x=\frac{\pi}{12}$ or $x=\frac{5\pi}{12}$	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.

13(a) (i)	$f(x) = x\log_e x - 1$ $f'(x) = x \times \frac{1}{x} + \log_e x \times 1$ $= \log_e x + 1$ Stationary points $f'(x) = 0$ $\log_e x + 1 = 0$ $\log_e x = -1$ $x = \frac{1}{e}$ When $x = \frac{1}{e}$ $y = \frac{1}{e}\log_e \frac{1}{e} - 1$ $= -\frac{1}{e} - 1$ Stationary point is $(\frac{1}{e}, -\frac{1}{e} - 1)$ $f''(x) = \frac{1}{x}$ At $(\frac{1}{e}, -\frac{1}{e} - 1)$, $f''(\frac{1}{e}) = e > 0$, Minimum stationary point	2 Marks: Correct answer. 1 Mark: Determines the stationary point or makes similar progress.
13(a) (ii)	$f(x) = x\log_e x - 1$ $f'(x) = \log_e x + 1$ $f(2) = 2\log_e 2 - 1$ $f'(2) = \log_e 2 + 1$ $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$ $= 2 - \frac{2\log_e 2 - 1}{\log_e 2 + 1} = 1.771848327...$	1 Mark: Correct answer.
13(a) (iii)	$f''(x) = \frac{1}{x} > 0$ as $x > 0$ Curve is always concave up.	1 Mark: Correct answer.
13(a) (iv)		2 Marks: Correct answer. 1 Mark: Shows the x or y intercepts.
13(b) (i)	$T = 22 + Ae^{-kt}$ or $Ae^{-kt} = T - 22$ $\frac{dT}{dt} = -kAe^{-kt}$ $= -k(T - 22)$	1 Mark: Correct answer.

13(b) (ii)	Initially $t = 0$ and $T = 80$, $T = 22 + Ae^{-kt}$ $80 = 22 + Ae^{-k \times 0}$ $A = 58$ Also $t = 10$ and $T = 60$ $60 = 22 + 58e^{-k \times 10}$ $e^{-10k} = \frac{38}{58}$ $-10k = \log_e \frac{38}{58}$ $k = -\frac{1}{10}\log_e \frac{38}{58}$ $= \frac{1}{10}\log_e \frac{58}{38}$ $= 0.042285685...$	2 Marks: Correct answer. 1 Mark: Finds the value of A or shows similar understanding of the problem.
13(b) (iii)	We need to find t when $T = 30$ $30 = 22 + 58e^{-kt}$ $e^{-kt} = \frac{8}{58}$ $-kt = \log_e \frac{4}{29}$ $t = -\frac{1}{k}\log_e \frac{4}{29}$ $= 46.84804.. \approx 47$ minutes It will take about 47 minutes for the tea to cool to 30°C	2 Marks: Correct answer. 1 Mark: Determines the value of e^{-kt}.
13(c) (i)	Number of possible outcomes $= 6 \times 6 \times 6 \times 6 \times 6 = 6^5$ Different scores $= 6 \times 5 \times 4 \times 3 \times 2$ P(all different) $= \frac{6 \times 5 \times 4 \times 3 \times 2}{6^5}$ $= \frac{5}{54}$	2 Marks: Correct answer. 1 Mark: Shows some understanding.
13(c) (ii)	$P(6) = \frac{1}{6}$ and $P(\text{Not } 6) = \frac{5}{6}$ P(0 or 1 six) $= {}^5C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^5 + {}^5C_1\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^4$ $= \frac{3125}{3888}$ ≈ 0.804	2 Marks: Correct answer. 1 Mark: Calculates the probability for no 6 and exactly one 6.

14(a) (i)	$a = 2x^3 + 2x$ $v^2 = 2\int(2x^3+2x)\,dx$ $v^2 = 2(\frac{x^4}{2}+x^2)+c$ When $x=2$, $v=5$ then $5^2 = 2(\frac{2^4}{2}+2^2)+c$ $25 = 16+8+c$ $c=1$ $v^2 = 2(\frac{x^4}{2}+x^2)+1$ $= x^4+2x^2+1$ $=(x^2+1)^2$ $v = x^2+1$ Note: initial value of v is positive hence $v \neq -(x^2+1)$	2 Marks: Correct answer. 1 Mark: Determines $v^2 = 2(\frac{x^4}{2}+x^2)+c$ or makes some progress towards the solution.
14(a) (ii)	$\frac{dx}{dt} = x^2+1$ $\frac{dt}{dx} = \frac{1}{x^2+1}$ $t = \int \frac{1}{x^2+1}\,dx$ $t = \tan^{-1}x + c$ When $x=2$, $t=0$ $0 = \tan^{-1}2 + c$ $c = -\tan^{-1}2$ $t = \tan^{-1}x - \tan^{-1}2$ $\tan^{-1}x = t + \tan^{-1}2$ $x = \tan(t+\tan^{-1}2)$ $x = \frac{t+\tan^{-1}2}{1-2\tan t}$	2 Marks: Correct answer. 1 Mark: Determines $t = \tan^{-1}x + c$ or makes some progress towards the solution.
14(b) (i)	$\text{RHS} = 2(p^2+q^2)-(p+q)^2$ $= 2p^2+2q^2-p^2-2pq-q^2$ $= p^2-2pq+q^2$ $=(p-q)^2$ $=$ LHS	1 Mark: Correct answer.

14(b) (ii)	Coordinates of $M\left(\frac{2p+2q}{2}, \frac{p^2+q^2}{2}\right)$ or $\left(p+q, \frac{p^2+q^2}{2}\right)$ Using the result in (i) and $p-q=1$ (given) $(p-q)^2 = 2(p^2+q^2)-(p+q)^2$ $1 = 2(2y)-(x)^2$ $x^2 = 4y-1$ Therefore the locus of M is the parabola $x^2 = 4y-1$	2 Marks: Correct answer. 1 Mark: Determines the coordinates of M or makes some progress.
14(b) (iii)	Now $x^2 = 4y-1$ $= 4(y-\frac{1}{4})$ Focal length is 1, vertex $(0,\frac{1}{4})$ and parabola is concave up Focus is $(0,\frac{5}{4})$	1 Mark: Correct answer.
14(c) (i)	Step 1: To prove the statement true for $n=1$ $7^n - 1 = 7^1 - 1 = 6$ Divisible by 16 Result is true for $n=1$ Step 2: Assume the result true for $n=k$ $7^n - 1 = 6P$ where P is an integer. Hence $7^k = 6P+1$ - (1) To prove the result is true for $n=k+1$ $7^{(k+1)} - 1 = 6Q$ where Q is an integer. $\text{LHS} = 7^{(k+1)} - 1$ $= 7^k \times 7 - 1$ $= 7 \times (6P+1) - 1$ from (1) $= 7(6P) + 7 - 1$ $= 6(7P+1)$ $= 6Q$ $=$ RHS Q is an integer as P and k are integers. Result is true for $n=k+1$ if true for $n=k$ Step 3: Result true by principle of mathematical induction.	3 Marks: Correct answer. 2 Marks: Proves the result true for $n=1$ and attempts to use the result of $n=k$ to prove the result for $n=k+1$. 1 Mark: Proves the result true for $n=1$.

14(d) (i)	$\left(x+\frac{1}{x}\right)^5 = x^5 + 5x^4\left(\frac{1}{x}\right) + 10x^3\left(\frac{1}{x}\right)^2 + 10x^2\left(\frac{1}{x}\right)^3 + 5x\left(\frac{1}{x}\right)^4 + \left(\frac{1}{x}\right)^5$ $= x^5 + 5x^3 + 10x + 10\left(\frac{1}{x}\right) + 5\left(\frac{1}{x}\right)^3 + \left(\frac{1}{x}\right)^5$	1 Mark: Correct answer.
14(d) (ii)	$\left(x+\frac{1}{x}\right)^5 = x^5 + \left(\frac{1}{x}\right)^5 + 5\left[x^3 + \left(\frac{1}{x}\right)^3\right] + 10\left(x+\frac{1}{x}\right)$ Now $x^3 + \left(\frac{1}{x}\right)^3 = \left(x+\frac{1}{x}\right)\left(x^2 - 1 + \left(\frac{1}{x}\right)^2\right)$ $= \left(x+\frac{1}{x}\right)\left(\left(x+\frac{1}{x}\right)^2 - 3\right)$ If $x+\frac{1}{x} = n$ $\left(x+\frac{1}{x}\right)^5 = x^5 + \left(\frac{1}{x}\right)^5 + 5\left[x^3 + \left(\frac{1}{x}\right)^3\right] + 10\left(x+\frac{1}{x}\right)$ $n^5 = x^5 + \left(\frac{1}{x}\right)^5 + 5\left[n\left(n^2-3\right)\right] + 10n$ $x^5 + \left(\frac{1}{x}\right)^5 = n^5 - 5n^3 + 5n$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Groups in pairs $x^a + \left(\frac{1}{x}\right)^a$