ABBOTSLEIGH

AUGUST 2006

YEAR 12
ASSESSMENT 4
TRIAL HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes.
- Working time – 3 hours.
- Write using blue or black pen.
- Board-approved calculators may be used.
- A table of standard integrals is provided.
- All necessary working should be shown in every question.

Total marks (120)
- Attempt Questions 1-10.
- All questions are of equal value.

Marks

Question 1 (12 marks)
Use a SEPARATE writing booklet

(a) Simplify $6-(4x+2)$. 1

(b) Solve $x^2+3x-10=0$. 2

(c) Evaluate, correct to 3 significant figures

$$\pi\sqrt{\frac{2.2^3}{6.7-5.1}}$$ 2

(d) Simplify $\dfrac{x^3-8}{2x-4}$. 2

(e) Goods and Service Tax (GST) is added to goods for sale by increasing the cost price by 10%. If the selling price of an item is \$231, what was the GST? 1

(f) (i) Solve the equation $|2x+3|\le 7$. 2

(ii) Graph the solution to part (i) on the number line. 1

(g) Solve $5^x=\dfrac{1}{125}$. 1

Question 2 (12 marks)
Use a SEPARATE writing booklet.

(a) Differentiate:

(i) $\dfrac{1}{6x^3}$ 1

(ii) $e^{\cos x}$ 1

(iii) $\log_e(x+5)$ 1

(b) Use the list of standard integrals to find $\int \sec 4x \tan 4x \, dx$. 1

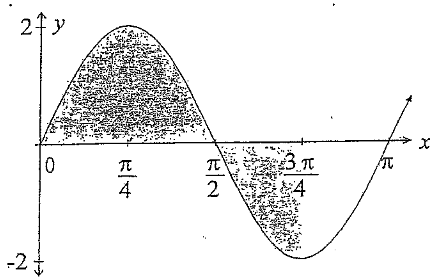
(c) Find the primitive function for:

(i) $\sec^2 \frac{x}{2}$ 1

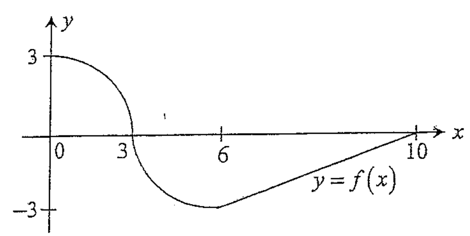
(ii) $\frac{x^2}{x^3 - 4}$ 2

(iii) e^{3x} 1

(d) Find the shaded area shown for the curve $y = 2\sin 2x$: 2



(e) The following illustrates the function, $y = f(x)$ for $x = 0$ to $x = 10$.



The curve from $x = 0$ to $x = 3$ is a part of a circle of radius 3 units, similarly for that part of the curve from $x = 3$ to $x = 6$. From $x = 6$ to $x = 10$, the function is a straight line.

Explain why $\int_0^{10} f(x)\,dx = -6$. 2

Question 3 (12 marks) Marks
Use a SEPARATE writing booklet.

(a) (i) On at least one-third of a page, plot the points $A(4,4)$ and $B(0,7)$ on a number plane. 1

(ii) Find the length of AB. 1

(iii) Find the gradient of AB. 1

(iv) Show that the equation of AB is $3x + 4y - 28 = 0$. 1

(v) Given the point $D(-1,1)$, find the perpendicular distance from D to the line AB. 2

(vi) Find the y-value of the point $C(-9, y)$ such that AB is parallel to CD. 1

(vii) Find the area of the trapezium $ABCD$. 2

(b) (i) On the same set of axes sketch the graphs of $y = \frac{4x}{\pi}$ and $y = \tan x$ for $0 \le x \le 2\pi$ (indicate all essential features). 2

(ii) Use your graph to determine how many solutions there are to the equation $\tan x = \frac{4x}{\pi}$ for $0 \le x \le 2\pi$. 1

Question 4 (12 marks)
Use a SEPARATE writing booklet.

(a) If α and β are the roots of the quadratic equation $3x^2 + 6x + 1 = 0$, find the value of

(i) $\alpha + \beta$ 1

(ii) $\alpha\beta$ 1

(iii) $\alpha^2 + \beta^2$ 2

Question 4 (cont) **Marks**

(b) The curve $y=f(x)$ has a gradient function of $\frac{dy}{dx}=2x+3$. The curve passes through the point $(2,-1)$. Find the equation of the curve. **3**

(c) A soccer goal mouth is 8 metres wide. If a player is standing in front of the goal mouth 4 metres from one goal post and 10 metres from the other goal post, through what angle must she shoot to be successful in scoring the goal? (Answer to the nearest degree.) **2**

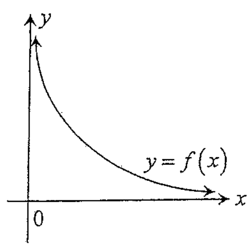
(d) (i) Differentiate $y=\cos^2 x$. **1**

(ii) Hence, or otherwise, evaluate $\int_0^{\frac{\pi}{4}} \cos x \sin x \, dx$. **2**

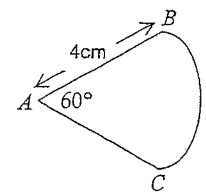
Question 5 (12 marks)
Use a SEPARATE writing booklet.

(a) Find the value(s) of k, such that $x^2+4kx+8=0$ has real roots. **3**

(b) For the following curve $y=f(x)$, describe its shape using $f'(x)$ and $f''(x)$. **2**



(c)



NOT TO SCALE

(i) Calculate in terms of π, the length of the arc BC of the sector with centre A. **1**

(ii) Calculate the area of the segment between the straight line BC and the arc of the sector, correct to nearest 0.1cm^2. **2**

(d) For the parabola $(y-2)^2=-12(x+4)$:

(i) state the coordinates of the vertex. **1**

(ii) state the coordinates of the focus. **2**

(iii) state the equation of the directrix. **1**

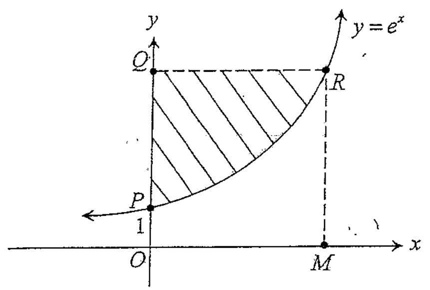
Question 6 (12 marks)
~~Use a SEPARATE writing booklet.~~ **Marks**

(a) Find the equation of the tangent to the curve $y = e^{4x+1}$ at the point where $x = 0$. (Give your answer in general form.) **2**

(b) Sketch the curve $y = \ln(x-2)$, clearly showing any intercepts and asymptotes. **2**

(c) Use Simpson's Rule with 3 function values to evaluate $\int_0^{\frac{\pi}{4}} \sec x \, dx$. (Answer correct to two decimal places.) **2**

(d)



NOT TO SCALE

(i) Find the area of the rectangle $OQRM$ in terms of e, if the coordinates of M are (3,0). **1**

(ii) Find the shaded area PQR in terms of e. **2**

(iii) The section of the curve $y = e^x$ from P to R is rotated 360° about the x-axis to form a solid. Calculate the exact volume of the solid generated. **3**

~~Question 7 (12 marks)~~
Use a SEPARATE writing booklet. **Marks**

(a) A referendum was held in the city of Toowoomba and the people asked if they would consider having recycled water for drinking. 40% were in favour of the proposal, while 60% were against. If two people were chosen at random, what is the probability that

(i) both agree to use recycled water for drinking? **1**

(ii) one person is against using recycled water for drinking? **2**

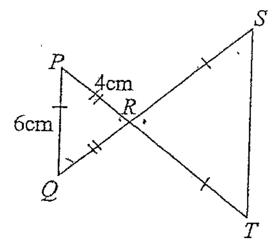
(b) For the curve $y = \frac{1}{3}x^3 - 9x + 2$

(i) find the stationary points and determine their nature. **3**

(ii) By solving $y'' = 0$ we can see that there may be an inflexion point at $x = 0$. What else must be done to show that $(0,2)$ is an inflexion point? **1**

(iii) Sketch the curve in the domain $-4 \le x \le 4$, showing all essential features (there is no need to find the x-intercept). **2**

(c)



NOT TO SCALE

In the diagram above, $PR = RQ = 4\text{cm}$ and $PQ = RS = RT = 6\text{cm}$.

(i) Prove that $\Delta PRQ \parallel\!| \Delta RST$. **2**

(ii) Find the length of ST. **1**

Question 8 (12 marks)
Use a SEPARATE writing booklet. Marks

(a) A student decides to save money over one year. In her first week she puts aside \$0.10. In the second week, \$0.40, then in the third week, \$0.70, and so on with constant increases over time.

(i) What amount will she deposit in her 52nd week? 2

(ii) How much has she saved altogether over the year? 2

(b) ABC is an isosceles triangle with $AC = BC$. The side AC is produced to D and CX is parallel to AB such that $\angle XCD$ is acute. $\angle BCA = 80°$.

(i) Draw a neat diagram to illustrate the information given above. 2

(ii) Explain why $\angle XCD = 50°$. 2

(c) For $0 \le x \le 2\pi$, solve $\tan x = \frac{-1}{\sqrt{3}}$. 2

(d) Solve $3\log_8 2 = \log_8 x - \log_8 3$. 2

Question 9 (12 marks)
Use a SEPARATE writing booklet.

(a) Josephine borrows \$300 000 to buy a unit. Interest is calculated monthly at the rate of 6% per annum compounded monthly. She agrees to repay the loan with equal monthly instalments of M at the end of each month for 20 years. Let A_n be the amount owing after n months.

(i) Find an expression for A_1. 1

(ii) Show that $A_2 = 300\,000(1.005)^2 - M(1.005) - M$. 1

(iii) Find the amount of the monthly repayment (to nearest 5 cents). 3

Question 9 CONT Marks

(b) A particle moves along a straight line so that its distance x metres from a fixed point O is given by $x = 4 - 3t + 12\ln(t+3)$ where the time t is measured in seconds.

(i) What is the exact initial position? 1

(ii) Find expressions for the velocity and acceleration of the particle at time t. 2

(iii) Find the time when the velocity of the particle is zero. 2

(iv) What is the exact distance that the particle has travelled in the first 2 seconds? 2

Question 10 (12 marks)
Use a SEPARATE writing booklet.

(a) The perimeter of an isosceles triangle is 16cm. The length of each of the equal sides is $(8-x)\text{cm}$.

(i) Show the height, h, of the triangle is $\left(4\sqrt{4-x}\right)\text{cm}$. 2

(ii) Show that the area of the isosceles triangle, A, is $A = 4x\sqrt{4-x}$. 1

(iii) For what value of x is the area of the triangle maximised? 3

(b) A Geiger counter is taken into a region after a nuclear accident and gives a reading of 40 000. One year later, the same Geiger counter gives a reading of 36 000. It is known that the reading N is given by the formula

$$N = N_0 e^{-kt}$$

where N_0 and k are constants and t is the time measured in years.

(i) Evaluate the constants N_0 and k. 3

(ii) It is known that the region will become safe when the reading reaches 400. After how many years will the region become safe? (Answer to the nearest whole year.) 3

END OF PAPER

Abbotsleigh Trial HSC - Mathematics, 2006

Q1(a) $6-(4x+2)$

$=6-4x-2$

$=4-4x$

(b) $x^2+3x-10=0$

$(x+5)(x-2)=0$

$\therefore x=-5, 2$

(c) $\pi\sqrt{\dfrac{2.2^3}{6.7-5.1}}$

$=8.1044\ldots$

$=8.10$ (to 3.s.f.)

(d) $\dfrac{x^3-8}{2x-4}=\dfrac{\cancel{(x-2)}(x^2+2x+4)}{2\cancel{(x-2)}}$

$=\dfrac{x^2+2x+4}{2}$

e) $S.P. = C.P. + 10\% \times C.P$

$= 110\% \times C.P$

$\$231 = 1.10 \times C.P$

$CP = \$210$

$GST = SP - CP$

$= 231-210$

$= \$21$

OR $110\% = \$231$

$\div 11 \quad \div 11$

$10\% = \$21$

$\therefore GST = \$21$

f)(i) $|2x+3| \le 7$

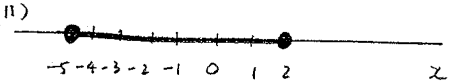
$2x+3 \ge -7$ $\qquad$ $2x+3 \le 7$

$2x \ge -10$ $\qquad$ $2x \le 4$

$x \ge -5$ $\qquad$ $x \le 2$

$\therefore -5 \le x \le 2$

(ii)



(g) $5^x = \dfrac{1}{125}$

$= \dfrac{1}{5^3}$

$= 5^{-3}$

$\therefore x=-3$

Q2(a)(i) $y=\dfrac{1}{6x^3}$

$=\dfrac{1}{6}\times\dfrac{1}{x^3}$

$=\dfrac{1}{6}\times x^{-3}$

$y'=-\dfrac{3}{6}\times x^{-4}$

$=-\dfrac{1}{2x^4}$

(ii) $y=e^{\cos x}$

$y'=(-\sin x)e^{\cos x}$

(iii) $y=\log_e(x+5)$

$\therefore y'=\dfrac{1}{x+5}$

(b) $\int \sec 4x \tan 4x \, dx =$

$=\dfrac{1}{4}\sec 4x + C$

(c)(i) $\int \sec^2\left(\dfrac{x}{2}\right) dx$

$=\int \sec^2\left(\left(\dfrac{1}{2}\right)x\right) dx$

$=\dfrac{1}{\left(\frac{1}{2}\right)}\tan\dfrac{x}{2} + C$

$=2\tan\dfrac{x}{2} + C$

(c)(ii) $\int \dfrac{x^2}{x^3-4}\,dx$

$=\dfrac{1}{3}\int\dfrac{3x^2}{x^3-4}\,dx$

$=\dfrac{1}{3}\ln(x^3-4)+C$

(iii) $\int e^{3x}\,dx$

$=\dfrac{1}{3}e^{3x}+C$

) $A=\int_0^{\frac{\pi}{2}} 2\sin 2x\,dx + \left|\int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} 2\sin 2x\,dx\right|$

$R = 3\int_0^{\frac{\pi}{4}} 2\sin 2x\,dx$

$=6\int_0^{\frac{\pi}{4}}\sin 2x\,dx$

$=\dfrac{-6}{2}\Big[\cos 2x\Big]_0^{\frac{\pi}{4}}$

$=-3\left(\cos\left(2\times\frac{\pi}{4}\right)-\cos(2\times 0)\right)$

$=-3\left(\cos\frac{\pi}{2}-\cos 0\right)$

$=-3(0-1)$

$=3$ units^2

(e) The area between the curve and x-axis from $x=0$ to $x=3$ is the SAME as the area between the curve and x-axis from $x=3$ to $x=6$, but since it is below the x-axis here, the value $\int_3^6 f(x)\,dx = -\int_0^3 f(x)\,dx$

and so they cancel to zero when added.

Similarly, the area between the curve (line) $y=f(x)$ from $x=6$ to $x=10$, is below (hence negative) the x-axis and forms a triangle:

area $=\frac{1}{2}\times$ base $\times$ height

$=\frac{1}{2}\times(10-6)\times 3$

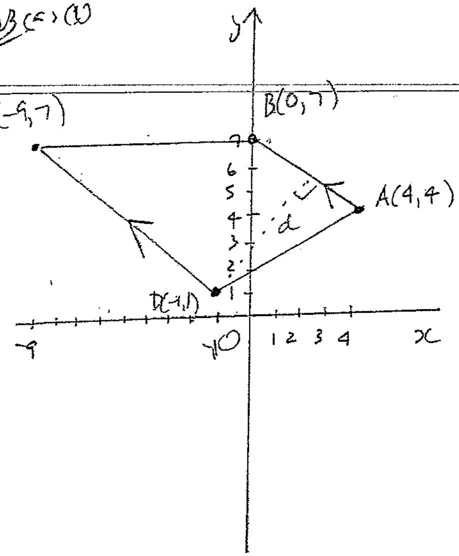
$=6$ units

$\therefore \int_0^{10} f(x)\,dx = \underbrace{\int_0^3 f(x)\,dx + \int_3^6 f(x)\,dx}_{0} + \int_6^{10} f(x)\,dx$

$= 0 + (-6)$

$= -6$

Q3(a)(i)



(ii) $AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(4-7)^2+(4-0)^2}$

$= \sqrt{(-3)^2+4^2}$

$= \sqrt{9+16}$

$= \sqrt{25}$

$\therefore AB = 5$ units

(iii) $m_{AB} = \frac{4-7}{4-0}$

$\therefore m_{AB} = \frac{-3}{4}$

(iv) $y-y_1 = m(x-x_1)$

$y-4 = \frac{-3}{4}(x-4)$

$4(y-4) = -3(x-4)$

$4y-16 = -3x+12$

$\therefore 3x+4y-28=0$, as required.

(v) $d = \left|\frac{Ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$

$= \left|\frac{3(-1)+4(1)-28}{\sqrt{3^2+4^2}}\right|$

$= \left|\frac{-3+4-28}{5}\right|$

$= \frac{27}{5}$ units or 5·4 units or $5\frac{2}{5}$ units

(vi) $m_{AB} = m_{CD} = \frac{-3}{4}$

$m_{CD} = \frac{y_2-y_1}{x_2-x_1} = -\frac{3}{4}$

$\Rightarrow \frac{y-1}{-9-(-1)} = -\frac{3}{4}$

$\frac{y-1}{-8} = -\frac{3}{4}$

$y-1 = 6$

$\therefore y = 7$ $\therefore C(-9,7)$

(vii) Area of Trapezium $= \frac{1}{2} \times$ height $\times$ (sum of parallel sides)

$A = \frac{1}{2} \times \frac{27}{5} \times \underbrace{(AB + CD)}_{15}$

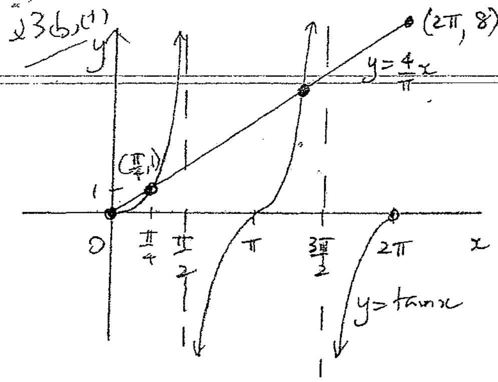
$CD = \sqrt{6^2+8^2}$

$\therefore CD = 10$

$= \frac{1}{2} \times \frac{27}{5} \times (5+10)$

$= 40.5$ units^2

Q3(b)(i)



(ii) 3 solutions (since 3 points of intersection)

Q4(a) $3x^2+6x+1=0$

(i) $\alpha+\beta = -\frac{b}{a}$

$= \frac{-6}{3}$

$\therefore \alpha+\beta = -2$

(ii) $\alpha\beta = \frac{c}{a}$

$\therefore \alpha\beta = \frac{1}{3}$

(iii) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2(\alpha\beta)$

$= (-2)^2 - 2(\frac{1}{3})$

$= 4 - \frac{2}{3}$

$\therefore \alpha^2+\beta^2 = 3\frac{1}{3}$

(b) $\frac{dy}{dx} = 2x+3$

$y = x^2+3x+k$

[at $x=2$, $y=-1$]

$-1 = 2^2+3(2)+k$

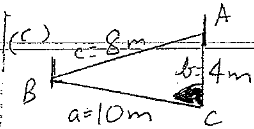
$-1 = 4+6+k$

$-1 = 10+k$

$k = -11$

$y = x^2+3x-11$

(c)



$\cos C = \frac{(a^2+b^2-c^2)}{(2ab)}$

$= \frac{(10^2+4^2-8^2)}{(2\times 10\times 4)}$

$= 0.65$

$\hat{C} = 49.45...^\circ$

$\hat{C} = 49^\circ$ (to nearest degree)

The angle through which the player must shoot is 49°.

(d)(i) $y = \cos^2 x$

$= (\cos x)^2$

$\frac{dy}{dx} = -2(\cos x)\sin x$

(ii) hence $\int -2\cos x \sin x\, dx = \cos^2 x + C$

$\Rightarrow \int_0^{\frac{\pi}{4}} \cos x.\sin x\, dx = \left[-\frac{1}{2}\cos^2 x\right]_0^{\frac{\pi}{4}}$

$= -\frac{1}{2}\left(\cos\frac{\pi}{4}\right)^2 + \frac{1}{2}(\cos 0)^2$

$= -\frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)^2 + \frac{1}{2}(1)^2$

$= -\frac{1}{2} \times \frac{1}{2} + \frac{1}{2}$

$= -\frac{1}{4} + \frac{1}{2}$

$= \frac{1}{4}$

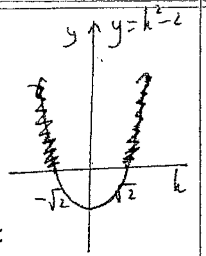
Q.5 (a) Real Roots: $\Delta \geq 0$

$b^2 - 4ac \geq 0$

$(4k)^2 - 4(1)(8) \geq 0$

$16k^2 - 32 \geq 0$

$k^2 - 2 \geq 0$



$\therefore k \leq -\sqrt{2},\ k \geq \sqrt{2}$

(b) $f'(x) < 0$ (decreasing)

$f''(x) > 0$ (concave up)

(c) (i) $l = \theta r$ $\quad \left[60^\circ = \frac{\pi}{3}\right]$

$= \frac{4\pi}{3}$ cm

(ii) Area of segment $= \frac{1}{2} r^2 (\theta^c - \sin\theta^c)$

$= \frac{1}{2}(4)^2\left(\frac{\pi}{3} - \sin\frac{\pi}{3}\right)$

$= \frac{1}{2} \times 16 \times \left(\frac{\pi}{3} - \sin 60^\circ\right)$

$= 1.449\ldots$

$= 1.4\text{ cm}^2$ (to nearest 0.1cm^2)

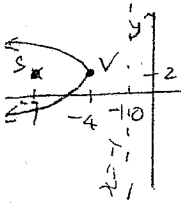
(d) $(y-2)^2 = -12(x+4)$

$\boxed{(y-h)^2 = -4a(x-h)}$ $\boxed{V(h,k)}$

(i) $V(-4, 2)$

(ii) $-4a = -12$

$a = 3$ (N.B. 'a' is always positive)



$S(-7, 2)$

(iii) $x = -1$

Q6

(a) $y = e^{4x+1}$

$y' = 4e^{4x+1}$

[at $x = 0$]

$m_T = 4e^{1}$

$= 4e$

Find y: $y = e^{4(0)+1}$

$= e$ $\quad (0, e)$

Find equation:

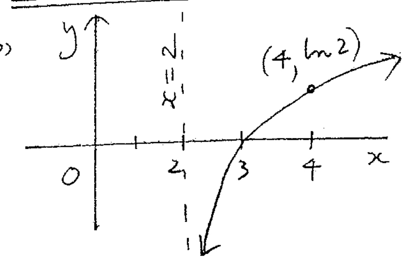
$y - y_1 = m(x - x_1)$

$y - e = 4e(x - 0)$

$y - e = 4ex$

$\therefore 4ex - y + e = 0$

(b)



(c) $\int_a^b f(x)\,dx \doteqdot \frac{b-a}{6}\left\{f(a) + 4\left(f\left(\frac{a+b}{2}\right)\right) + f(b)\right\}$

$\doteqdot \frac{\frac{\pi}{4} - 0}{6}\left\{f(0) + 4f\left(\frac{\pi}{8}\right) + f\left(\frac{\pi}{4}\right)\right\}$

$= \frac{\pi}{24}\left\{\sec 0 + 4\sec\frac{\pi}{8} + \sec\frac{\pi}{4}\right\}$

$= 0.8827\ldots$

$\doteqdot 0.88$

Q6

(d) (i) $A = (3e^3)$ units^2

(ii) Shaded area

= Area of Rect − Area under curve from $x = 0$ to $x = 3$

$= 3e^3 - \int_0^3 e^x\,dx$

$= 3e^3 - \left[e^x\right]_0^3$

$= 3e^3 - \{e^3 - e^0\}$

$= 3e^3 - e^3 + 1$

$= (2e^3 + 1)$ units^2

(iii) $V = \pi\int_a^b [f(x)]^2\,dx$

$= \pi\int_0^3 (e^x)^2\,dx$

$= \pi\int_0^3 e^{2x}\,dx$

$= \frac{\pi}{2}\left[e^{2x}\right]_0^3$

$= \frac{\pi}{2}\left\{e^6 - e^0\right\}$

$= \frac{\pi}{2}(e^6 - 1)$ unit^3

7(a) $P(F, F) = \frac{40}{100} \times \frac{40}{100}$

$= \frac{16}{100}$ or 0.16

$P(F, \bar{F}) + P(\bar{F}F)$

$= 0.4 \times 0.6 + 0.6 \times 0.4$

$= 0.48$

(b) (i) $y = \frac{1}{3}x^3 - 9x + 2$

$y' = x^2 - 9$

$y'' = 2x$

For stationary points $\Rightarrow y' = 0$

$x^2 - 9 = 0$

$(x-3)(x+3) = 0$

$x = -3$ or $x = 3$

$y = 20$ $\qquad y = -16$

$\therefore (-3, 20)$ $\qquad (3, -16)$

Check Max/Min:

$y'' = 2(-3)$ $\qquad y'' = 2(3)$

$= -6 < 0$ $\qquad = 6 > 0$

$\therefore$ MAXIMUM $\qquad \therefore$ MINIMUM

hence $(-3, 20)$ is a (local) maximum point while $(3, -16)$ is a (local) minimum point.

(ii) Need to check that concavity changes either side of the point where $x = 0$:

e.g. $y'' = 2x$

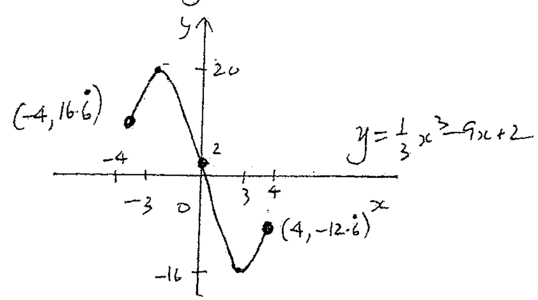
x	0^-	0	0^+
y''	$-$	0	$+$

since concavity has changed

hence $(0, 2)$ is an inflexion point.

(iii) $x = -4 \Rightarrow y = 16.\dot{6}$

$x = 4 \Rightarrow y = -12.\dot{6}$



Q7

(a)(i) $\angle PRQ = \angle SRT$ (vertically opposite angles are equal)

$\frac{PR}{RS} = \frac{RQ}{RT} = \frac{4}{6} = \frac{2}{3}$

$\therefore \triangle PRQ \,|||\, \triangle RST$ (with two sets sides in proportion and equal included angle)

(ii) $\frac{ST}{PQ} = \frac{SR}{RQ}$ (similar $\triangle$s)

$\frac{x}{6} = \frac{6}{4}$

$x = \frac{36}{4}$

$\therefore ST = 9\text{cm}$

Q8 (i) $T_1 = a = 10$, $T_2 = 40$, $T_3 = 70$ } $d = 30$ {NB all in cents}

$T_n = a + (n-1)d$

$T_{52} = a + 51 \times d$

$= 10 + 51 \times 30$

$= 1540$

Her 52nd deposit is $15.40.

(ii) $S_n = \frac{n}{2}(a + l)$

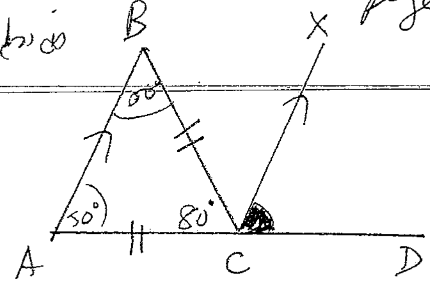
$S_{52} = \frac{52}{2}(10 + 1540)$

$= 40300$

Her total for the year was $403

page 7

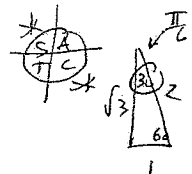
(b)(i)



(ii) $\angle CAB = \angle CBA = 50°$ (equal angles opposite equal sides in an isosceles $\triangle$, and angle sum of a $\triangle$)

$\angle XCD = 50°$ (corresponding angles in between parallel lines)

(c) $\tan x = -\frac{1}{\sqrt{3}}$



$x = \pi - \frac{\pi}{6},\ 2\pi - \frac{\pi}{6}$

$\therefore x = \frac{5\pi}{6},\ \frac{11\pi}{6}$

(d) $3\log_8 2 = \log_8 x - \log_8 3$

$\log_8 2^3 = \log_8\left(\frac{x}{3}\right)$

(equating logs)

$\therefore 8 = \frac{x}{3}$

$\therefore x = 24$

page 8

Q9 (a) $r = 6\%$ p.a.

$= 0.06$ pa

$= 0.005$ per month

(i) $A_1 = P\left(1 + \frac{r}{100}\right) - M$

$\therefore A_1 = 300\,000(1.005)^1 - M$

(ii) $A_2 = A_1\left(1 + \frac{r}{100}\right) - M$

$A_2 = [300\,000(1.005)^1 - M](1.005) - M$

$= 300\,000(1.005)^2 - M(1.005) - M$, as required.

(iii) A_n = amount owing at end of n months

hence $A_{240} = 0$.

$A_{240} = 300\,000(1.005)^{240} - M(1.005)^{239} - M(1.005)^{238} - \ldots - M(1.005)^1 - M$

$= 300\,000(1.005)^{240} - M[1 + 1.005 + 1.005^2 + \ldots + 1.005^{239}]$

G.P. $a = 1$, $r = 1.005$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$= \frac{1(1.005^{240} - 1)}{(1.005 - 1)}$

$= 300\,000(1.005)^{240} - M\left(\frac{1.005^{240} - 1}{1.005 - 1}\right)$

$\therefore M = \frac{300\,000(1.005)^{240}(1.005 - 1)}{(1.005^{240} - 1)}$

(b)(i) ~~$x = 4 - 3t + 12\ln(t+3)$~~

let $t = 0$: $x = 4 - 3(0) + 12\ln(0+3)$

$\therefore x = 4 + 12\ln 3$

(ii) $x = 4 - 3t + 12\ln(t+3)$

$v = \dot{x} = -3 + \frac{12}{t+3}$ or $-3 + 12(t+3)^{-1}$

$a = \ddot{x} = -12(t+3)^{-2}$ or $\frac{-12}{(t+3)^2}$

(iii) $v = 0 \Longrightarrow -3 + \frac{12}{t+3} = 0$

$\frac{12}{t+3} = 3$

$12 = 3(t+3)$

$4 = t + 3$

$\therefore t = 1$

(iv) $x_0 = 4 + 12\ln 3$ ($\doteqdot 17.18$)

$x_1 = 4 - 3(1) + 12\ln(1+3)$

$= 4 - 3 + 12\ln 4$

$= 1 + 12\ln 4$ (approx $\doteqdot 17.63\ldots$)

$x_2 = 4 - 3(2) + 12\ln(2+3)$

$= 4 - 6 + 12\ln 5$

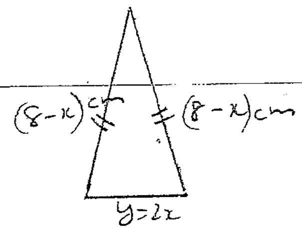
$= -2 + 12\ln 5$ ($\doteqdot 17.31\ldots$)

Distance = (Dist. between x_0 & x_1) + (Dist. betw. x_1 & x_2)

$= [(1 + 12\ln 4) - (4 + 12\ln 3)] + [(1 + 12\ln 4) - (-2 + 12\ln 5)]$

$= 1 + 12\ln 4 - 4 - 12\ln 3 + 1 + 12\ln 4 + 2 - 12\ln 5$

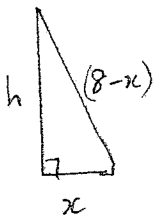
Q14 (a) (i)



$P = y + 8 - x + 8 - x = 16$

$y - 2x + 16 = 16$

$\therefore \underline{y = 2x}$



$h^2 + x^2 = (8-x)^2$

$h^2 + \cancel{x^2} = 64 - 16x + \cancel{x^2}$

$h^2 = 64 - 16x$

$h^2 = 16(4-x)$

$h = 4\sqrt{4-x}$, as required.

(ii) Area, $A = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{\cancel{2}} \times \cancel{2}x \times 4\sqrt{4-x}$

$= 4x\sqrt{4-x}$, as required.

(iii) $A = (4x)(\sqrt{4-x}) = (4x)(4-x)^{\frac{1}{2}}$

$\frac{dA}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

$= (4-x)^{\frac{1}{2}} \cdot 4 + \overset{2}{\cancel{4}}x\left(\frac{1}{\cancel{2}}(4-x)^{\frac{1}{2}-1} \cdot -1\right)$

$= 4(4-x)^{\frac{1}{2}} - 2x(4-x)^{-\frac{1}{2}}$

$= 2(4-x)^{-\frac{1}{2}}\left[2(4-x) - x\right]$

$= 2(4-x)^{-\frac{1}{2}}(8-2x-x)$

$= 2(4-x)^{-\frac{1}{2}}(8-3x)$

$= 2(8-3x)$

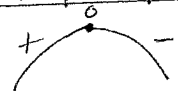
Let $A' = 0 \Rightarrow 8 - 3x = 0$

$3x = 8$

$x = \frac{8}{3}$ or $2\frac{2}{3}$ or $2.\dot{6}$

check maximum:

x	$2\frac{2}{3}^-$	$2\frac{2}{3}$	$2\frac{2}{3}^+$
A'	$+$	0	$-$



To the left of $x = 2\frac{2}{3}$, $A' > 0$

& to the right of $x = 2\frac{2}{3}$, $A' < 0$,

hence "maximum".

The value of x for the area to be maximised is $2\frac{2}{3}$ cm

b) $N = N_0 e^{-kt}$

(i) let $t = 0 \Rightarrow$ $40\,000 = N_0 e^0$

$\therefore N_0 = 40\,000$

$N = 40\,000\, e^{-kt}$

[at $t = 1$, $N = 36\,000$]

$36\,\cancel{000} = 40\,\cancel{000}\, e^{-k}$

$\frac{36}{40} = e^{-k}$

$0.9 = e^{-k}$

$\therefore -k = \log_e 0.9$

$\underline{k = -\log_e 0.9 \text{ or } \log\left(\frac{10}{9}\right)}$

(ii) $N = 40\,000\, e^{-kt}$

$400 = 40\,000\, e^{(+\log_e 0.9)t}$

$0.01 = e^{t \log_e 0.9}$

$\log_e 0.01 = \log_e\left(e^{t \log_e 0.9}\right) \Rightarrow t = \frac{\log_e 0.01}{\log_e 0.9}$

$t \log_e 0.9 = \log_e 0.01$

$= 43.70\ldots$