ACE Examinations

Student Name: ______________________

2011

YEAR 12

TRIAL HSC EXAMINATION

Mathematics

General Instructions

- Reading time - 5 minutes
- Working time - 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks - 120

- Attempt Questions 1-10
- All questions are of equal value

Total marks - 120
Attempt Questions 1 - 10
All questions are of equal value

Answer each question in the appropriate writing booklet.

Question 1 (12 marks) **Marks**

(a) Express $\frac{1}{\sqrt{7}-2}$ with a rational denominator. **2**

(b) Solve $x^2 - 3x = 0$ **2**

(c) Solve $|x-1| = 4$ **2**

(d) The line $6x - ky = 2$ passes through the point $(3, 2)$. Find the value of k. **2**

(e) Differentiate $\sin x + x^2$ with respect to x. **2**

(f) Solve $x^2 + 4x + 3 \geq 0$ **2**

Question 2 (12 marks) **Marks**

(a) Differentiate with respect to x.

(i) $2e^x \cos x$ **2**

(ii) $\dfrac{\tan x}{x}$ **2**

(b) Find

(i) $\int e^{4x}\,dx$ **1**

(ii) $\int_0^{\frac{\pi}{4}} (\sec^2 x - x)\,dx$ **2**

(c) Evaluate $\sum_{r=1}^{3} 2^{1-r}$ **1**

(d) Ten kilograms of chlorine is placed in water and begins to dissolve.
After t hours the amount A kg of undissolved chlorine is given by $A = 10e^{-kt}$.

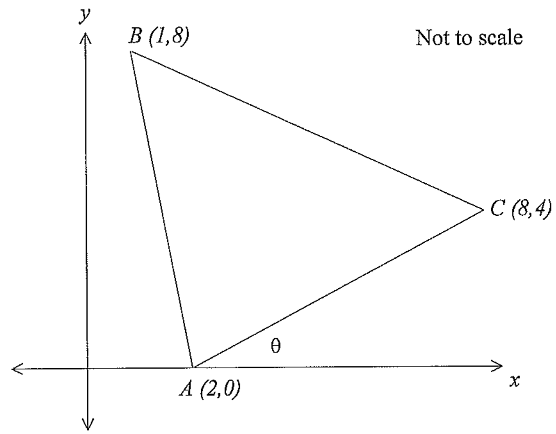
(i) Calculate the value of k given that $A = 3.6$ and $t = 5$. Answer correct to three decimal places. **2**

(ii) After how many hours does one kilogram of chlorine remain undissolved? Answer correct to one decimal place. **2**

Question 3 (12 marks) **Marks**

(a) Solve for x if $4^x = 32$ **1**

(b)



The points A, B and C have coordinates (2,0), (1,8) and (8,4) respectively.
The angle between the line AC and the x-axis is θ.

Copy this diagram.

(i) Find the gradient of the line AC. **1**

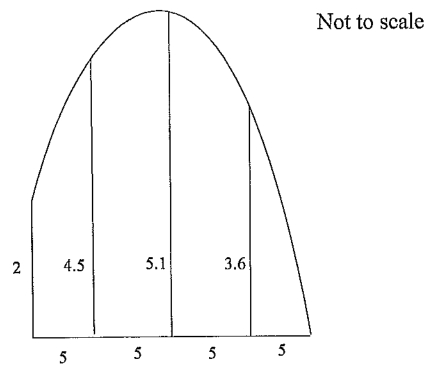
(ii) Calculate the size of angle θ to the nearest minute. **1**

(iii) Find the equation of the line AC. **2**

(iv) Find the coordinates of D, the midpoint of AC. **2**

(v) Show that AC is perpendicular to BD. **2**

(c) The diagram below shows a native garden. All measurements are in metres.



(i) Use the Trapezoidal Rule with 4 intervals to find an approximate value for the area of the native garden. **2**

(ii) If 25 millimetres of rain fell overnight, how many litres of rain fell on the native garden. Assume $1\text{ m}^3 = 1000$ litres **1**

Question 4 (12 marks) **Marks**

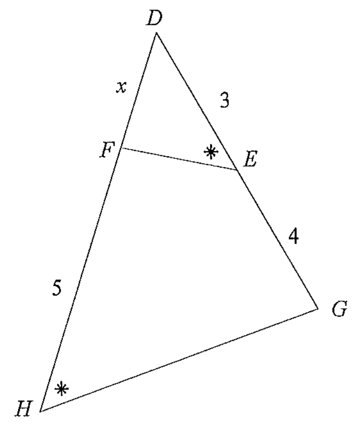
(a) Boxes are stacked in layers, where each layer contains one box less than the layer below. There are six boxes in the top layer, seven boxes in the next layer, and so on. There are n layers altogether.

(i) Write down the number of boxes in the bottom layer. **2**

(ii) Show that there are $\frac{1}{2}n(n+11)$ boxes. **2**

(b) Find the value of k if the sum of the roots of $x^2-(k-1)x+2k=0$ is equal to the product of the roots. **2**

(c) In the diagram below $\angle DEF = \angle DHG$, $DE = 3$, $EG = 4$, $FH = 5$ and $DF = x$. Copy the diagram.



(i) Prove that $\triangle DEF$ is similar to $\triangle DGH$. **2**

(ii) Hence find the value of x. **2**

(d) The line $y = mx+b$ is a tangent to the curve $y = x^3-3x+2$ at the point $(-2, 0)$. Find the value of m and b. **2**

Question 5 (12 marks) **Marks**

(a) Three markers are placed out to sea. Marker B is 4 km north of marker A. However to sail from A to B a boat must first sail from A to C on a bearing 025° and then turn and sail from C to B on a bearing of 335°.

(i) What is the distance from A to C? **2**

(ii) Calculate the distance from A to B through C. **2**

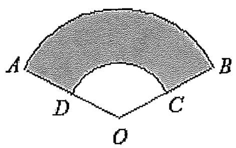
(b) In a school the student population is 45% male and 55% female. Two students are selected at random to represent their school.

(i) What is the probability that both are female? Answer correct to 2 decimal places. **1**

(ii) What is the probability that one is female and the other is male? Answer correct to 2 decimal places. **2**

(iii) What is the probability that neither student is female? Answer correct to 2 decimal places. **1**

(c) A car windscreen wiper traces out the area $ABCD$ where AB and CD are arcs of circles with a centre O and radii 40 cm and 20 cm respectively. Angle AOB measures 120°.



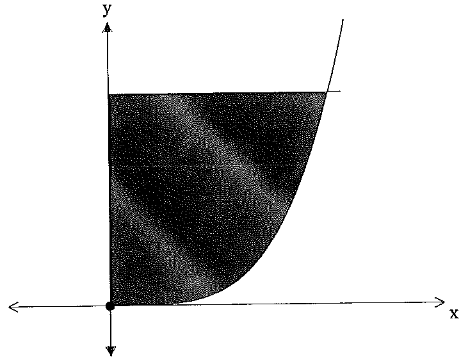
Not to scale

(i) What is the exact length of arc AB? **1**

(ii) What is the area of $ABCD$? Answer to the nearest whole number. **3**

Question 6 (12 marks) **Marks**

(a) The shaded region in the diagram is bounded by the curve $y = x^4$, the y-axis and the line $y = 4$.



Calculate the volume of the solid of revolution when this region is rotated about the y-axis. **3**

(b) The third and seventh terms of a geometric series are 2.5 and 40 respectively.

(i) Find the common ratio. **2**

(ii) Find the first term. **1**

(c) The equation of a parabola is given by $y = x^2 - 4x + 7$.

(i) Find the coordinates of its vertex. **2**

(ii) What is its focal length? **1**

(iii) Find the equation of the tangent at the point $P(3,4)$. **2**

(iv) For what values of x is the parabola concave upwards? **1**

Question 7 (12 marks) **Marks**

(a) A particle moves along a straight line so that its distance x, in metres from a fixed point O is given by $x = \cos t + t$, where t is the time measured in seconds.

(i) Where is the particle initially? **1**

(ii) When does the particle first come to rest? **2**

(iii) Where does the particle first come to rest? **1**

(iv) When does the particle next come to rest? **1**

(v) What is the acceleration of the particle after $\frac{\pi}{3}$ seconds? **2**

(b) A flat circular disc is being heated so that the rate of increase of the area (A in m^2), after t hours, is given by $\frac{dA}{dt} = \frac{1}{8}\pi t$. Initially the disc has a radius of 2 metres. Leave your answers in exact form.

(i) Find the initial area. **1**

(ii) Find an expression for the area after t hours. **2**

(iii) Calculate the radius after 2 hours. **1**

(iv) How long does it take for the area to increase by 25%? **1**

Question 8 (12 marks) **Marks**

(a) Consider the curve given by $y = \frac{1}{2}x^4 - x^3$.

(i) Find any turning points and determine their nature. **2**

(ii) Find any points of inflexion. **2**

(iii) Sketch the curve and indicate where the curve cuts the x-axis. **2**

(iv) For what values of x is the curve concave down? **1**

(b) Monique has set up her superannuation fund and after 10 years she has accumulated \$134 000. However due to an accident she is no longer able to work and make further contributions to the fund. Monique is leaving the money in the superannuation fund to accumulate interest at 8% p.a. compounded annually. However she needs to withdraw \$24 000 at the end of each year for normal living expenses.

(i) Show that at the end of the first year she has $\$(134000 \times 1.08 - 24000)$ in the superannuation fund. **1**

(ii) Find a similar expression for the amount in the fund after 3 years. **2**

(iii) Hence find how many years the fund will last before there is no money in it. **2**

Question 9 (12 marks) **Marks**

(a) (i) Express $\sin\theta\cos\theta+\dfrac{\cos^3\theta}{\sin\theta}$ as a single trigonometric ratio. **1**

(ii) Hence solve $\sin\theta\cos\theta+\dfrac{\cos^3\theta}{\sin\theta}=1$ for $0\le\theta\le 2\pi$. **1**

(b) A can of soup is the shape of a closed cylinder with a height h cm and a radius r cm. The volume of the can of soup is 400 cm^3.

(i) Find an expression for h in terms of r. **1**

(ii) Show that the surface area SA cm^2 of the can is given by the formula: **2**

$$SA = 2\pi r^2+\frac{800}{r}$$

(iii) If the area of the metal used to make the can of soup is to be minimized, find the radius of the can. **3**

(c) It is assumed that the number $N(t)$ of ants in a certain nest at time $t\ge 0$ is given by

$$N(t)=\frac{A}{1+e^{-t}}$$

where A is a constant and t is measured in months.

(i) At time $t = 0$, $N(t)$ is estimated at 2×10^5 ants. What is the value of A? **1**

(ii) What is the value of $N(t)$ after one month? **1**

(iii) How many ants would you expect to find in the nest when t is very large? **1**

(iv) Find an expression for the rate at which the number of ants increases an any time t. **1**

Question 10 (12 marks) **Marks**

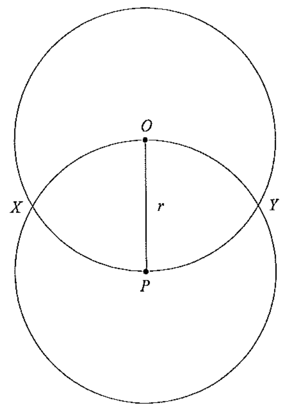
(a) (i) Sketch on the same number plane the graphs of $y=\sin 2x$ and $y=1-\cos 2x$ over the domain $0\le x\le\frac{\pi}{2}$. **2**

(ii) Write down the values of x for which $\sin 2x = 1-\cos 2x$ in the domain $0\le x\le\frac{\pi}{2}$. **1**

(iii) Evaluate the integral $\int_0^{\frac{\pi}{2}}(1-\cos 2x-\sin 2x)\,dx$. **2**

(iv) Calculate the area between $y=\sin 2x$ and $y=1-\cos 2x$ over the domain $0\le x\le\frac{\pi}{2}$. **2**

(b) Two equal circles with centres O and P intersect at X and Y as shown in the diagram. The centres of each circle lie on the circumference of the other circle.



(i) Calculate the exact area of the region $XOYP$. **3**

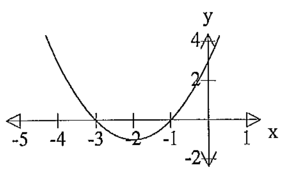
(ii) What fraction of the circle centre O lies outside the region $XOYP$. **2**

End of paper

ACE Examination 2011

Trial HSC Mathematics Examination

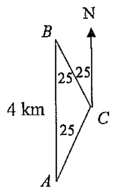
Worked solutions and marking guidelines

	Solution	Criteria
1(a)	$\frac{1}{\sqrt{7}-2}=\frac{1}{\sqrt{7}-2}\times\frac{\sqrt{7}+2}{\sqrt{7}+2}$ $=\frac{\sqrt{7}+2}{7-4}$ $=\frac{\sqrt{7}+2}{3}$	2 Marks: Correct answer. 1 Mark: Multiplies by the conjugate
1(b)	$x^2-3x=0$ $x(x-3)=0$ Therefore $x=0$ or $x=3$.	2 Marks: Correct answer. 1 Mark: Correctly factorises or one solution.
1(c)	$\lvert x-1\rvert=4$ $x-1=4$ or $x-1=-4$ $x=5$ $x=-3$	2 Marks: Correct answer. 1 Mark: Finds one solution
1(d)	$(3,2)$ satisfies the equation $6x-ky=2$ $6\times3-k\times2=2$ $18-2k=2$ $-2k=-16$ $k=8$	2 Marks: Correct answer. 1 Mark: Substitutes $(3,2)$ into the equation
1(e)	$\frac{d}{dx}(\sin x+x^2)=\cos x+2x$	2 Marks: Correct answer. 1 Mark: Differentiates one term correctly
1(f)	$x^2+4x+3\ge0$ $(x+3)(x+1)\ge0$  Hence $x\ge-1$ or $x\le-3$	2 Marks: Correct answer. 1 Mark: Finds one solution

2(a) (i)	$\frac{d}{dx}2e^x\cos x=2e^x(-\sin x)+\cos x2e^x$ $=2e^x(\cos x-\sin x)$	2 Marks: Correct answer. 1 Mark: Applies the product rule
2(a) (ii)	$\frac{d}{dx}\left(\frac{\tan x}{x}\right)=\frac{x\sec^2x-\tan x\times1}{x^2}$ $=\frac{x\sec^2x-\tan x}{x^2}$	2 Marks: Correct answer. 1 Mark: Applies the quotient rule
3(b) (i)	$\int e^{4x}dx=\frac{1}{4}e^{4x}+c$	1 Mark: Correct answer.
3(b) (ii)	$\int_0^{\frac{\pi}{4}}(\sec^2x-x)dx=\left[\tan x-\frac{x^2}{2}\right]_0^{\frac{\pi}{4}}$ $=\left(\tan\frac{\pi}{4}-\frac{\left(\frac{\pi}{4}\right)^2}{2}\right)-\left(\tan0-\frac{0^2}{2}\right)$ $=1-\frac{\pi^2}{32}$	2 Marks: Correct answer. 1 Mark: Integrates correctly.
2(c)	$\sum_{r=1}^{3}2^{1-r}=2^0+2^{-1}+2^{-2}$ $=1+\frac{1}{2}+\frac{1}{4}=1\frac{3}{4}$	1 Mark: Correct answer.
2(d) (i)	$A=10e^{-kt}$ $3.6=10e^{-k\times5}$ $e^{-5k}=0.36$ $-5k\log_e e=\log_e 0.36$ $k=\frac{\log_e 0.36}{-5}$ $=0.2043302495\approx0.204$	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution
2(d) (ii)	$A=10e^{-kt}$ $1=10e^{-0.204...\times t}$ $e^{-0.204...\times t}=0.1$ $-0.204...\times t\times\log_e e=\log_e 0.1$ $t=\frac{\log_e 0.1}{-0.204...}$ $=11.26893888..\approx11.3$ hours	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution

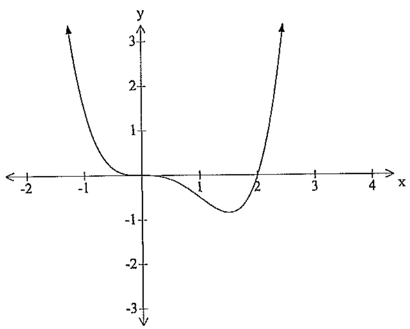
3(a)	$4^x = 32$ $(2^2)^x = 2^5$ $2x = 5$ $x = 2.5$	1 Mark: Correct answer.
3(b) (i)	Gradient of AC $M = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{4-0}{8-2} = \frac{4}{6} = \frac{2}{3}$	1 Mark: Correct answer.
3(b) (ii)	Gradient of AC $\tan\theta = \frac{2}{3}$ $\theta = 33°41'$	1 Mark: Correct answer.
3(b) (iii)	Point slope formula $y - y_1 = m(x - x_1)$ $y - 0 = \frac{2}{3}(x-2)$ $3y = 2(x-2)$ $2x - 3y - 4 = 0$	2 Marks: Correct answer. 1 Mark: Substitutes into point slope form
3(b) (iv)	Mid-point formula $x = \frac{x_1 + x_2}{2} = \frac{2+8}{2} = 5$ $y = \frac{y_1 + y_2}{2} = \frac{0+4}{2} = 2$ Midpoint is D(5, 2)	2 Marks: Correct answer. 1 Mark: Finds one solution
3(b) (v)	AC is perpendicular to BD if $m_1 m_2 = -1$. Gradient of AC is $^2/_3$ Gradient of BD $M = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-2}{1-5} = \frac{6}{-4} = -\frac{3}{2}$ Now $m_1 m_2 = -1$ $\frac{2}{3} \times -\frac{3}{2} = -1$ (True)	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution
3(c) (i)	$A = \frac{h}{2}(d_f + 2d_m + d_l) + \frac{h}{2}(d_f + 2d_m + d_l)$ $= \frac{5}{2}(2 + 2\times 4.5 + 5.1) + \frac{5}{2}(5.1 + 2\times 3.6 + 0)$ $= 71 \text{ m}^2$	2 Marks: Correct answer. 1 Mark: Uses trapezoidal rule
3(c) (ii)	Now 25 mm = 0.025 m $V = Ah$ $= 71 \times 0.025$ $= 1.775 \text{ m}^3 = 1775$ Litres	1 Mark: Correct answer.

4(a) (i)	Number of boxes in each layer from the top are an AP: 6, 7, 8... $T_n = a + (n-1)d$ $= 6 + (n-1)\times 1$ $= 6 + n - 1$ $= n + 5$	2 Marks: Correct answer. 1 Mark: Recognises AP and using nth term formula
4(a) (ii)	Sum the boxes in each layer. ($a = 6$ and $l = n+5$) $S_n = \frac{n}{2}(a + l)$ $= \frac{n}{2}(6 + n + 5) = \frac{1}{2}n(n+11)$	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution
4(b)	$\alpha + \beta = -\frac{b}{a} = -\frac{-(k-1)}{1} = (k-1)$ $\alpha\beta = \frac{c}{a} = \frac{2k}{1} = 2k$ Now $(k-1) = 2k$ $k = -1$	2 Marks: Correct answer. 1 Mark: Correctly calculates the sum or product
4(c) (i)	In $\triangle DEF$ and $\triangle DGH$ $\angle DEF = \angle DHG$ (given data) $\angle FDE = \angle HDG$ (common angle) $\angle DFE = \angle DGH$ (angle sum of a triangle is 180) $\triangle DEF$ is similar to $\triangle DGH$ (equiangular)	2 Marks: Correct answer. 1 Mark: Shows some understanding
4(c) (ii)	$\frac{3}{x+5} = \frac{x}{7}$ (corresponding sides in similar triangles) $x^2 + 5x = 21$ $x^2 + 5x - 21 = 0$ $x = \frac{-5 \pm \sqrt{5^2 - 4\times 1\times -21}}{2\times 1}$ $= \frac{-5 \pm \sqrt{109}}{2}$ ≈ 2.72 or -7.72 (ignore this answer)	2 Marks: Correct answer. 1 Mark: Correctly matches the corresponding sides
4(d)	$y = x^3 - 3x + 2$ At the point $(-2, 0)$ $\frac{dy}{dx} = 3\times(-2)^2 - 3 = 9$ $\frac{dy}{dx} = 3x^2 - 3$ Point slope formula $y - y_1 = m(x - x_1)$ $y - 0 = 9(x - -2)$ $y = 9x + 18$ Hence $m = 9$ and $b = 18$	2 Marks: Correct answer. 1 Mark: Finds the gradient of the tangent.

5(a) (i)	$\dfrac{AC}{\sin 25}=\dfrac{4}{\sin 130}$ $AC=\dfrac{4\sin 25}{\sin 130}$ $=2.206755838\ldots$ ≈ 2.2 km  	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution
5(a) (ii)	$\square ABC$ is an isosceles triangle $\angle CAB=\angle CBA$ Total distance travelled $=2\times AC$ $=4.413511676\ldots$ ≈ 4.4 km	2 Marks: Correct answer. 1 Mark: Recognises an isosceles triangle or calculates *BC*
5(b) (i)	$P(FF)=0.55\times 0.55$ $=0.3025$	1 Mark: Correct answer.
5(b) (ii)	$P(MF \text{ or } FM)=0.45\times 0.55+0.55\times 0.45$ $=0.495$	2 Marks: Correct answer. 1 Mark: Calculates *P(MF)* or *P(FM)* only
5(b) (iii)	$P(MM)=0.45\times 0.45$ $=0.2025$	1 Mark: Correct answer.
5(c) (i)	$120^\circ=\dfrac{120\pi}{180}$ $=\dfrac{2\pi}{3}$ radians $l=r\theta$ $=40\times\dfrac{2\pi}{3}$ $=\dfrac{80\pi}{3}$	1 Mark: Correct answer.
5(c) (ii)	Area of sector $ABO=\dfrac{1}{2}\times 40^2\times\dfrac{2\pi}{3}$ $=\dfrac{1600\pi}{3}$ Area of sector $CDO=\dfrac{1}{2}\times 20^2\times\dfrac{2\pi}{3}$ $=\dfrac{400\pi}{3}$ Area of $ABCD=\dfrac{1600\pi}{3}-\dfrac{400\pi}{3}$ $=400\pi$ ≈ 1257 cm^2	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution 1 Mark: Uses the area of a sector formula

6(a)	Now $y=x^4$ or $y^{\frac{1}{2}}=x^2$ $V=\pi\int_0^4 x^2\,dy$ $=\pi\int_0^4 y^{\frac{1}{2}}\,dy$ $=\pi\left[\dfrac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4$ $=\dfrac{2\pi}{3}\left(4^{\frac{3}{2}}-0^{\frac{3}{2}}\right)$ $=\dfrac{16\pi}{3}$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Correctly sets up the integral
6(b) (i)	$T_3=ar^2=2.5$ and $T_7=ar^6=40$ Divide the two equations $\dfrac{ar^6}{ar^2}=\dfrac{40}{2.5}$ $r^4=16$ $r=\pm 2$	2 Marks: Correct answer. 1 Mark: Uses the formula for the nth term of a GP.
6(b) (ii)	Substitute ± 2 for r into the equation $ar^6=40$ $a(\pm 2)^6=40$ $a=\dfrac{40}{64}=\dfrac{5}{8}$	1 Mark: Correct answer.
6(c) (i)	$y=x^2-4x+7$ $y=(x-2)^2+3$ $y-3=(x-2)^2$ Vertex is (2, 3)	2 Marks: Correct answer. 1 Mark: Completes the square
6(c) (ii)	$y-k=4a(x-h)^2$ $y-3=4\times\dfrac{1}{4}(x-2)^2$ Focal length is $^1/_4$	1 Mark: Correct answer.
6(c) (iii)	$\dfrac{dy}{dx}=2x-4$ At the point $(3,4)$ $\dfrac{dy}{dx}=2\times 3-4=2$ Point slope formula $y-y_1=m(x-x_1)$ $y-4=2(x-3)$ $y=2x-2$	2 Marks: Correct answer. 1 Mark: Finds gradient of the tangent
6(c) (iv)	$\dfrac{d^2y}{dx^2}=2>0$ Parabola is concave up for all real x	1 Mark: Correct answer.

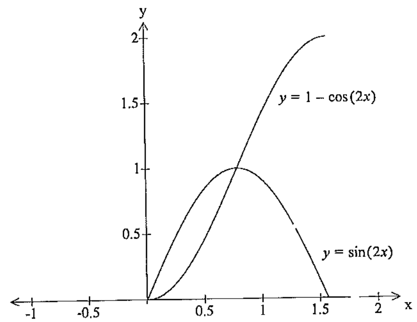
7(a) (i)	Initially $t=0$ $x=\cos t+t=\cos 0+0=1$	1 Mark: Correct answer.
7(a) (ii)	Particle comes to rest when $v=0$ $v=\frac{dx}{dt}$ $0=-\sin t+1$ $\sin t=1$ $t=\frac{\pi}{2}$ seconds	2 Marks: Correct answer. 1 Mark: Finds an expression for the velocity
7(a) (iii)	When $t=\frac{\pi}{2}$ $x=\cos\frac{\pi}{2}+\frac{\pi}{2}=\frac{\pi}{2}$ metres	1 Mark: Correct answer.
7(a) (iv)	$\sin t=1$ Next comes to rest at $\frac{5\pi}{2}$ seconds $t=\frac{\pi}{2},\frac{5\pi}{2},$...seconds	1 Mark: Correct answer.
7(a) (v)	$a=\frac{dv}{dt}$ $=-\cos t$ When $t=\frac{\pi}{3}$ $a=-\cos\frac{\pi}{3}=-0.5$	2 Marks: Correct answer. 1 Mark: Finds an expression for the acceleration
7(b) (i)	$A=\pi r^2=\pi\times 2^2=4\pi$ m^2	1 Mark: Correct answer.
7(b) (ii)	$A=\int\frac{1}{8}\pi t\,dt$ When $t=0$, $A=4\pi$ $4\pi=\frac{1}{16}\pi 0^2+c$ $c=4\pi$ $=\frac{1}{16}\pi t^2+c$ Hence $A=\frac{1}{16}\pi t^2+4\pi$	2 Marks: Correct answer. 1 Mark: Integrates to find A
7(b) (iii)	When $t=2$ $A=\frac{1}{16}\pi\times 2^2+4\pi=\frac{1}{4}\pi+4\pi=\frac{17\pi}{4}$ Hence $A=\pi r^2$ $r=\frac{\sqrt{17}}{2}$ $\frac{17\pi}{4}=\pi\times r^2$	1 Mark: Correct answer.
7(b) (iv)	25% increase in area $A=1.25\times 4\pi=5\pi$ $5\pi=\frac{1}{16}\pi t^2+4\pi$ $t=4$ hours	1 Mark: Correct answer.

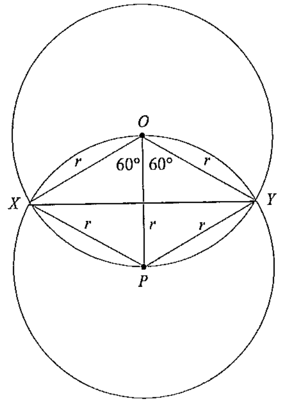
8(a) (i)	$y=\frac{1}{2}x^4-x^3$ Turning points $\frac{dy}{dx}=0$ $\frac{dy}{dx}=2x^3-3x^2$ $2x^3-3x^2=0$ $x^2(2x-3)=0$ $\frac{d^2y}{dx^2}=6x^2-6x$ $x=0,\ x=\frac{3}{2}$ When $x=0$, $y=0$ and $\frac{d^2y}{dx^2}=0$ Possible point of inflexion When $x=\frac{3}{2}$, $y=-\frac{27}{32}$ and $\frac{d^2y}{dx^2}=\frac{9}{2}>0$ Minimum	2 Marks: Correct answer. 1 Mark: Obtains the first derivative and uses $\frac{dy}{dx}=0$ to find turning points
8(a) (ii)	Possible points of inflexion $\frac{d^2y}{dx^2}=0$ $6x^2-6x=0$ $6x(x-1)=0$ $x=0,\ x=1$ Check for change in concavity When $x=-0.1$ then $\frac{d^2y}{dx^2}=6\times -0.1\times(-0.1-1)>0$ When $x=0.1$ then $\frac{d^2y}{dx^2}=6\times 0.1\times(0.1-1)<0$ When $x=1.1$ then $\frac{d^2y}{dx^2}=6\times 1.1\times(1.1-1)>0$ Hence (0, 0) and $(1,-\frac{1}{2})$ are points of inflexion.	2 Marks: Correct answer. 1 Mark: Determines the possible points of inflexion but does not test for concavity
8(a) (iii)	Cuts the x-axis when $y=0$ or $0=\frac{1}{2}x^4-x^3$ $0=\frac{1}{2}x^3(x-2)$ $x=0, x=2$ 	2 Marks: Correct answer. 1 Mark: Obtains the x-intercepts

8(a) (iv)	Concave down for $0<x<1$	1 Mark: Correct answer.
8(b) (i)	$A=P(1+r)^n$ $=134000\times(1+0.08)^1$ $=134000(1.08)$ After 1 year $A_1=134000\times1.08-24000$	1 Mark: Correct answer.
8(b) (ii)	After 2 years $A_2=(134000(1.08)-24000)\times1.08-24000$ $=134000\times1.08^2-24000(1.08+1)$ After 3 years $A_3=\left[(134000\times1.08^2-24000(1.08+1)\right]\times1.08-24000$ $=134000\times1.08^3-24000(1.08^2+1.08+1)$	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution
8(b) (iii)	After n years $A_n=134000\times1.08^n-24000(1.08^{n-1}+1.08^{n-2}+1)$ To find n when $A_n=0$ $0=134000\times1.08^n-24000\left(\frac{1.08^n-1}{1.08-1}\right)$ $134000\times1.08^n\times0.08-24000(1.08^n-1)=0$ $10720\times1.08^n-24000\times1.08^n+24000=0$ $13280\times1.08^n=24000$ $1.08^n=\frac{24000}{13280}$ $n\times\log1.08=\log\left(\frac{24000}{13280}\right)$ $n\approx7.69$ years	2 Marks: Correct answer. 1 Mark: Obtains a correct expression for A_n and uses $A_n=0$

9(a) (i)	$\sin\theta\cos\theta+\frac{\cos^3\theta}{\sin\theta}=\frac{\cos\theta}{\sin\theta}(\sin^2\theta+\cos^2\theta)$ $=\cot\theta$	1 Mark: Correct answer.
9(a) (ii)	$\cot\theta=1$ $\theta=\frac{\pi}{4}$ or $\frac{5\pi}{4}$	1 Mark: Correct answer.
9(b) (i)	$V=\pi r^2h$ $400=\pi r^2\times h$ $h=\frac{400}{\pi r^2}$	1 Mark: Correct answer.
9(b) (ii)	$SA=2\pi r^2+2\pi rh$ $=2\pi r^2+2\pi r\times\frac{400}{\pi r^2}$ $=2\pi r^2+\frac{800}{r}$	2 Marks: Correct answer. 1 Mark: Applies the formula for the SA of a cylinder
9(b) (iii)	$SA=2\pi r^2+\frac{800}{r}$ $\frac{dSA}{dr}=4\pi r-800r^{-2}$ Minimal SA occurs when $\frac{dSA}{dr}=0$ $4\pi r-800r^{-2}=0$ $4r\left(\pi-\frac{200}{r^3}\right)=0$ Hence $r=0$ (no can) or $\pi-\frac{200}{r^3}=0$ $r=\sqrt[3]{\frac{200}{\pi}}$ ≈4 cm Check $\frac{d^2SA}{dr^2}=4\pi+1600r^{-3}$ $=4\pi+\frac{1600}{r^3}$ At $r=4$ $\frac{d^2SA}{dr^2}>0$ and is a minima	3 Marks: Correct answer. 2 Marks: Determines the radius is approximately 4cm 1 Mark: Differentiates the SA formula with respect to r

9(c) (i)	$N(t)=\dfrac{A}{1+e^{-t}}$ $2\times10^5=\dfrac{A}{1+e^0}$ $A=4\times10^5$	1 Mark: Correct answer.
9(c) (ii)	When $t=1$ $N(t)=\dfrac{A}{1+e^{-t}}$ $=\dfrac{4\times10^5}{1+e^{-1}}$ $=292423$ ants	1 Mark: Correct answer.
9(c) (iii)	When $t\to\infty$ $N(t)=\dfrac{A}{1+e^{-t}}$ $=\dfrac{4\times10^5}{1+e^{-\infty}}$ $=400000$ ants	1 Mark: Correct answer.
9(c) (iv)	$N(t)=\dfrac{4\times10^5}{1+e^{-t}}=4\times10^5\times(1+e^{-t})^{-1}$ $\dfrac{dN(t)}{dt}=4\times10^5\times-1\times(1+e^{-t})^{-2}\times-1e^{-t}$ $=\dfrac{(4\times10^5)e^{-t}}{(1+e^{-t})^2}$ or $\dfrac{Ae^{-t}}{(1+e^{-t})^2}$	1 Mark: Correct answer.

10(a) (i)		2 Marks: Correct answer. 1 Mark: Graphs one of the equation correctly or shows some understanding
10(a) (ii)	$\theta=0$ or $\dfrac{\pi}{4}$	1 Mark: Correct answer.
10(a) (iii)	$\int_0^{\frac{\pi}{2}}(1-\cos2x-\sin2x)dx$ $=\left[x-\dfrac{1}{2}\sin2x+\dfrac{1}{2}\cos2x\right]_0^{\frac{\pi}{2}}$ $=\left(\dfrac{\pi}{2}-\dfrac{1}{2}\sin2\times\dfrac{\pi}{2}+\dfrac{1}{2}\cos2\times\dfrac{\pi}{2}\right)-\left(0-\dfrac{1}{2}\sin2\times0+\dfrac{1}{2}\cos2\times0\right)$ $=\left(\dfrac{\pi}{2}-\dfrac{1}{2}\right)-\left(\dfrac{1}{2}\right)$ $=\dfrac{\pi}{2}-1$	2 Marks: Correct answer. 1 Mark: Correct answer.
10(a) (iv)	$\int_0^{\frac{\pi}{2}}(1-\cos2x-\sin2x)dx$ $=\int_0^{\frac{\pi}{4}}(\sin2x-(1-\cos2x))dx+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(1-\cos2x-\sin2x)dx$ $=\left[-\dfrac{1}{2}\cos2x-x+\dfrac{1}{2}\sin2x+\right]_0^{\frac{\pi}{4}}+\left[x-\dfrac{1}{2}\sin2x+\dfrac{1}{2}\cos2x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ $=\left(0-\dfrac{\pi}{4}+\dfrac{1}{2}-(-\dfrac{1}{2})\right)+\left(\dfrac{\pi}{2}-\dfrac{1}{2}-(\dfrac{\pi}{4}-\dfrac{1}{2})\right)$ $=-\dfrac{\pi}{4}+\dfrac{1}{2}+\dfrac{1}{2}+\dfrac{\pi}{2}-\dfrac{1}{2}-\dfrac{\pi}{4}+\dfrac{1}{2}$ $=1$	2 Marks: Correct answer. 1 Mark: Makes significant progress towards the solution

<table>
<tr><td>10(b)
(i)</td><td>

Area of segment XYP

$A = \frac{1}{2} r^2 (\theta - \sin\theta)$

$= \frac{1}{2} r^2 (\frac{2\pi}{3} - \sin\frac{2\pi}{3})$

$= \frac{1}{2} r^2 \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$

Area of the region XOYP is twice the area of segment XYP

$A = 2 \times \frac{1}{2} r^2 \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$

$= r^2 \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$</td><td>3 Marks: Correct answer.

2 Marks: Makes significant progress towards the solution

1 Mark: Recognises equilateral triangles or similar understanding</td></tr>
<tr><td>10(b)
(ii)</td><td>Area outside the region XOYP

$A = \pi r^2 - r^2 \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$

$= r^2 \left(\frac{\pi}{3} + \frac{\sqrt{3}}{2}\right)$

Fraction required is $= \dfrac{r^2 \left(\frac{\pi}{3} + \frac{\sqrt{3}}{2}\right)}{\pi r^2}$

$= \frac{1}{3} + \frac{\sqrt{3}}{2\pi}$</td><td>2 Marks: Correct answer.

1 Mark: Makes progress towards the solution</td></tr>
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