ACE Examinations

Student Name: ____________________

2013

YEAR 12

YEARLY EXAMINATION

Mathematics

General Instructions

- Reading time - 5 minutes
- Working time - 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Show all necessary working in Questions 11-16

Total marks - 100

Section I

10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II

90 marks

- Attempt Questions 11-16
- Allow about 2 hour 45 minutes for this section

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Section I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

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1 What is $\dfrac{1+\sqrt{5}}{7-2\sqrt{5}}$ as a fraction with a rational denominator?

(A) $\dfrac{12+9\sqrt{5}}{29}$

(B) $\dfrac{17+9\sqrt{5}}{29}$

(C) $\dfrac{12+9\sqrt{5}}{69}$

(D) $\dfrac{17+9\sqrt{5}}{69}$

2 What values of x is the curve $f(x)=x^3+x^2$ concave down?

(A) $x<-\dfrac{1}{3}$

(B) $x>-\dfrac{1}{3}$

(C) $x<-3$

(D) $x>3$

3 What is the solution to the equation $9^x-28(3^x)+27=0$?

(A) $x=1$ and $x=3$
(B) $x=1$ and $x=4$
(C) $x=0$ and $x=3$
(D) $x=0$ and $x=4$

4 In a business the employees are 55% male and 45% female. Two employees are selected at random. What is the probability that both are male?

(A) 0.2025
(B) 0.2475
(C) 0.3025
(D) 0.5555

5 The table below shows the values of a function $f(x)$ for five values of x.

x	1	1.5	2	2.5	3
$f(x)$	4	1	−2	3	8

What value is an estimate for $\int_1^3 f(x)dx$ using Simpson's rule with these five values?

(A) 4
(B) 6
(C) 8
(D) 12

6 What is the value of $\sum_{r=1}^{50}(2r-4)$?

(A) 96
(B) 2350
(C) 2450
(D) 4700

7 What is the derivative of $\dfrac{\tan x}{x}$ with respect to x?

(A) $\dfrac{-\sec^2 x}{x^2}$

(B) $\dfrac{\sec^2 x}{x^2}$

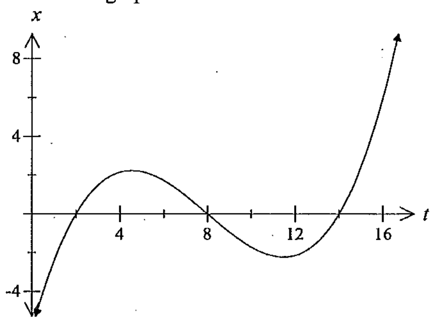
(C) $\dfrac{x\sec^2 x-\tan x}{x^2}$

(D) $\dfrac{\tan x-x\sec^2 x}{x^2}$

8 What is the value of k if the equation $5x^2+2x+k=0$ has −2 as one of its roots?

(A) $k=-24$
(B) $k=-16$
(C) $k=16$
(D) $k=24$

9 The displacement, x metres, from the origin of a particle moving in a straight line at any time (t seconds) is shown in the graph.



When was the particle at rest?

(A) $t = 4.5$ and $t = 11.5$

(B) $t = 0$

(C) $t = 2$, $t = 8$ and $t = 14$

(D) $t = 1.5$ and $t = 8$

10 What is solution to the equation $\dfrac{\cos\theta}{\sqrt{3}} = -\dfrac{1}{2}$ for $0^\circ \le \theta \le 360^\circ$?

(A) $\theta = 30^\circ$ or 330°

(B) $\theta = 60^\circ$ or 300°

(C) $\theta = 150^\circ$ or 210°

(D) $\theta = 120^\circ$ or 240°

Section II

90 marks
Attempt Questions 11 □ 16
Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet.

All necessary working should be shown in every question.

Question 11 (15 marks) **Marks**

(a) Graph on a number line the values of x for which $|x-2| \le 1$. **2**

(b) Find integers a and b such that $(5-\sqrt{3})^2 = a + b\sqrt{3}$. **2**

(c) Differentiate with respect to x.

(i) $(x^2+3)^4$ **1**

(ii) $x^2 \log_e x$ **2**

(d) Find the equation of the tangent to the curve $y = x^2 - 5x$ at the point $(1,-4)$? **2**

(e) For the parabola $(y-1)^2 = 8x + 16$.

(i) Find the coordinates of the vertex and the focus. **2**

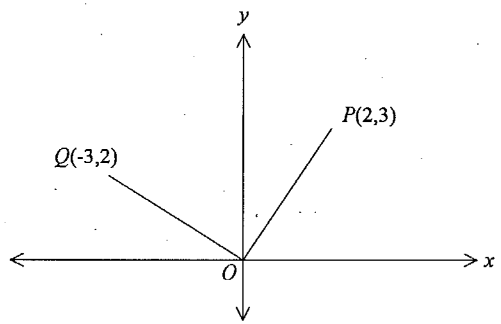
(ii) Sketch the graph of the parabola showing the intercepts. **2**

(iii) What is the axis of symmetry? **1**

(iv) Determine the equation of the directrix. **1**

Question 12 (15marks) **Marks**

(a)



The diagram shows two points $P(2,3)$ and $Q(-3,2)$ on the number plane.

(i) What is the gradient of the line QO? **1**

(ii) Show that PO is perpendicular to QO **1**

(iii) $OPRQ$ is a square in which OQ is parallel to PR. Show that the equation of PR is $2x+3y-13=0$. **2**

(iv) The point R lies on the line $x=-1$. What are the coordinates of R? **1**

(v) Show that the length of the line PR is $\sqrt{13}$ units. **1**

(vi) Find the area of the square $OPRQ$. **1**

(b) The third term of an arithmetic series is 32 and the seventh term is 12.

(i) What is the common difference? **1**

(ii) Find the sum of the first ten terms. **2**

(c) Find:

(i) $\int \sec^2 7x dx$ **1**

(ii) $\int_{-1}^{2} \frac{5}{x+2} dx$ **2**

(d) Sketch the curve $y=1-\cos 2x$ for $0 \le x \le \pi$ **2**

Question 13 (15 marks) **Marks**

(a) The roots of the equation $2x^2-5x+12=0$ are α and β. Find the value of:

(i) $\alpha+\beta$ **1**

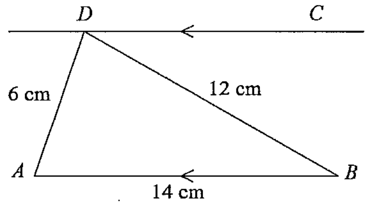
(ii) $\alpha\beta$ **1**

(iii) $\frac{1}{\alpha}+\frac{1}{\beta}$ **1**

(b) Given that $f''(x)=\frac{2}{x^2}+2e^{2x}$ and when $x=1$, $f'(x)=e^2$ and $f(x)=\frac{e^2}{2}$. **3**

Find an expression for $f(x)$ in terms of x.

(c)



Not to scale

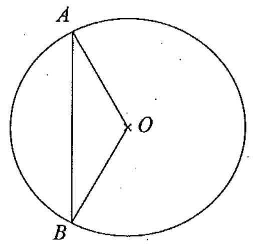
In the diagram, AB is parallel to CD, BD is 12 cm, AD is 6 cm and AB is 14 cm

(i) Use the cosine rule to find the size of $\angle BAD$ to nearest degree. **2**

(ii) Hence find the size of $\angle CDA$, giving reasons for your answer. **2**

(d) The area bounded by $y^2=3-2x-x^2$, $y \ge 0$ and between $x=-3$ and $x=1$ is revolved about the x axis. Calculate the volume of the solid formed if this area is rotated about the x axis. **2**

(e)



Not to scale

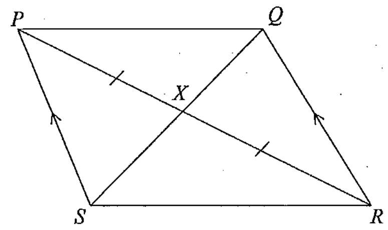
A circle has centre O and radius of 12 cm. The length of arc AB is 8π cm.

(i) What is the size of $\angle AOB$? Answer in radians. **1**

(ii) Find the area of the minor segment cut off by the chord AB. **2**

Question 14 (15 marks) **Marks**

(a)



In the diagram above, $PX = XR$ and PS is parallel to QR.

(i) Prove that $\Delta PXS \equiv \Delta RXQ$. **3**

(ii) Hence show that $PQ = RS$. **1**

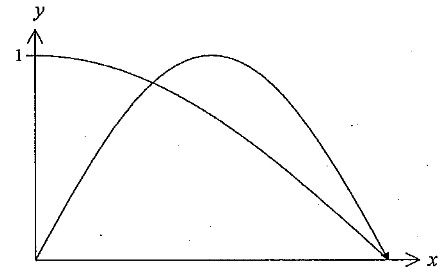
(b) A function $f(x)$ is defined by $f(x) = -x^2(2x-3)$.

(i) Find all solutions for $f(x) = 0$. **1**

(ii) Find the stationary points for the curve $y = f(x)$ and determine their nature. **3**

(iii) Sketch the graph of $y = f(x)$ showing the stationary points and x-intercepts. **2**

(c) The diagram below shows the graph of $y = \sin 2x$ and $y = \cos x$ $(0 \le x \le \frac{\pi}{2})$



(i) Where do the two curves intersect? **2**

(ii) Calculate the area between $y = \sin 2x$, $y = \cos x$ and the x-axis. **3**

Question 15 (15 marks) **Marks**

(a) The population P of a certain insect is falling according to the formula $P = 3000e^{-kt}$, where t is in days.

(i) What is the initial population of the insects? **1**

(ii) Show that $\frac{dP}{dt} = -kP$. **1**

(iii) What is the value of k if it takes 4 days for the number of insects to fall to 1000? **2**

(iv) How long will it take for the number of insects to fall to 10% of the initial number? **2**

(b) A bag contains two blue balls, one red ball, and one white ball. Sandra selects one ball from the bag and keeps it hidden. She then selects a second ball, also keeping it hidden.

(i) Find the probability that both the selected balls are blue. **1**

(ii) Find the probability that at least one of the selected balls is blue. **2**

(iii) Sandra drops one of the selected balls and she sees that it is blue. What is the probability that the ball that is hidden is also blue? **1**

(c) Colin invests \$$P$ at 8% per annum compounded annually. He plans to withdraw \$5000 at the end of each year for six years to cover university fees.

(i) Write down an expression for the amount \$$A_1$ remaining in the account following the withdrawal of the first \$5000. **1**

(ii) Find an expression for the amount \$$A_3$ remaining in the account after the third withdrawal. **2**

(iii) How much does Colin need to invest if the account balance is to be \$0 at the end of the six years? **2**

Question 16 (15 marks) **Marks**

(a) Top soil is tipped from a truck onto a pile. The rate, R kg/s, at which the top soil is flowing is given by the expression $R = t(100 - t^2)$ for $0 \le t \le A$, where t is the time in seconds after the top soil begins to flow.

(i) What is the rate of flow after 5 seconds? **1**

(ii) What is the largest value of A for which the expression for R is physically reasonable? **1**

(iii) Find the maximum rate of flow of the top soil. **3**

(iv) When the top soil starts to flow, the pile already contains 200 kg of top soil. Show that the expression for the mass M of top soil in the pile at time t is given by $M = 50t - \frac{t^4}{4} + 200$. **1**

(v) Calculate the total weight of the top soil that was tipped from the truck. **2**

(b) A particle moves so that its acceleration, a ms^{-2} after t seconds is given by $a = -2$. Initially the particle is located at $x = -3$ and its velocity is 4 ms^{-1}

(i) Find an expression for the velocity of the particle after t seconds. **1**

(ii) Find an expression for the position of the particle after t seconds. **1**

(iii) Determine when the particle is at rest. **1**

(iv) At what time will the particle first be at the origin? **2**

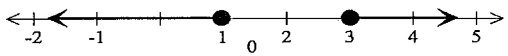
(v) Sketch displacement (x) as a function of time (t) **2**

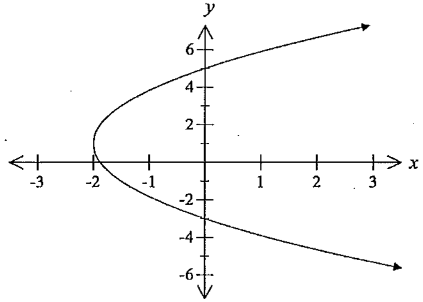
End of paper

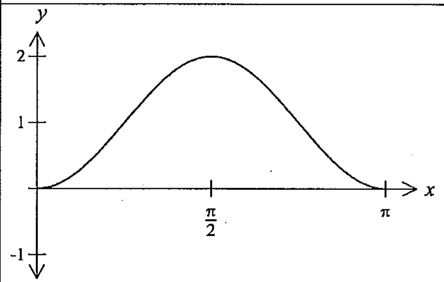
ACE Examination 2013
HSC Mathematics Yearly Examination
Worked solutions and marking guidelines

Section I

	Solution	Criteria
1	$\frac{1+\sqrt{5}}{7-2\sqrt{5}}=\frac{1+\sqrt{5}}{7-2\sqrt{5}}\times\frac{7+2\sqrt{5}}{7+2\sqrt{5}}$ $=\frac{7+2\sqrt{5}+7\sqrt{5}+10}{49-20}$ $=\frac{17+9\sqrt{5}}{29}$	1 Mark: B
2	Concave down when $f''(x)<0$ $f'(x)=3x^2+2x$ $f''(x)=6x+2$ $6x+2<0$ $x<-\frac{1}{3}$	1 Mark: A
3	Let $m=3^x$ then $m^2-28m+27=0$ $(m-27)(m-1)=0$ $m=27 \quad m=1$ $3^x=27 \quad 2^x=1$ $x=3 \quad x=0$	1 Mark: C
4	$P(MM)=0.55\times0.55$ $=0.3025$	1 Mark: C
5	$\int_2^4 f(x)dx=\frac{h}{3}[y_0+y_4+4(y_1+y_3)+2y_2]$ $=\frac{\frac{1}{2}}{3}[4+8+4\times(1+3)+2\times-2]$ $=4$	1 Mark: A
6	$\sum_{r=1}^{50}(2r-4)=-2+0+2+4+...+96$ Arithmetic series with $a=-2$, $l=96$ and $n=50$ $S_n=\frac{n}{2}(a+l)$ $=\frac{50}{2}(-2+96)$ $=2350$	1 Mark: B
7	$\frac{d}{dx}\left(\frac{\tan x}{x}\right)=\frac{x\sec^2 x-\tan x\times1}{x^2}$ $=\frac{x\sec^2 x-\tan x}{x^2}$	1 Mark: C
8	Substitute $x=-2$ into $5x^2+2x+k=0$ $5x^2+2x+k=0$ $5\times(-2)^2+2\times-2+k=0$ $20-4+k=0$ $k=-16$	1 Mark: B
9	Particle at rest if $v=0$ (stationary points of the curve) $t=4.5$ and $t=11.5$	1 Mark: A
10	$\cos\theta=-\frac{\sqrt{3}}{2}$ $\theta=150^\circ$ or 210°	1 Mark: C

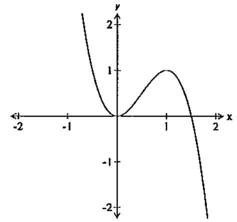
Section II		
11(a)	$\lvert x-2\rvert \geq 1$ $x-2\geq 1$ and $x-2\leq -1$ $x\geq 3$ $x\leq 1$ 	2 Marks: Correct answer. 1 Mark: Determines one of the solutions
11(b)	$(5-\sqrt{3})^2 = 25-10\sqrt{3}+3$ $= 28-10\sqrt{3}$ $= a+b\sqrt{3}$ Therefore $a=28$ and $b=-10$	2 Marks: Correct answer. 1 Mark: Expands the binomial or finds *a* or *b*.
11(c) (i)	$\frac{d\left(x^2+3\right)^4}{dx} = 4\left(x^2+3\right)^3\times 2x$ $= 8x\left(x^2+3\right)^3$	1 Mark: Correct answer.
11(c) (ii)	$\frac{d(x^2\log_e x)}{dx} = x^2\times\frac{1}{x}+\log_e x\times 2x$ $= x+2x\log_e x$	2 Marks: Correct answer. 1 Mark: Applies the product rule or differentiates $\log_e x$.
11(d)	$y=x^2-5x$ At the point $(1,-4)$ $\frac{dy}{dx}=2\times 1-5=-3$ $\frac{dy}{dx}=2x-5$ Point slope formula $y-y_1=m(x-x_1)$ $y-(-4)=-3(x-1)$ $y=-3x-1$	2 Marks: Correct answer. 1 Mark: Finding the gradient of the tangent.
11(e) (i)	$(y-1)^2=8x+16$ $(y-1)^2=4\times 2\times(x+2)$ Vertex is $(-2,1)$, focal length is 2 and focus is $(0,1)$	2 Marks: Correct answer. 1 Mark: Finds the vertex or focus
11(e) (ii)	*x*-intercepts ($y=0$) $(0-1)^2=8x+16$ $8x=-15$ $x=-\frac{15}{8}$ *y*-intercepts ($x=0$) $(y-1)^2=8\times 0+16$ $(y-1)^2=16$ $y-1=\pm 4$ $y=5$ or $y=-3$	2 Marks: Correct answer. 1 Mark: Identifies the intercepts.

		
11(e) (iii)	Axis of symmetry is $y=1$	1 Mark: Correct answer.
11(e) (iv)	Directrix is $x=-4$	1 Mark: Correct answer.
12(a) (i)	Gradient of QO $m=\frac{y_2-y_1}{x_2-x_1}$ $=\frac{2-0}{-3-0}=-\frac{2}{3}$	1 Mark: Correct answer.
12(a) (ii)	Gradient of PO $m=\frac{y_2-y_1}{x_2-x_1}$ $=\frac{3-0}{2-0}=\frac{3}{2}$ Perpendicular lines $m_1m_2=-1$ $-\frac{2}{3}\times\frac{3}{2}=-1$ True	1 Mark: Correct answer.
12(a) (iii)	Parallel lines have the same gradient. Gradient of PR is $-\frac{2}{3}$ (same as QO) $y-y_1=m(x-x_1)$ $y-3=-\frac{2}{3}(x-2)$ $3y-9=-2x-4$ $2x+3y-13=0$	2 Marks: Correct answer. 1 Mark: Recognises the gradient of PR is $-\frac{2}{3}$ or substitutes (2,3) into formula
12(a) (iv)	$x=-1$ satisfies the equation $2x+3y-13=0$ $2\times -1+3y-13=0$ $3y=15$ $y=5$ There coordinates of R are $(-1,5)$	1 Mark: Correct answer.

12(a) (v)	$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $=\sqrt{(-1-2)^2+(5-3)^2}$ $=\sqrt{9+4}=\sqrt{13}$	1 Mark: Correct answer.
12(a) (vi)	$A=s^2$ $=(\sqrt{13})^2=13$ square units	1 Mark: Correct answer.
12(b) (i)	$T_n=a+(n-1)d$ $T_3=a+2d=32$........(1) $T_7=a+6d=12$........(2) Eqn (2) – (1) $4d=-20$ $d=-5$	1 Mark: Correct answer.
12(b) (ii)	Required to find the first term Substitute -5 for d into eqn (1) $a+2\times-5=32,\ a=42$ $S_n=\frac{n}{2}\left[2a+(n-1)d\right]$ $S_{10}=\frac{10}{2}\left[2\times42+9\times-5\right]$ $=195$	2 Marks: Correct answer. 1 Mark: Finds the value of a or makes some progress towards the solution.
12(c) (i)	$\int\sec^2 7x\,dx=\frac{1}{7}\tan 7x+c$	1 Mark: Correct answer.
12(c) (ii)	$\int_{-1}^{2}\frac{5}{x+2}dx=5\left[\log_e(x+2)\right]_{-1}^{2}$ $=5\left[(\log_e 4)-(\log_e 1)\right]$ $=5\log_e 4$	2 Marks: Correct answer. 1 Mark: Determines the primitive.
12(d)		2 Marks: Correct answer. 1 Mark: Shows some understanding of the problem.

13(a) (i)	$\alpha+\beta=-\frac{b}{a}$ $=-\frac{-5}{2}=\frac{5}{2}$	1 Mark: Correct answer.
13(a) (ii)	$\alpha\beta=\frac{c}{a}$ $=\frac{12}{2}=6$	1 Mark: Correct answer.
13(a) (iii)	$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}$ $=\frac{\frac{5}{2}}{6}=\frac{5}{12}$	1 Mark: Correct answer.
13(b)	$f''(x)=2x^{-2}+2e^{2x}$ $f'(x)=-2x^{-1}+e^{2x}+c$ When $x=1$ then $f'(x)=e^2$ $e^2=-2\times1+e^2+c$ $c=2$ $f'(x)=-2x^{-1}+e^{2x}+2$ $f(x)=-2\log_e x+\frac{e^{2x}}{2}+2x+k$ When $x=1$ then $f(x)=\frac{e^2}{2}$ $\frac{e^2}{2}=-2\log_e 1+\frac{e^2}{2}+2+k$ $k=-2$ $f(x)=-2\log_e x+\frac{e^{2x}}{2}+2x-2$	3 Marks: Correct answer. 2 Marks: Integrates to find $f(x)$ or makes significant progress. 1 Mark: Integrates to find $f'(x)$ or shows some understanding of the problem.
13(c) (i)	$\text{Cos}\angle BAD=\frac{14^2+6^2-12^2}{2\times14\times6}$ $\angle BAD=58.41186449...$ $\approx 58^\circ$	2 Marks: Correct answer. 1 Mark: Uses cosine with at least one correct value.
13(c) (ii)	$\angle CDA+58=180$ (co-interior angles supplementary $AB//DC$) $\angle CDA=180-58$ $=122^\circ$	2 Marks: Correct answer. 1 Mark: Finds $\angle CDA=122^\circ$.

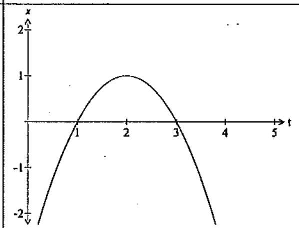
13(d)	$y^2 = 3 - 2x - x^2$ $V = \pi \int_a^b y^2 dx$ $V = \pi \int_{-3}^{1} 3 - 2x - x^2 dx$ $= \pi \left[3x - x^2 - \frac{x^3}{3} \right]_{-3}^{1}$ $= \pi \left[\left(3 - 1 - \frac{1}{3} \right) - (-9 - 9 + 9) \right]$ $= \frac{32\pi}{3}$	2 Marks: Correct answer. 1 Mark: Sets up the integral to determine the volume.
13(e) (i)	$l = r\theta$ $8\pi = 12\theta$ $\theta = \frac{2\pi}{3}$	1 Mark: Correct answer.
13(e) (ii)	$A = \frac{1}{2} \times r^2 \times \theta$ $= \frac{1}{2} \times 12^2 \times \frac{2\pi}{3}$ $= 48\pi \text{ cm}^2$ $A = \frac{1}{2} ab \sin O$ $= \frac{1}{2} \times 12 \times 12 \times \sin \frac{2\pi}{3}$ $= 72 \times \frac{\sqrt{3}}{2} = 36\sqrt{3} \text{ cm}^2$ Area of segment $= 48\pi - 36\sqrt{3} \approx 88.44 \text{ cm}^2$	2 Marks: Correct answer. 1 Mark: Determines the area of the sector or triangle.
14(a) (i)	In ΔPXS and ΔRXQ $\angle PXS = \angle RXQ$ (vertically opposite angles are equal) $\angle PSX = \angle XQR$ (alternate angles are equal, parallel lines) $PX = XR$ (given) $\therefore \Delta PXS \equiv \Delta RXQ$ (AAS)	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: One relevant statement and reason.
14(a) (ii)	$PS = RQ$ (matching sides in congruent triangles) $\therefore PQRS$ is a parallelogram (a pair of opposite sides parallel and equal) $\therefore PQ = RS$ (opposite sides in a parallelogram are equal)	1 Mark: Correct answer.
14(b) (i)	$-x^2(2x + 3) = 0$ $x = 0,\ x = -\frac{3}{2}$	1 Mark: Correct answer.

14(b) (ii)	Stationary points $f'(x) = 0$ $-6x^2 + 6x = 0$ $-6x(x - 1) = 0$ $x = 0,\ x = 1$ Stationary points are $(0, 0)$ and $(1, -1)$. $f''(x) = -12x + 6$ At $(0, 0)$, $f''(0) = 6 > 0$, Minimum stationary point At $(1, 1)$, $f''(1) = -6 < 0$, Maximum stationary point	3 Marks: Correct answer. 2 Marks: Finds the turning points. 1 Mark: Obtains the first derivative and recognises $f'(x) = 0$.
14(b) (iii)		2 Marks: Correct answer. 1 Mark: Makes some progress towards sketching the curve.
14(c) (i)	Solve the equations simultaneously $\sin 2x = \cos x$ $2 \sin x \cos x = \cos x$ $2 \sin x \cos x - \cos x = 0$ $\cos x(2 \sin x - 1) = 0$ Therefore $\cos x = 0$ $2 \sin x = 0$ $x = \frac{\pi}{2},\ y = 0$ $x = \frac{\pi}{6},\ y = \frac{\sqrt{3}}{2}$	2 Marks: Correct answer. 1 Mark: Finds one of the points of intersection or the x-coordinates of both points.
14(c) (ii)	$A = \int_0^{\frac{\pi}{6}} \sin 2x dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos x dx$ $= \left[-\frac{\cos 2x}{2} \right]_0^{\frac{\pi}{6}} + [\sin x]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$ $= \left(-\frac{1}{4} - \left(-\frac{1}{2} \right) \right) + \left(1 - \frac{1}{2} \right)$ $= \frac{3}{4}$ square units	3 Marks: Correct answer. 2 Marks: Makes significant progress. 1 Mark: Correctly sets up one integral

15(a) (i)	Initial population occurs when $t=0$. $P=3000e^{-kt}$ $=3000e^{-k\times 0}=3000$	1 Mark: Correct answer.
15(a) (ii)	$P=3000e^{-kt}$ $\frac{dP}{dt}=-k\times 3000e^{-kt}$ $=-kP$	1 Mark: Correct answer.
15(a) (iii)	When $t=4$ then $P=1000$ $1000=3000e^{-4k}$ $e^{-4k}=\frac{1}{3}$ $\log_e e^{-4k}=\log_e 0.\dot{3}$ $-4k=\log_e 0.\dot{3}$ $k=\frac{\log_e 0.\dot{3}}{-4}=0.2746530722...$	2 Marks: Correct answer. 1 Mark: Substitutes $t=4$ and $P=1000$ into the formula.
15(a) (iv)	We need to find t when $P=300$. $300=3000e^{-k\times t}$ $e^{-kt}=0.1$ $-kt=\log_e 0.1$ $t=-\frac{1}{k}\log_e 0.1$ $=-\frac{\log_e 0.1}{0.274653..}$ $=8.383613097..\approx 8.4$ days	2 Marks: Correct answer. 1 Mark: Determines $e^{-kt}=0.1$ or shows some understanding of the problem.
15(b) (i)	$\text{P(BB)}=\frac{2}{4}\times\frac{1}{3}$ $=\frac{1}{6}$	1 Mark: Correct answer.
15(b) (ii)	Probability of at least one blue ball is every outcome except selecting red and white balls. $P(\text{at least 1 blue})=1-P(RW+WR)$ $=1-(\frac{1}{4}\times\frac{1}{3}+\frac{1}{4}\times\frac{1}{3})$ $=\frac{5}{6}$	2 Marks: Correct answer. 1 Mark: Shows some understanding of the problem.

15(b) (iii)	If one of the selected balls is blue, there are 5 possible outcomes only one of which will yield another blue. Sample space $=\{\text{BB,BR,BW,RB,WB}\}$ $\text{P(E)}=\frac{1}{5}$	1 Mark: Correct answer.
15(c) (i)	$A=P(1+r)^n$ $=P\times(1+0.08)^1$ $=P(1.08)$ After 1 year $A_1=P(1.08)-5000$	1 Mark: Correct answer.
15(c) (ii)	After 2 years $A_2=(P(1.08)-5000)\times 1.08-5000$ $=P\times 1.08^2-5000(1.08+1)$ After 3 years $A_3=\left[(P\times 1.08^2-5000(1.08+1)\right]\times 1.08-5000$ $=P\times 1.08^3-5000(1.08^2+1.08+1)$	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.
15(c) (iii)	After 6 years $A_6=P\times 1.08^6-5000(1.08^5+1.08^4+...+1)$ To find P when $A_6=0$ $0=P\times 1.08^6-5000\left(\frac{1.08^6-1}{1.08-1}\right)$ $0=P\times 1.08^6\times 0.08-5000(1.08^6-1)$ $P\times 1.08^6\times 0.08=5000(1.08^6-1)$ $P=\frac{5000(1.08^6-1)}{1.08^6\times 0.08}$ $=\$23\ 114.40$ Colin needs to invest $23 114.40	2 Marks: Correct answer. 1 Mark: Obtains a correct expression for A_6 and uses $A_6=0$.

16(a) (i)	When $t = 5$ $R = t(100 - t^2)$ $= 5 \times (100 - 5^2) = 375$ Flow rate is 375 kg/s	1 Mark: Correct answer.
16(a) (ii)	The expression is reasonable until $R = 0$ $t(100 - t^2) = 0$ $t = 0, t = 10$ ($t = -10$ is not possible) The largest value of t is 10.	1 Mark: Correct answer.
16(a) (iii)	$R = t(100 - t^2) = 100t - t^3$ $\frac{dR}{dt} = 100 - 3t^2$, $\frac{d^2R}{dt^2} = -6t$ Maximum occurs at the stationary point ($\frac{dR}{dt} = 0$). $100 - 3t^2 = 0$ $t^2 = \frac{100}{3}$ $t = \frac{10}{\sqrt{3}}$, $(t \geq 0)$ When $t = \frac{10}{\sqrt{3}}$ then $\frac{d^2R}{dt^2} = -6 \times \frac{10}{\sqrt{3}} < 0$ (maximum value) When $t = \frac{10}{\sqrt{3}}$ $R = 100 \times \frac{10}{\sqrt{3}} - \left(\frac{10}{\sqrt{3}}\right)^3$ $= \frac{1000}{\sqrt{3}} - \frac{1000}{3\sqrt{3}}$ $= \frac{2000}{3\sqrt{3}}$ $= \frac{2000\sqrt{3}}{9} \approx 384.9001795...$ Therefore the maximum flow rate is $\frac{2000\sqrt{3}}{9}$ kg/s	3 Marks: Correct answer. 2 Marks: Determines $t = \frac{10}{\sqrt{3}}$ and tests for maximum value. 1 Mark: Calculates the first derivative or has some understanding of the problem.
16(a) (iv)	Let the mass of the top soil be M. Then $\frac{dM}{dt} = 100t - t^3$ $M = 50t - \frac{t^4}{4} + c$ When $t = 0$ then $M = 200$ hence $c = 200$ $M = 50t - \frac{t^4}{4} + 200$	1 Mark: Correct answer.

16(a) (v)	When $t = 10$ Total weight $= \int_0^{10} (100t - t^3)\,dt$ $= \left[50t^2 - \frac{t^4}{4}\right]_0^{10}$ $= \left[\left(50 \times 10^2 - \frac{10^4}{4}\right) - 0\right]$ $= 2500$ kg	2 Marks: Correct answer. 1 Mark: Shows some understanding.
16(b) (i)	$a = -2$ $v = \int -2\,dt = -2t + c$ When $t = 0$ then $v = 4$ $4 = -2 \times 0 + c$ or $c = 4$ $\therefore v = -2t + 4$	1 Mark: Correct answer.
16(b) (ii)	$x = \int -2t + 4\,dt$ $= -t^2 + 4t + c$ When $t = 0$ then $x = -3$ $-3 = 0 + 0 = c$ or $c = -3$ $\therefore x = -t^2 + 4t - 3$	1 Mark: Correct answer.
16(b) (iii)	Particle at rest when $v = 0$ $v = -2t + 4$ $0 = -2t + 4$ $t = 2$ Therefore the particle is at rest after 2 seconds.	1 Mark: Correct answer.
16(b) (iv)	At the origin when $x = 0$ $x = -t^2 + 4t - 3$ $0 = -t^2 + 4t - 3$ $0 = -(t^2 - 4t + 3)$ $0 = -(t - 3)(t - 1)$ $\therefore t = 1$ or $t = 3$ First at the origin when $t = 1$	2 Marks: Correct answer. 1 Mark: Determines that $0 = -t^2 + 4t - 3$
16(b) (v)		2 Marks: Correct answer. 1 Mark: Intercepts or correct shape.