ACE Examinations   Student Name: ____________________

2016

YEAR 12

YEARLY EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time - 5 minutes
- Working time - 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A reference sheet is provided at the back of this paper
- Show relevant mathematical reasoning and/or calculations in Questions 11-14

Total marks - 70

Section I
10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II
60 marks

- Attempt Questions 11-14
- Allow about 1 hour 45 minutes for this section

Section I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

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1 What is the acute angle between the lines $x-2y+1=0$ and $2x-y-1=0$?

(A) 37°
(B) 45°
(C) 90°
(D) 143°

2 What is the number of asymptotes on the graph of $y=\frac{1}{x^2-1}$?

(A) 1
(B) 2
(C) 3
(D) 4

3 Which of the following is the correct expression for $\int\frac{dx}{\sqrt{4-x^2}}$?

(A) $\cos^{-1}\frac{x}{2}+C$

(B) $\cos^{-1}2x+C$

(C) $\sin^{-1}\frac{x}{2}+C$

(D) $\sin^{-1}2x+C$

4 A curve has parametric equations $x=t-3$ and $y=t^2+2$. What is the Cartesian equation of this curve?

(A) $y=x^2-x-1$
(B) $y=x^2+x-1$
(C) $y=x^2-6x+11$
(D) $y=x^2+6x+11$

5 A particle is moving in a straight line with $v^2 = 36 - 4x^2$ and undergoing simple harmonic motion. If the particle is initially at the origin, which of the following is the correct equation for its displacement in terms of t?

(A) $x = 2\sin(3t)$

(B) $x = 3\sin(2t)$

(C) $x = 2\sin(9t)$

(D) $x = 3\sin(4t)$

6 Solve the inequality $\frac{x^2-4}{x} \geq 0$.

(A) $-2 \leq x < 0$ or $x \geq 2$

(B) $-2 \geq x > 0$ or $x \leq 2$

(C) $-4 \leq x < 0$ or $x \geq 4$

(D) $-4 \geq x > 0$ or $x \leq 4$

7 What is the value of $\int_0^1 \frac{4x}{2x+1}dx$? Use the substitution $u = 2x+1$.

(A) $2 - \log_e 2$

(B) $2 - \log_e 3$

(C) $4 - 2\log_e 2$

(D) $4 - 2\log_e 3$

8 What is the correct expression for the indefinite integral $\int \left(\cos^2 x + 2\sec^2 x\right) dx$?

(A) $\frac{1}{2}x + \frac{1}{4}\sin 2x + \tan x + C$

(B) $\frac{1}{2}x - \frac{1}{4}\sin 2x + \tan x + C$

(C) $\frac{1}{2}x + \frac{1}{4}\sin 2x + 2\tan x + C$

(D) $\frac{1}{2}x - \frac{1}{4}\sin 2x + 2\tan x + C$

9 What is the term independent of x in the expansion of $\left(x^2 - \frac{2}{x}\right)^9$

(A) ${}^9C_3(-2)^3$

(B) ${}^9C_6(-2)^6$

(C) ${}^9C_3(2)^3$

(D) ${}^9C_6(2)^6$

10 A particle moves in a straight line with a displacement of x and velocity of v. When $t = 0$ the acceleration is $3x^2$, velocity $-\sqrt{2}$ and displacement is 1. Which of the following is the correct equation for x as a function of t?

(A) $x = \frac{-2}{(t+\sqrt{2})^2}$

(B) $x = \frac{-2}{(t-\sqrt{2})^2}$

(C) $x = \frac{2}{(t+\sqrt{2})^2}$

(D) $x = \frac{2}{(t-\sqrt{2})^2}$

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hour and 45 minutes for this section

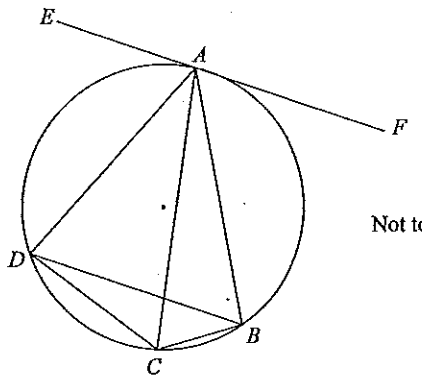
Answer each question in the appropriate writing booklet.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **Marks**

(a) Use Newton's method to find a second approximation to the positive root of $x - 2\sin x = 0$. Take $x = 1.6$ as the first approximation. **2**

(b) $ABCD$ is a cyclic quadrilateral. EAF is a tangent at A to the circle. CA bisects $\angle BCD$. **3**



Not to scale

Show that $EAF \parallel DB$.

(c) What is the exact value of the definite integral $\int_0^{\frac{\pi}{3}} \sin^2 x\,dx$? **2**

(d) A particle moves in a straight line and its position at any time is given by:

$$x = 1 + \sqrt{3}\cos 4t + \sin 4t$$

(i) Prove the motion is simple harmonic. **2**

(ii) Find the amplitude of the motion. **1**

(iii) When does the particle first reach maximum speed after time $t = 0$? **2**

(e) The velocity V of a particle decreases according to the equation:

$$\frac{dV}{dt} = -k(V - P)$$

where t is the time in seconds and k is a positive constant. The initial velocity of the particle is 0 ms^{-1} and the terminal velocity or P is 60 ms^{-1}.

(i) Verify that $V = P + Ae^{-kt}$ is a solution of the above equation, where A is a constant. **1**

(ii) What is the value of k if the velocity of the particle after 10 seconds is 35 ms^{-1}? Answer correct to two significant places. **2**

Question 12 (15 marks) **Marks**

(a) $P(2p, p^2)$ and $Q(2q, q^2)$ are two points on the parabola $x^2 = 4y$.
M is the midpoint of PQ.

(i) Show that $(p-q)^2 = 2(p^2+q^2)-(p+q)^2$. **1**

(ii) If P and Q move on the parabola so that $p-q=4$, show that the locus of M is the parabola $x^2 = 4y-16$. **2**

(iii) What is the focus of the locus of M? **1**

(b) A stone is projected from the top of an 80 metre high vertical cliff with an initial velocity of V ms^{-1} at an angle of projection of θ. It reaches its greatest height after 3 seconds and hits the ground at a horizontal distance of 320 m from the foot of the cliff. Assume $g = 10$ ms^{-2}.

(i) Determine the parametric equations of the path. Use the top of the vertical cliff as the origin. **2**

(ii) Show that $V\sin\theta = 30$. **1**

(iii) Show that the stone reaches the ground after 8 seconds. **1**

(iv) Show that $V\cos\theta = 40$. **1**

(v) Find the value of V and the angle of projection θ. **2**

(c) The angle between the lines $y = mx$ and $y = \frac{1}{2}x$ is 45°. Find the value of m. **2**

(d) State the domain and range of $y = 4\cos^{-1}\left(\frac{3x}{2}\right)$. **2**

Question 13 (15 marks) **Marks**

(a) Use the substitution $u = x+1$ to evaluate $\int_0^{15} \frac{x}{\sqrt{x+1}}\,dx$. **3**

(b) Use the principle of mathematical induction to prove for $n \geq 1$ that **3**

$$\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\ldots+\frac{n}{(n+1)!} = 1-\frac{1}{(n+1)!}$$

(c) (i) Express $\cos x - \sqrt{3}\sin x$ in the form $R\cos(x+\alpha)$ where $R > 0$ and α is an acute angle. **2**

(ii) Hence or otherwise, solve the equation $\cos x - \sqrt{3}\sin x = 2$ for $0 \leq x \leq 2\pi$. **1**

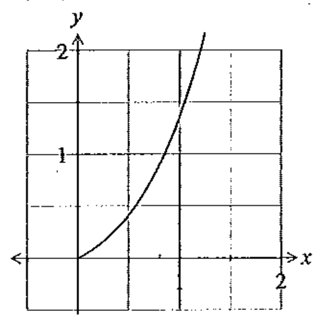
(d) What are the roots of the equation $x^3 + 6x^2 - x - 30 = 0$ given one root is the sum of the other two roots? **3**

(e) Consider the function $f(x) = \frac{xe^x}{2}$ for $x \geq 0$.

(i) Show that $f'(x) > 0$ for all x in the domain. **1**

(ii) Explain why $f(x)$ has an inverse function $f^{-1}(x)$. **1**

(iii) Copy the sketch of $y = f(x)$ below and insert a sketch of $y = f^{-1}(x)$. **1**



Question 14 (15 marks) **Marks**

(a) A particle is moving in a straight line with simple harmonic motion.
At time t seconds it has displacement x metres from a fixed point O on the line, velocity v ms^{-1} and acceleration a ms^{-2} is given by $a=-4x+4$.
Initially, the particle is 2 m to the right of O and is moving away from O with a speed of $2\sqrt{3}$ ms^{-1}.

(i) Use integration to show that $v^2=-4(x-3)(x+1)$. **2**

(ii) Hence find the centre and amplitude of the motion. **1**

(b) (i) Expand $(1+x)^n$ using the binomial theorem. **1**

(ii) Show that $\sum_{r=1}^{n} {}^nC_r = 2^n - 1$ **2**

(iii) Show that $\frac{1}{n+1}\sum_{r=1}^{n+1} {}^{n+1}C_r = \sum_{r=0}^{n}\frac{1}{r+1}{}^nC_r$ **2**

Hint: Integrate the identity in part (i) between the limits of 0 and 1.

(c) Five players are selected at random from four sporting teams.
Each of the teams consists of ten players numbered from 1 to 10.

(i) What is the probability that of the five selected players, three are numbered '1' and two are numbered '7'? **2**

(ii) What is the probability that the five selected players contain at least four players from the same team? **2**

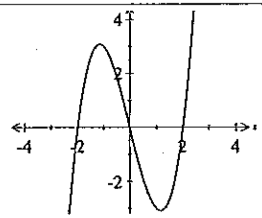
(d) Find all real x such that $|4x-1| > 2\sqrt{x(1-x)}$ **3**

End of paper

ACE Examination 2016

HSC Mathematics Extension 1 Yearly Examination

Worked solutions and marking guidelines

Section I		
	Solution	**Criteria**
1	$x-2y+1=0$, $y=\frac{1}{2}x+\frac{1}{2}$, $m_1=\frac{1}{2}$ $2x-y-1=0$, $y=2x-1$, $m_2=2$ $\tan\theta=\left\|\frac{m_1-m_2}{1+m_1m_2}\right\|=\left\|\frac{\frac{1}{2}-2}{1+\frac{1}{2}\times 2}\right\|=\frac{3}{4}$ $\theta=36.86989765...$ $\approx 37^\circ$	1 Mark: A
2	$x\neq 1$ or $x\neq -1$ $x\to\infty$ $y=\frac{1}{(x+1)(x-1)}=\frac{1}{(x+1)}\times\frac{1}{(x-1)}\to 0$ $\therefore x=1$, $x=-1$ and $y=0$ are asymptotes. Number of asymptotes is 3.	1 Mark: C
3	$\int\frac{dx}{\sqrt{4-x^2}}=\sin^{-1}\frac{x}{2}+C$	1 Mark: C
4	$x=t-3$ or $t=x+3$ Substitute $x+3$ for t into $y=t^2+2$ $y=(x+3)^2+2=x^2+6x+11$	1 Mark: D
5	$v^2=36-4x^2$ $=2^2(9-x^2)=n^2(a^2-x^2)$ $a^2=9$ or $a=3$, $n=2$ and $\alpha=0$ (initially at the origin) $x=a\sin(nt+\alpha)$ $=3\sin(2t)$	1 Mark: B
6	$x^2\times\frac{x^2-4}{x}\geq 0\times x^2$ $x\neq 0$ $x(x^2-4)\geq 0$ $x(x-2)(x+2)\geq 0$ $-2\leq x<0$ or $x\geq 2$ 	1 Mark: A

7	$u=2x+1$ $\quad x=1$ then $u=3$ $du=2dx$ $\quad x=0$ then $u=1$ $\int_0^1 \frac{4x}{2x+1}dx=\int_1^3 \frac{2(u-1)}{u}\times\frac{1}{2}du$ $=\int_1^3 1-\frac{1}{u}du$ $=[u-\log_e u]_1^3$ $=[(3-\log_e 3)-(1-\log_e 1)]=2-\log_e 3$	1 Mark: B
8	$\int\left(\cos^2 x+2\sec^2 x\right)dx=\int\left(\frac{1}{2}(1+\cos 2x)+2\sec^2 x\right)dx$ $=\frac{1}{2}x+\frac{1}{4}\sin 2x+2\tan x+C$	1 Mark: C
9	$T_{r+1}={}^9C_r(x^2)^{9-r}\left(-\frac{2}{x}\right)^r$ $={}^9C_r x^{18-2r}(-2)^r x^{-r}={}^9C_r(-2)^r x^{18-3r}$ Term independent of x $18-3r=0$ $r=6$ $T_7={}^9C_6(-2)^6 x^{18-3\times 6}={}^9C_6(-2)^6$	1 Mark: B
10	$a=3x^2$ $v^2=2\int\left(3x^2\right)dx=2x^3+C$ When $x=1$, $v=-\sqrt{2}$ then $C=0$ $v=-\sqrt{2x^3}$ ($v<0$ when $x=1$) $\frac{dx}{dt}=-\sqrt{2x^3}$ $\frac{dt}{dx}=-\frac{1}{\sqrt{2}}x^{-\frac{3}{2}}$ $t=\frac{2}{\sqrt{2}}x^{-\frac{1}{2}}+C$ When $t=0$, $x=1$ then $C=-\sqrt{2}$ $t=\sqrt{2}x^{-\frac{1}{2}}-\sqrt{2}$ $x^{-\frac{1}{2}}=\frac{t+\sqrt{2}}{\sqrt{2}}$ $x=\frac{2}{(t+\sqrt{2})^2}$	1 Mark: C

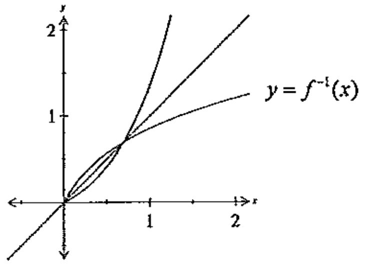
Section II		
11(a)	$f(x)=x-2\sin x$ $f'(x)=1-2\cos x$ $x_1=x_0-\frac{f(x_0)}{f'(x_0)}$ $=1.6-\frac{1.6-2\sin 1.6}{1-2\cos 1.6}$ $=1.977123551...$ ≈ 1.98	2 Marks: Correct answer. 1 Mark: Finds $f(1.6), f'(1.6)$ or shows some understanding of Newton's method.
11(b)	$\angle BAF=\angle BCA$ (Angle between a tangent and a chord is equal to the angle in the alternate segment) $\angle BCA=\angle DCA$ (CA bisects $\angle BCD$) $\angle DCA=\angle DBA$ (Angles in the same segment of a circle are equal) $\angle BAF=\angle DBA$ (from above) $\therefore EAF$ □ DB (Alternate angles $\angle BAF=\angle DBA$ are only equal when two lines are parallel)	3 Marks: Correct answer. 2 Marks: Makes some progress towards the solution. 1 Mark: States one relevant statement and circle theorem.
11(c)	$\int_0^{\frac{\pi}{3}}\sin^2 x\,dx=\int_0^{\frac{\pi}{3}}\frac{1}{2}(1-\cos 2x)dx$ $=\left[\frac{1}{2}x-\frac{1}{4}\sin 2x\right]_0^{\frac{\pi}{3}}$ $=\left[\left(\frac{\pi}{6}-\frac{1}{4}\sin\frac{2\pi}{3}\right)-\left(0-\frac{1}{4}\sin 0\right)\right]$ $=\frac{\pi}{6}-\frac{\sqrt{3}}{8}$ $=\frac{4\pi-3\sqrt{3}}{24}$	2 Marks: Correct answer. 1 Mark: Applies the double angle trig identity.
11(d) (i)	Simple harmonic motion occurs when $\ddot{x}=-n^2x$ Now $x=1+\sqrt{3}\cos 4t+\sin 4t$ $\dot{x}=-\sqrt{3}\times 4\sin 4t+4\cos 4t$ $\ddot{x}=-\sqrt{3}\times 4^2\cos 4t-4^2\sin 4t$ $=-4^2\left(\sqrt{3}\cos 4t+\sin 4t\right)$ $\ddot{x}=-4^2(x-1)$	2 Marks: Correct answer. 1 Mark: Recognises the condition for SHM.

11(d) (ii)	$x = 1+\sqrt{3}\cos 4t + \sin 4t$ $= 1+2\sin\frac{\pi}{3}\cos 4t + 2\cos\frac{\pi}{3}\sin 4t$ $= 1+2\left[\sin 4t\cos\frac{\pi}{3} + \cos 4t \sin\frac{\pi}{3}\right]$ $= 1+2\sin(4t+\frac{\pi}{3})$ (In the form $x = b + a\sin(nt+\alpha)$) Amplitude is 2.	1 Mark: Correct answer.
11(d) (iii)	Maximum speed at $\ddot{x}=0$ or $x=0$ (centre of motion) $\ddot{x} = -4^2\left(\sqrt{3}\cos 4t + \sin 4t\right) = 0$ $\sin 4t + \sqrt{3}\cos 4t = 0$ $\frac{\sin 4t}{\cos 4t} = -\sqrt{3}$ $\tan 4t = -\sqrt{3}$ $4t = \frac{2\pi}{3}, \frac{5\pi}{3}, ...$ $t = \frac{\pi}{6}, \frac{5\pi}{12}, ...$ Particle first reaches maximum speed at $t = \frac{\pi}{6}$	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.
11(e) (i)	$V = P + Ae^{-kt}$ or $Ae^{-kt} = V - P$ $\frac{dV}{dt} = -kAe^{-kt}$ $= -k(V-P)$	1 Mark: Correct answer.
11(e) (ii)	Initially $t=0$ and $V=0$, $P=60$ $V = P + Ae^{-kt}$ $0 = 60 + Ae^{-k\times 0}$ $A = -60$ Also $t=10$ and $V=35$ $35 = 60 - 60e^{-k\times 10}$ $e^{-10k} = \frac{25}{60}$ $-10k = \log_e \frac{25}{60}$ $k = -\frac{1}{10}\log_e\frac{25}{60} = \frac{1}{10}\log_e\frac{60}{25}$ $= 0.087546873... \approx 0.088$	2 Marks: Correct answer. 1 Mark: Finds the value of A.

12(a) (i)	$\text{RHS} = 2(p^2+q^2) - (p+q)^2$ $= 2p^2 + 2q^2 - p^2 - 2pq - q^2$ $= p^2 - 2pq + q^2$ $= (p-q)^2$ $= \text{LHS}$	1 Mark: Correct answer.
12(a) (ii)	Coordinates of $M\left(\frac{2p+2q}{2}, \frac{p^2+q^2}{2}\right)$ or $\left(p+q, \frac{p^2+q^2}{2}\right)$ Using the result in (i) and $p-q=4$ (given) $(p-q)^2 = 2(p^2+q^2) - (p+q)^2$ $4^2 = 2(2y) - (x)^2$ $x^2 = 4y - 16$ Therefore the locus of M is the parabola $x^2 = 4y - 16$	2 Marks: Correct answer. 1 Mark: Determines the coordinates of M or makes some progress.
12(a) (iii)	Now $x^2 = 4y - 16$ $= 4(y-4)$ Focal length is 1, vertex $(0,4)$ and parabola is concave up. Focus is $(0,5)$	1 Mark: Correct answer.
12(b) (i)	Horizontal $\ddot{x} = 0$ $\dot{x} = V\cos\theta$ $x = V\cos\theta t + C$ When $t=0$, $x=0$ implies $C=0$ $x = V\cos\theta t$ Vertical $\ddot{y} = -10$ $\dot{y} = -10t + V\sin\theta$ $y = -5t^2 + V\sin\theta t + C$ When $t=0$, $y=0$ implies $C=0$ $y = -5t^2 + V\sin\theta t$	2 Marks: Correct answer. 1 Mark: Finds horizontal or vertical parametric equations or shows some understanding of the problem.
12(b) (ii)	Greatest height when $\dot{y}=0$ at $t=3$ $\dot{y} = -10t + V\sin\theta$ $0 = -10\times 3 + V\sin\theta$ $\therefore V\sin\theta = 30$	1 Mark: Correct answer.

12(b) (iii)	Stone reaches the ground when $y=-80$ $y=-5t^2+V\sin\theta t$ $-80=-5t^2+30t$ $t^2-6t-16=0$ $(t-8)(t+2)=0$ $\therefore t=8$ (t must be positive)	1 Mark: Correct answer.
12(b) (iv)	Stone reaches the ground when $x=320$ at $t=8$ $x=V\cos\theta t$ $320=V\cos\theta\times 8$ $\therefore V\cos\theta=40$	1 Mark: Correct answer.
12(b) (v)	$V^2=\dot{x}^2+\dot{y}^2$ $=(V\cos\theta)^2+(V\sin\theta)^2$ $=40^2+30^2=2500$ $V=50$ $\tan\theta=\dfrac{V\sin\theta}{V\cos\theta}$ $=\dfrac{30}{40}$ $\theta=36°52'$	2 Marks: Correct answer. 1 Mark: Finds V or θ
12(c)	For $y=mx$ then $m_1=m$. For $y=\frac{1}{2}x$ then $m_2=\frac{1}{2}$ $\tan 45°=\left\lvert\dfrac{m-\frac{1}{2}}{1+m\times\frac{1}{2}}\right\rvert$ $1=\left\lvert\dfrac{2m-1}{2}\times\dfrac{2}{2+m}\right\rvert$ or $\left\lvert\dfrac{2m-1}{2+m}\right\rvert=1$ $\dfrac{2m-1}{2+m}=1$ or $\dfrac{2m-1}{2+m}=-1$ $2m-1=2+m$ $2m-1=-2-m$ $m=3$ $m=-\frac{1}{3}$	2 Marks: Correct answer. 1 Mark: Uses the formula with one correct gradient.
12(d)	Domain: $-1\le(\dfrac{3x}{2})\le 1$. $-\dfrac{2}{3}\le x\le\dfrac{2}{3}$ Range: $0\le\cos^{-1}\left(\dfrac{3x}{2}\right)\le\pi$ $0\le\dfrac{y}{4}\le\pi$ $0\le y\le 4\pi$	2 Marks: Correct answer. 1 Mark: Finds the domain or the range.
13(a)	$u=x+1$ when $x=0$ then $u=1$ $\dfrac{du}{dx}=1$ or $du=dx$ $x=15$ then $u=16$ $\int_0^{15}\dfrac{x}{\sqrt{x+1}}dx=\int_1^{16}\dfrac{u-1}{\sqrt{u}}du=\int_1^{16}(u^{\frac{1}{2}}-u^{-\frac{1}{2}})du$ $=\left[\dfrac{2}{3}u^{\frac{3}{2}}-2u^{\frac{1}{2}}\right]_1^{16}$ $=\left(\dfrac{2}{3}\times 16^{\frac{3}{2}}-2\times 16^{\frac{1}{2}}\right)-\left(\dfrac{2}{3}\times 1^{\frac{3}{2}}-2\times 1^{\frac{1}{2}}\right)=36$	3 Marks: Correct answer. 2 Marks: Finds the primitive as a function of u. 1 Mark: Sets up the integration using the substitution

13(b)	Step 1: To prove the statement true for $n=1$ LHS $=\dfrac{1}{2!}=\dfrac{1}{2}$ RHS $=1-\dfrac{1}{2!}=1-\dfrac{1}{2}=\dfrac{1}{2}$ Result is true for $n=1$ Step 2: Assume the result true for $n=k$ $\dfrac{1}{2!}+\dfrac{2}{3!}+\dfrac{3}{4!}+\ldots+\dfrac{k}{(k+1)!}=1-\dfrac{1}{(k+1)!}$ To prove the result is true for $n=k+1$ $\dfrac{1}{2!}+\dfrac{2}{3!}+\dfrac{3}{4!}+\ldots+\dfrac{k}{(k+1)!}+\dfrac{(k+1)}{(k+2)!}=1-\dfrac{1}{(k+2)!}$ LHS $=\dfrac{1}{2!}+\dfrac{2}{3!}+\dfrac{3}{4!}+\ldots+\dfrac{k}{(k+1)!}+\dfrac{(k+1)}{(k+2)!}$ $=1-\dfrac{1}{(k+1)!}+\dfrac{(k+1)}{(k+2)!}$ $=1-\dfrac{(k+2)}{(k+2)(k+1)!}+\dfrac{(k+1)}{(k+2)!}$ LHS $=1-\dfrac{(k+2)-(k+1)}{(k+2)!}$ $=1-\dfrac{1}{(k+2)!}=$ RHS Result is true for $n=k+1$ if true for $n=k$ Step 3: Result true by principle of mathematical induction.	3 Marks: Correct answer. 2 Marks: Proves the result true for $n=1$ and attempts to use the result of $n=k$ to prove the result for $n=k+1$. 1 Mark: Proves the result true for $n=1$.
13(c) (i)	$R\cos(x+\alpha)=\cos x-\sqrt{3}\sin x$ $R\cos(x+\alpha)=R\cos x\cos\alpha-R\sin x\sin\alpha$ Hence $R\cos\alpha=1$ and $R\sin\alpha=\sqrt{3}$ Dividing these equations $\tan\alpha=\dfrac{\sqrt{3}}{1}$ or $\alpha=\tan^{-1}\sqrt{3}$ or $\dfrac{\pi}{3}$ Squaring and adding the equations $R^2=1^2+\sqrt{3}^2$ or $R=2$ $\cos x-\sqrt{3}\sin x=2\cos(x+\dfrac{\pi}{3})$	2 Marks: Correct answer. 1 Mark: Determines the value of α or R
13(c) (ii)	$2\cos(x+\dfrac{\pi}{3})=2$ from (i) $\cos(x+\dfrac{\pi}{3})=1$ $x+\dfrac{\pi}{3}=0,2\pi,4\pi,\ldots$ $x=\dfrac{5\pi}{3}$	1 Mark: Correct answer.

13(d)	Let the roots be α, β and γ with $\alpha = \beta + \gamma$ $x^3 + 6x^2 - x - 30 = 0$ $\alpha + (\beta + \gamma) = -\frac{b}{a} = -\frac{6}{1} = -6$ $\alpha + \alpha = -6$ $\alpha = -3$ $\alpha\beta\gamma = -\frac{d}{a} = -\frac{-30}{1} = 30$ $-3 \times \beta\gamma = 30$ $\beta\gamma = -10 \quad (1)$ Also $\beta + \gamma = -3 \quad (2)$ By inspection (or solving the equations 1 & 2 simultaneously) $\beta = -5$ and $\gamma = 2$ Roots are $x = -5$, $x = -3$ and $x = 2$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Finds the sum or product of the roots.
13(e) (i)	$f'(x) = \frac{x}{2}e^x + e^x\frac{1}{2}$ $= \frac{e^x(x+1)}{2}$ $> 0 \quad$ when $x \geq 0$ then $e^x \geq 1$	1 Mark: Correct answer.
13(e) (ii)	The function $y = f(x)$ is an increasing function defined for $x \geq 0$. It has no turning points in this domain. Hence an inverse function $y = f^{-1}(x)$ exists as the function $y = f(x)$ is a one-to-one increasing function (it satisfies the horizontal line test).	1 Mark: Correct answer.
13(e) (iii)		1 Mark: Correct answer.

14(a) (i)	$\frac{d}{dx}(\frac{1}{2}v^2) = -4x + 4$ $\frac{1}{2}v^2 = -2x^2 + 4x + C$ When $t = 0$, $x = 2$, $v = 2\sqrt{3}$ $\frac{1}{2} \times (2\sqrt{3})^2 = -2 \times 2^2 + 4 \times 2 + C$ $C = 6$ $\frac{1}{2}v^2 = -2x^2 + 4x + 6$ $v^2 = -4x^2 + 8x + 12$ $= -4(x^2 - 2x - 3)$ $v^2 = -4(x-3)(x+1)$	2 Marks: Correct answer. 1 Mark: Finds an expression for $\frac{1}{2}v^2$ in terms of x.
14(a) (ii)	When $v = 0$ indicates the boundaries for displacement. $\{x : -1 \leq x \leq 3\}$ Therefore motion is centred at 1 m to the right of O with an amplitude of 2 m.	1 Mark: Correct answer.
14(b) (i)	$(1+x)^n = {}^nC_0 1^n + {}^nC_1 1^{n-1}x^1 + {}^nC_2 1^{n-2}x^2 + \ldots + {}^nC_n 1^1 x^n$ $= {}^nC_0 + {}^nC_1 x^1 + {}^nC_2 x^2 + \ldots + {}^nC_n x^n$	1 Mark: Correct answer.
14(b) (ii)	Substitute $x = 1$ into the above identity. $(1+1)^n = {}^nC_0 + {}^nC_1 \times 1^1 + {}^nC_2 \times 1^2 + \ldots + {}^nC_n \times 1^n$ $2^n = 1 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n \qquad ({}^nC_0 = 1)$ $\sum_{r=1}^{n} {}^nC_r = 2^n - 1$	2 Marks: Correct answer. 1 Mark: Shows some understanding.
14(b) (iii)	$\int_0^1 (1+x)^n dx = \int_0^1 {}^nC_0 + {}^nC_1 x^1 + {}^nC_2 x^2 + \ldots + {}^nC_n x^n dx$ $\left[\frac{(1+x)^{n+1}}{n+1}\right]_0^1 = \left[{}^nC_0 x + \frac{1}{2}{}^nC_1 x^2 + \frac{1}{3}{}^nC_2 x^3 + \ldots + \frac{1}{n+1}{}^nC_n x^{n+1}\right]_0^1$ $\frac{1}{n+1}[2^{n+1} - 1] = {}^nC_0 + \frac{1}{2}{}^nC_1 + \frac{1}{3}{}^nC_2 + \ldots + \frac{1}{n+1}{}^nC_n$ $\frac{1}{n+1}\sum_{r=1}^{n+1} {}^{n+1}C_r = \sum_{r=0}^{n} \frac{1}{r+1}{}^nC_r$	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.

14(c) (i)	The 5 selected players come from 4 teams of 10 (40 players) of whom 4 have a number '1' and 4 have a number '7'. Number of possible selections is ${}^{40}C_5$ Number of possible selections of three '1' is ${}^{4}C_3$ Number of possible selections of two '7' is ${}^{4}C_2$ $P(E)=\dfrac{{}^4C_3\times{}^4C_2}{{}^{40}C_5}$ $=\dfrac{1}{27,417}$	2 Marks: Correct answer. 1 Mark: Shows some understanding.
14(c) (ii)	5 players from the same team (4 teams with 10 players in each) $P(5\text{ players})=\dfrac{{}^4C_1\times{}^{10}C_5}{{}^{40}C_5}=\dfrac{14}{9,139}$ 4 players from the same team (4 teams with 10 players in each) $P(4\text{ players})=\dfrac{{}^4C_1\times{}^{10}C_4\times{}^{30}C_1}{{}^{40}C_5}$ $=\dfrac{350}{9,139}$ $P(E)=\dfrac{14}{9,139}+\dfrac{350}{9,139}$ $=\dfrac{28}{703}$	2 Marks: Correct answer. 1 Mark: Finds the probability of 5 players or 4 players.
14(d)	Inequality is only defined for $x(1-x)\geq 0$ (Cannot find the square root of a negative number) $\therefore 0\leq x\leq 1$ (1) Using the result $\lvert x\rvert=\sqrt{x^2}$ or $\lvert 4x-1\rvert=\sqrt{(4x-1)^2}$ $\sqrt{(4x-1)^2}>2\sqrt{x(1-x)}16x$ $(4x-1)^2>4x(1-x)$ $16x^2-8x+1>4x-4x^2$ $20x^2-12x+1>0$ $(10x-1)(2x-1)>0$ $\therefore x<\dfrac{1}{10}$ or $x>\dfrac{1}{2}$ (2) Combining results (1) and (2) $\therefore 0\leq x<\dfrac{1}{10}$ and $\dfrac{1}{2}<x\leq 1$	3 Marks: Correct answer. 2 Marks: Finds one correct region or makes significant progress. 1 Mark: Finds $0\leq x\leq 1$ or uses $\lvert x\rvert=\sqrt{x^2}$ or shows some understanding.