ACE Examinations

Student Name: ____________________

2011

YEAR 12

TRIAL HSC EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time - 5 minutes
- Working time - 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks - 84

- Attempt Questions 1-7
- All questions are of equal value

Total marks - 84
Attempt Questions 1 - 7
All questions are of equal value

Answer each question in the appropriate writing booklet.

Question 1 (12 marks) **Marks**

(a) Factorise completely $x^4+2x^3-2x^2-4x$ **2**

(b) What is the exact value of $\cos 15^\circ$? **2**

(c) The interval joining the points $A(9,12)$ and $B(1,b)$ is divided internally in the ratio 2:1 by the point $(a,0)$. Find a and b. **3**

(d) Solve $\dfrac{3}{x-2}\le 4$ **3**

(e) Find $\displaystyle\int\frac{dx}{\sqrt{36-x^2}}$ **2**

Question 2 (12 marks) **Marks**

(a) If α, β and χ are the roots of $x^3 - 4x + 1 = 0$ find

(i) $\alpha + \beta + \chi$ **1**

(ii) $\alpha\beta\chi$ **1**

(iii) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\chi}$ **1**

(b) (i) Express $3\cos x - 4\sin x$ in the form $A\cos(x + \alpha)$ where $0 \le \alpha \le \frac{\pi}{2}$. **2**

(ii) Hence or otherwise, solve $3\cos x - 4\sin x = 0.5$ for $0 \le x \le 2\pi$. Give your answer, or answers, correct to two decimal places. **2**

(c) $P(2at, at^2)$ is any point on the parabola $x^2 = 4ay$. The line k is parallel to the tangent at P and passes through the focus S of the parabola.

(i) Show that the equation of the line k is $y = tx + a$ **2**

(ii) The line k intersects the x-axis at the point Q. Find the coordinates of the midpoint, M, of the interval QS. **2**

(iii) Find the equation of the locus of M. **1**

Question 3 (12 marks) **Marks**

(a) Let $f(x) = e^{x+2}$

(i) Find the inverse function $f^{-1}(x)$ **2**

(ii) Sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same number plane. Show all the important features. **2**

(b) (i) Show that $\frac{\pi}{4} + \tan^{-1} x - x^2 = 0$ has a root in the interval $1 < x < \sqrt{3}$. **2**

(ii) One approximate solution of the equation $\frac{\pi}{4} + \tan^{-1} x - x^2 = 0$ is $x = 1$. Use one application of Newton's method to find another approximation to this solution, Give your answer correct to four decimal places. **2**

(c) (i) Prove that $\frac{\sec^2 x}{\tan x} = \frac{\text{cosec}\, x}{\cos x}$ **2**

(ii) Use the substitution $u = \tan x$ to find the exact value of this integral **2**

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cos ecx}{\cos x}\, dx$$

Question 4 (12 marks) **Marks**

(a) A box contains seven identical balls except for their colour. Four are blue, two are white and one is red. Two balls are selected at random.

(i) What is the probability they are both blue? **1**

(ii) After replacing the two balls the selection is repeated several times.

(a) What is the probability of getting two blue balls on at least one occasion from five selections of two balls? Answer correct to four decimal places. **2**

(b) What is the probability of getting two blue balls on exactly three occasions from five selections of two balls? Answer correct to four decimal places. **1**

(b) Consider the function $f(x) = x - 2\cos x$ for $0 \le x \le 2\pi$

(i) Find all the stationary points for the graph $y = f(x)$. **3**

(ii) Determine the nature of the stationary points. **1**

(iii) Where are the points of inflexion? **2**

(iv) Sketch the graph $y = f(x)$ using the above essential features **2**

Question 5 (12 marks) **Marks**

(a) A particle moving in a straight line obeys $v^2 = -x^2 + 2x + 8$ where x is its displacement from the origin in metres and v is its velocity in ms^{-1}. Initially, the particle is 2.5 metres to the right of the origin.

(i) Prove that the motion is simple harmonic. **2**

(ii) Find the centre of motion, the period and the amplitude. **3**

(iii) The displacement of the particle at any t is given by the equation $x = a\cos(nt + \alpha) + b$. What is the value of b and α ? **1**

(b) A bottle of water has a temperature of 20°C and is placed in a refrigerator whose temperature is 2°C. The cooling rate of the bottle of water is proportional to the difference between the temperature of the refrigerator and the temperature T of the bottle of water. This is expressed by the equation:

$$\frac{dT}{dt} = -k(T - 2)$$

where k is a constant of proportionality and t is the number of minutes after the bottle of water is placed in the refrigerator.

(i) Show that $T = 2 + Ae^{-kt}$ satisfies the equation. **1**

(ii) After 20 minutes in the refrigerator the temperature of the bottle of water is 10°C. What is the value of A and k in the above equation? **3**

(iii) How long will it take for the bottle of water to cool down to 5°C? **2**

Question 6 (12 marks) **Marks**

(a) A ball is thrown from the origin O with a velocity V and angle of elevation of θ, where $\theta \neq \frac{\pi}{2}$. You may assume that

$$x = Vt\cos\theta \text{ and } y = -\frac{1}{2}gt^2 + Vt\sin\theta$$

where x and y are the horizontal and vertical displacements of the ball in metres from the O at time t seconds after being thrown.

(i) Let $h = \frac{V^2}{2g}$ and show the equation of flight of the ball is **3**

$$y = x\tan\theta - \frac{1}{4h}x^2(1+\tan^2\theta)$$

(ii) The point of intersection when two balls are thrown with an angle of elevation of θ_1 and θ_2 is (a,b). Show that **3**

$$a^2 < 4h(h-b)$$

(b) Consider the binomial expansion

$$(1+x)^n = 1 + {}^nC_1x + {}^nC_2x^2 + \ldots + {}^nC_nx^n$$

(i) Show that **1**

$$1 - {}^nC_1 + {}^nC_2 - \ldots + (-1)^n\,{}^nC_n = 0$$

(ii) Show that **2**

$${}^nC_1 + 2\,{}^nC_2 + \ldots + n\,{}^nC_n = n2^{n-1}$$

(c) A clay shooter hits the target 95% of the time. In a competition he will have forty shots at the target.

(i) What is the probability he hits 36 targets? Answer correct to 4 decimal places. **1**

(ii) What is the probability he misses at most two times? Answer correct to 4 decimal places. **2**

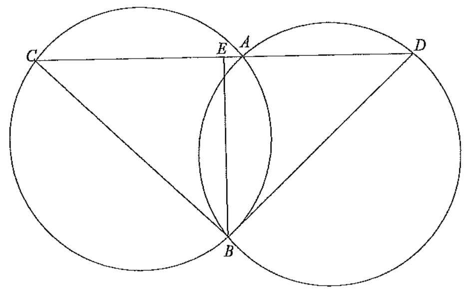
Question 7 (12 marks) **Marks**

(a) Use mathematical induction to prove that **3**

$$\frac{2}{1\times 2} + \frac{2}{2\times 3} + \frac{2}{3\times 4} + \ldots + \frac{2}{n\times(n+1)} = \frac{2n}{n+1}$$

for all positive integers n.

(b)



Two circles have equal radii and intersect at A and B. A line through A meets the two circles at C and D. E is the midpoint of CD.

(i) Why is $\angle DCB = \angle CDB$? **1**

(ii) Prove that BE is perpendicular to CD. **2**

(c) (i) Find all the solutions to the inequality $\frac{x}{1-x^2} \geq 0$ **3**

(ii) Show that $\tan x \sec x = \frac{\sin x}{1-\sin^2 x}$ **1**

(iii) Find all the solutions to $\tan x \sec x \geq 0$ when $0 \leq x \leq 2\pi$ **2**

End of paper

ACE Examination 2011

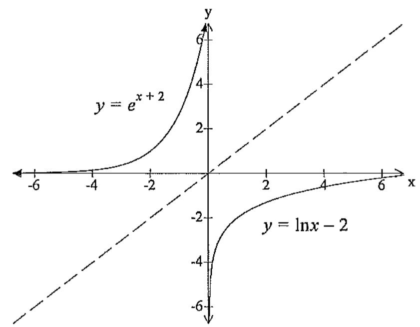
HSC Mathematics Extension 1 Yearly Examination

Worked solutions and marking guidelines

	Solution	Criteria
1(a)	$x^4+2x^3-2x^2-4x=x^3(x+2)-2x(x+2)$ $=(x^3-2x)(x+2)$ $=x(x^2-2)(x+2)$	2 Marks: Correct answer. 1 Mark: Finds one factor.
1(b)	$\cos(45-30)=\cos 45\cos 30+\sin 45\sin 30$ $=\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}\times\frac{1}{2}$ $=\frac{1}{\sqrt{2}}(\frac{\sqrt{3}}{2}+\frac{1}{2})$ $=\frac{\sqrt{3}+1}{2\sqrt{2}}=\frac{\sqrt{6}+\sqrt{2}}{4}$	2 Marks: Correct answer. 1 Mark: Uses difference formula or states one correct exact ratio.
1(c)	$x=\frac{mx_2+nx_1}{m+n}$ $y=\frac{my_2+ny_1}{m+n}$ $a=\frac{2\times1+1\times9}{2+1}$ $0=\frac{2\times b+1\times12}{2+1}$ $=\frac{11}{3}$ $0=2b+12$ $b=-6$	3 Marks: Correct answer. 2 Marks: Correctly finds either a or b. 1 mark: Uses ratio division formula.
1(d)	$(x-2)^2\times\frac{3}{(x-2)}\le 4\times(x-2)^2$ $(x-2)3\le 4(x-2)^2 \quad x\ne 2$ $(x-2)(3-4x+8)\le 0$ $(x-2)(11-4x)\le 0$ $x<2$ and $x\ge\frac{11}{4}$	3 Marks: Correct answer. 2 Marks: Finds one correct solution. 1 Mark: Multiplies both sides of the inequality by $(x-2)^2$.
1(e)	$\int\frac{dx}{\sqrt{36-x^2}}=\sin^{-1}\frac{x}{6}+C$	2 Marks: Correct answer. 1 Mark: Recognises inverse sine function.

2(a)(i)	$x^3+0x^2-4x+1=0$ $\alpha+\beta+\chi=-\frac{b}{a}=-\frac{0}{1}=0$	1 Mark: Correct answer.
2(a)(ii)	$\alpha\beta\chi=-\frac{d}{a}=-\frac{1}{1}=-1$	1 Mark: Correct answer.
2(a)(iii)	$\alpha\beta+\alpha\chi+\beta\chi=\frac{c}{a}=\frac{-4}{1}=-4$ $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\chi}=\frac{\alpha\beta+\alpha\chi+\beta\chi}{\alpha\beta\chi}$ $=\frac{-4}{-1}$ $=4$	1 Mark: Correct answer.
2(b)(i)	$A\cos(x+\alpha)=A\cos x\cos\alpha-A\sin x\sin\alpha$ $3\cos x-4\sin x=A\cos x\cos\alpha-A\sin x\sin\alpha$ $A\cos\alpha=3$ (1) $A\sin\alpha=4$ (2) Squaring and adding eqn(1) and (2) $A^2\cos^2\alpha+A^2\sin^2\alpha=9+16$ $A^2=25$ $A=5$ Dividing eqn(2) by eqn(1) $\tan\alpha=\frac{\sin\alpha}{\cos\alpha}$ $=\frac{4}{3}$ $\alpha=\tan^{-1}\frac{4}{3}$ $=0.927295218$ $=0.93$ $3\cos x-4\sin x=5(\cos(x+0.93))$	2 Marks: Correct answer. 1 Mark: Finds either A or α or states eqn (1) and (2).
2(b)(ii)	$3\cos x-4\sin x=0.5$ $5(\cos(x+\tan^{-1}\frac{4}{3}))=0.5$ $\cos(x+\tan^{-1}\frac{4}{3})=0.1$ $x+\tan^{-1}\frac{4}{3}=1.470628906,\ (2\pi-1.470628906)$ $x=0.543336876,\ 3.885261184$ $=0.54,\ 3.89$	2 Marks: Correct answer. 1 Mark: Finds one correct answer.

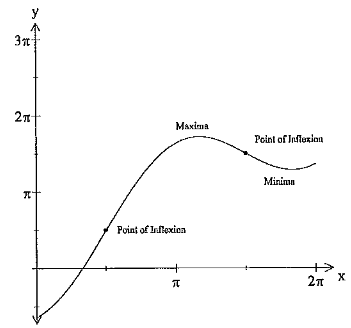
2(c) (i)	To find the gradient of the tangent $y=\frac{1}{4a}x^2$ $\frac{dy}{dx}=\frac{1}{2a}x$ At $P(2at,at^2)$ $\frac{dy}{dx}=\frac{1}{2a}\times 2at=t$ Line k has a gradient of t and passes through $S(0,a)$ $y-y_1=m(x-x_1)$ $y-a=t(x-0)$ $y=tx+a$	2 Marks: Correct answer. 1 Mark: Finds or states the gradient of the tangent at P.
2(c) (ii)	To find the coordinates of Q Substitute $y=0$ into $y=tx+a$ then $x=-\frac{a}{t}$ or $Q(-\frac{a}{t},0)$ To find the coordinates of M $x=\frac{x_1+x_2}{2}=\frac{-\frac{a}{t}+0}{2}=-\frac{a}{2t}$ $y=\frac{y_1+y_2}{2}=\frac{0+a}{2}=\frac{a}{2}$ $M(-\frac{a}{2t},\frac{a}{2})$	2 Marks: Correct answer. 1 Mark: Finds the coordinates of Q.
2(c) (iii)	To find the equation of the locus eliminate t. However y is independent of t. $y=\frac{a}{2}$	1 Mark: Correct answer.
3(a) (i)	$f(x)=e^{x+2}$ or $y=e^{x+2}$ Inverse function is $x=e^{y+2}$ $\log_e x=y+2$ $y=\log_e x-2$ $f^{-1}(x)=\log_e x-2$	2 Marks: Correct answer. 1 Mark: Interchanges the x and y values.

3(a) (ii)		2 Marks: Correct answer. 1 Mark: Draws one of the graphs correctly.
3(b) (i)	$f(x)=\frac{\pi}{4}+\tan^{-1}x-x^2$ $f(1)=\frac{\pi}{4}+\tan^{-1}1-1$ $=\frac{\pi}{4}+\frac{\pi}{4}-1$ $=\frac{\pi}{2}-1>0$ $f(\sqrt{3})=\frac{\pi}{4}+\tan^{-1}\sqrt{3}-3$ $=\frac{\pi}{4}+\frac{\pi}{3}-3$ $=\frac{7\pi}{12}-3<0$ $f(x)$ changes sign between $x=1$ and $x=\sqrt{3}$. Hence equation has a root in the interval $1<x<\sqrt{3}$.	2 Marks: Correct answer. 1 Mark: Correctly evaluates the value of the function at either $x=1$ or $x=\sqrt{3}$.
3(b) (ii)	$f'(x)=\frac{1}{1+x^2}-2x$ $f'(1)=\frac{1}{1+1}-2$ $=-1.5$ $f(1)=\frac{\pi}{2}-1$ $x_1=x_0-\frac{f(x_0)}{f'(x_0)}$ $=1-\frac{\frac{\pi}{2}-1}{-1.5}$ $=1.3805$	2 Marks: Correct answer. 1 Mark: Correctly evaluates the derivative of the function.

3(c) (i)	$\text{LHS} = \dfrac{\sec^2 x}{\tan x}$ $= \dfrac{1}{\cos^2 x} \div \tan x$ $= \dfrac{1}{\cos^2 x} \times \dfrac{\cos x}{\sin x}$ $= \dfrac{1}{\cos x} \times \operatorname{cosec} x$ $= \dfrac{\operatorname{cosec} x}{\cos x}$ $= \text{RHS}$	2 Marks: Correct answer. 1 Mark: Makes significant progress towards the solution.
3(c) (ii)	$u = \tan x$ $u = \tan\frac{\pi}{3} = \sqrt{3}$ $u = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$ $du = \sec^2 x dx$ $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cos ecx}{\cos x} dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sec^2 x}{\tan x} dx$ $= \int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{1}{u} du$ $= [\log_e u]_{\frac{1}{\sqrt{3}}}^{\sqrt{3}}$ $= \log_e \sqrt{3} - \log_e \frac{1}{\sqrt{3}}$ $= \log_e 3$	2 Marks: Correct answer. 1 Mark: Recognises the use of part (i) or makes progress in the substitution.

4(a) (i)	$P(BB) = \frac{4}{7} \times \frac{3}{6}$ $= \frac{2}{7}$	1 Mark: Correct answer.
4(a) (ii)a	P(Two blue at least once) = 1 – P(No blue 5 times) $= 1 - \left(\frac{5}{7}\right)^5$ $= 0.8141$	2 Marks: Correct answer. 1 Mark: Use of complementary event.
4(a) (ii)b	P(Two blue exactly 3 times) = ${}^5C_3 \left(\frac{2}{7}\right)^3 \left(\frac{5}{7}\right)^2$ $= 0.1190$	1 Mark: Correct answer.

4(b) (i)	$y = x - 2\cos x$ $\frac{dy}{dx} = 1 + 2\sin x$ $\frac{d^2y}{dx^2} = 2\cos x$ Stationary points occur when $\frac{dy}{dx} = 0$ $1 + 2\sin x = 0$ $2\sin x = -1$ $\sin x = -\frac{1}{2}$ $x = \frac{7\pi}{6}, \frac{11\pi}{6}$ When $x = \frac{7\pi}{6}$ and $x = \frac{11\pi}{6}$ $y = \frac{7\pi}{6} - 2\cos\frac{7\pi}{6}$ $y = \frac{11\pi}{6} - 2\cos\frac{11\pi}{6}$ $= \frac{7\pi}{6} - 2 \times -\frac{\sqrt{3}}{2}$ $= \frac{11\pi}{6} - 2 \times \frac{\sqrt{3}}{2}$ $= \frac{7\pi}{6} + \sqrt{3}$ $= \frac{11\pi}{6} - \sqrt{3}$ Stationary points are $(\frac{7\pi}{6}, \frac{7\pi}{6} + \sqrt{3})$ and $(\frac{11\pi}{6}, \frac{11\pi}{6} - \sqrt{3})$	3 Marks: Correct answer. 2 Marks: Finds one stationary point or only finds the x coordinates. 1 Mark: Finds the first derivative.
4(b) (ii)	At $(\frac{7\pi}{6}, \frac{7\pi}{6} + \sqrt{3})$ At $(\frac{11\pi}{6}, \frac{11\pi}{6} - \sqrt{3})$ $\frac{d^2y}{dx^2} = 2\cos x$ $\frac{d^2y}{dx^2} = 2\cos x$ $= 2\cos\frac{7\pi}{6} < 0$ $= 2\cos\frac{11\pi}{6} > 0$ Maxima at $(\frac{7\pi}{6}, \frac{7\pi}{6} + \sqrt{3})$ and Minima at $(\frac{11\pi}{6}, \frac{11\pi}{6} - \sqrt{3})$	1 Mark: Correct answer.

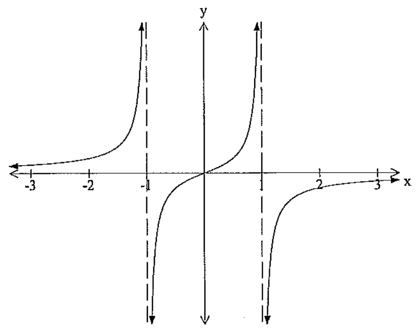
4(b) (iii)	Point of inflexion occurs when $\frac{d^2y}{dx^2}=0$ $2\cos x=0$ $\cos x=0$ $x=\frac{\pi}{2},\frac{3\pi}{2}$ when $x=\frac{\pi}{2}$ and $x=\frac{3\pi}{2}$ $y=\frac{\pi}{2}-2\cos\frac{\pi}{2}$ $y=\frac{3\pi}{2}-2\cos\frac{3\pi}{2}$ $=\frac{\pi}{2}$ $=\frac{3\pi}{2}$ Test for change in concavity When $x<\frac{\pi}{2}$ then $\frac{d^2y}{dx^2}=2\cos x>0$ When $x>\frac{\pi}{2}$ then $\frac{d^2y}{dx^2}=2\cos x<0$ When $x<\frac{3\pi}{2}$ then $\frac{d^2y}{dx^2}=2\cos x<0$ When $x>\frac{3\pi}{2}$ then $\frac{d^2y}{dx^2}=2\cos x>0$ Point of inflexion at $(\frac{\pi}{2},\frac{\pi}{2})$ and $(\frac{3\pi}{2},\frac{3\pi}{2})$	2 Marks: Correct answer. 1 Mark: Finds the point of inflexion but does not test for a change in concavity.
4(b) (iv)		2 Marks: Correct answer. 1 Mark: Basic shape of the curve or shows one essential feature.

5(a) (i)	Simple harmonic motion occurs when $\ddot{x}=-n^2(x-b)$ Now $\ddot{x}=\frac{d}{dx}(\frac{1}{2}v^2)$ $=\frac{d}{dx}(\frac{1}{2}\times(-x^2+2x+8))$ $=\frac{1}{2}\times(-2x+2)$ $=-x+1$ $=-1(x-1)$ SHM about the position $x=-1$ ($n=1$ and $b=1$)	2 Marks: Correct answer. 1 Mark: Recognises the formula for SHM.
5(a) (ii)	Centre of motion is $x=1$ Period $=\frac{2\pi}{n}$ $n=1$ (from part (i)) $=\frac{2\pi}{1}=2\pi$ To find the amplitude $v^2=-x^2+2x+8$ $=1^2(8+2x-x^2)$ $=1^2(9-(x-1)^2)$ $=n^2(a^2-x^2)$ Amplitude is 3 metres Alternatively amplitude occurs at the extremes when $v=0$ or $-x^2+2x+8=0$ $x^2-2x-8=0$ $(x-4)(x+2)=0$ Displacement occurs between $x=4$ and $x=-2$ (distance of 6) Amplitude is 3 metres.	3 Marks: Correct answer. 2 Marks: Finds two correct answers. 1 Mark: Finds one correct answer.
5(a) (iii)	Now $a=3$, $n=1$ and $b=1$ (centre of motion) $x=a\cos(nt+\alpha)+1$ $=3\cos(t+\alpha)+1$ Initially $t=0$ and $x=2.5$ $x=3\cos(t+\alpha)+1$ $2.5=3\cos(0+\alpha)+1$ $\cos\alpha=\frac{1.5}{3}$ $\alpha=\frac{\pi}{3}$ $x=3\cos(t+\frac{\pi}{3})+1$	1 Mark: Correct answer.

5(b) (i)	$T = 2 + Ae^{-kt}$ or $Ae^{-kt} = T - 2$ $\frac{dT}{dt} = -kAe^{-kt}$ $= -k(T-2)$	1 Mark: Correct answer.
5(b) (ii)	Initially $t = 0$ and $T = 20$ $T = 2 + Ae^{-kt}$ $20 = 2 + Ae^{-k\times 0}$ $A = 18$ Also $t = 20$ and $T = 10$ $T = 2 + 18e^{-kt}$ $10 = 2 + 18e^{-k\times 20}$ $e^{-k\times 20} = \frac{8}{18}$ $-20k = \log_e \frac{4}{9}$ $k = -\frac{1}{20}\log_e \frac{4}{9}$ $= \frac{1}{20}\log_e \frac{9}{4}$ $= 0.04054651081...$	3 Marks: Correct answer. 2 Marks: Finds the value of A and an expression for k. 1 Mark: Finds the value of A.
5(b) (iii)	We need to find t when $T = 5$ $T = 2 + 18e^{-kt}$ $5 = 2 + 18e^{-kt}$ $e^{-kt} = \frac{3}{18}$ $-kt = \log_e \frac{1}{6}$ $t = \frac{1}{k}\log_e 6$ $= 20\frac{\log_e 6}{\log_e \frac{9}{4}}$ $= 44.19022583$ It will take about 44 minutes for the bottle to cool to 5°C?	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.

6(a) (i)	$x = Vt\cos\theta$ (1) $y = -\frac{1}{2}gt^2 + Vt\sin\theta$ (2) From eqn (1) $t = \frac{x}{V\cos\theta}$ sub into eqn (2) $y = -\frac{1}{2}g(\frac{x}{V\cos\theta})^2 + V(\frac{x}{V\cos\theta})\sin\theta$ $= -\frac{gx^2}{2V^2\cos^2\theta} + \frac{\sin\theta x}{\cos\theta}$ $= -\frac{gx^2\sec^2\theta}{2V^2} + \tan\theta x$ $= -\frac{2gx^2\sec^2\theta}{4V^2} + \tan\theta x$ $= -\frac{x^2\sec^2\theta}{4h} + \tan\theta x$ $= x\tan\theta - \frac{1}{4h}x^2(1+\tan^2\theta)$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Makes t the subject of eqn (1) or equivalent progress.
6(a) (ii)	Now (a,b) satisfies the equation $y = x\tan\theta - \frac{1}{4h}x^2(1+\tan^2\theta)$ $b = a\tan\theta - \frac{1}{4h}a^2(1+\tan^2\theta)$ $4hb = 4ha\tan\theta - a^2(1+\tan^2\theta)$ $(1+\tan^2\theta)a^2 - 4ha\tan\theta + 4hb = 0$ $a^2\tan^2\theta - 4ha\tan\theta + 4hb + a^2 = 0$ Quadratic equation has 2 solutions if the discriminate is greater than zero. $b^2 - 4ac > 0$ $(-4ha)^2 - 4a^2(4hb + a^2) > 0$ $16h^2a^2 - 16a^2hb - 4a^4 > 0$ $4a^2(4h^2 - 4hb - a^2) > 0$ $4h^2 - 4hb - a^2 > 0$ $a^2 < 4h^2 - 4hb$ $a^2 < 4h(h-b)$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Substitutes (a,b) into equation of flight and simplifies.
6(b) (i)	Substitute $x = -1$ $(1+x)^n = 1 + {}^nC_1x + {}^nC_2x^2 + ... + {}^nC_nx^n$ $(1+-1)^n = 1 + {}^nC_1 - 1 + {}^nC_2(-1)^2 + ... + {}^nC_n(-1)^n$ $1 - {}^nC_1 + {}^nC_2 + ... + (-1)^n\,{}^nC_n = 0$	1 Mark: Correct answer.

6(b) (ii)	Differentiate both sides of the identity $n(1+x)^{n-1} = {}^nC_1 + 2\,{}^nC_2x + \ldots + n\,{}^nC_nx^{n-1}$ Substitute $x=1$ $n(1+1)^{n-1} = {}^nC_1 + 2\,{}^nC_2 1 + \ldots + n\,{}^nC_n 1^{n-1}$ $n2^{n-1} = {}^nC_1 + 2\,{}^nC_2 + \ldots + n\,{}^nC_n$ ${}^nC_1 + 2\,{}^nC_2 + \ldots + n\,{}^nC_n = n2^{n-1}$	2 Marks: Correct answer. 1 Mark: Differentiates both sides of the identity.
6(c) (i)	Let p be the probability of hitting the target ($p = 0.95$) Let q be the probability of hitting the target ($q = 0.05$) $P(k \text{ successes}) = {}^nC_k(0.95)^k(0.05)^{n-k}$ $P(36 \text{ targets}) = {}^{40}C_{36}(0.95)^{36}(0.05)^4$ $= 0.0901$	1 Mark: Correct answer.
6(c) (ii)	Misses at most 2 targets then $k = 38$, 39 and 40 $P(\text{At most 2 misses})$ $= {}^{40}C_{38}0.95^{38}0.05^2 + {}^{40}C_{39}0.95^{39}0.05^1 + {}^{40}C_{40}0.95^{40}$ $= 0.6767$	2 Marks: Correct answer. 1 Mark: Makes some progress.
7(a)	Step 1: To prove the statement true for $n=1$ $\text{LHS} = \frac{2}{1\times 2} = 1 \quad \text{RHS} = \frac{2\times 1}{1+1} = 1$ Result is true for $n=1$ Step 2: Assume the result true for $n=k$ $\frac{2}{1\times2}+\frac{2}{2\times3}+\frac{2}{3\times4}+\ldots+\frac{2}{k\times(k+1)} = \frac{2k}{k+1}$ To prove the result is true for $n=k+1$ $\frac{2}{1\times2}+\frac{2}{2\times3}+\frac{2}{3\times4}+\ldots+\frac{2}{k\times(k+1)}+\frac{2}{(k+1)(k+2)} = \frac{2(k+1)}{(k+2)}$ $\text{LHS} = \frac{2}{1\times2}+\frac{2}{2\times3}+\frac{2}{3\times4}+\ldots+\frac{2}{k\times(k+1)}+\frac{2}{(k+1)(k+2)}$ $= \frac{2k}{k+1}+\frac{2}{(k+1)(k+2)}$ $= \frac{2k(k+2)+2}{(k+1)(k+2)}$ $= \frac{2k^2+4k+2}{(k+1)(k+2)}$ $= \frac{2(k+1)(k+1)}{(k+1)(k+2)}$ $= \frac{2(k+1)}{(k+2)}$ $= \text{RHS}$	3 Marks: Correct answer. 2 Marks: Proves the result true for $n=1$ and attempts to use the result of $n=k$ to prove the result for $n=k+1$. 1 Mark: Proves the result true for $n=1$.

	Result is true for $n=k+1$ if true for $n=k$ Step 3: Result true by principle of mathematical induction.	
7(b) (i)	Two equal circles containing the same chord AB. $\angle DCB = \angle CDB$ (Two angles at the circumference standing the same or equal arcs are equal)	1 Mark: Correct answer.
7(b) (ii)	$\triangle DCB$ is isosceles (Two angles are equal from part (i)) $BC = BD$ (Two sides opposite the equal angles in an isosceles triangle) Consider $\triangle CBE$ and $\triangle DBE$ $BC = BD$ (from above) $BE = BE$ (common side) $CE = DE$ (E is the midpoint of CD) $\triangle CBE \equiv \triangle DBE$ (SSS) $\angle CEB = \angle DEB$ (corresponding angles in congruent triangles) $\angle CEB + \angle DEB = 180$ (straight angle measures 180°) $\angle CEB = \angle DEB = 90°$ BE is perpendicular to CD	2 Marks: Correct answer. 1 Mark: Makes some progress towards the proof.
7(c) (i)	$(1-x^2)^2 \times \frac{x}{1-x^2} \geq 0 \times (1-x^2)^2 \qquad x \neq \pm 1$ $(1-x^2)x \geq 0$ $(1-x)(1+x)x \geq 0$  From the graph $0 \leq x < 1$ and $x < -1$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Finds part of the solution or demonstrates some understanding.

7(c) (ii)	$\text{LHS} = \tan x \sec x$ $= \dfrac{\sin x}{\cos x} \times \dfrac{1}{\cos x}$ $= \dfrac{\sin x}{\cos^2 x}$ $= \dfrac{\sin x}{1 - \sin^2 x}$ $= \text{RHS}$	1 Mark: Correct answer.
7(c) (iii)	Using parts (i) and (ii) $0 \le \sin x < 1$ and $\sin x < -1$ $0 \le x < \dfrac{\pi}{2}$ No solution Solution also in the second quadrant $0 \le x < \dfrac{\pi}{2}$ and $\dfrac{\pi}{2} < x \le \pi$	2 Marks: Correct answer. 1 mark: Makes some progress using parts (i) and (ii).