ACE Examinations

Student Name: ____________________

2016

YEAR 12

YEARLY EXAMINATION

Mathematics

General Instructions

- Reading time - 5 minutes
- Working time - 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A reference sheet is provided at the back of this paper
- Show relevant mathematical reasoning and/or calculations in Questions 11-16

Total marks - 100

Section I

10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II

90 marks

- Attempt Questions 11-16
- Allow about 2 hour 45 minutes for this section

Section I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

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1 What is the value of $\lim\limits_{x\to 7}\dfrac{x^2+5x-84}{x-7}$?

(A) -5
(B) 0
(C) 12
(D) 19

2 What are the conditions for the expression $ax^2+bx+c=0$ to be positive definite?

(A) $c>0$ and $\Delta<0$
(B) $c>0$ and $\Delta>0$
(C) $a>0$ and $\Delta<0$
(D) $a>0$ and $\Delta>0$

3 What is the value of $\sum\limits_{r=1}^{4}2^{1-r}$?

(A) $\dfrac{1}{64}$
(B) $\dfrac{9}{8}$
(C) $\dfrac{15}{8}$
(D) $\dfrac{9}{4}$

4 What is the greatest value of the function $y=4-2\cos x$?

(A) 2
(B) 4
(C) 6
(D) 8

5 What is the solution to the equation $\log_e(x+2)-\log_e x=\log_e 4$?

(A) $\frac{2}{5}$

(B) $\frac{2}{3}$

(C) $\frac{3}{2}$

(D) $\frac{5}{2}$

6 What is the derivative of $y=\frac{e^{2x}+4}{x^3}$?

(A) $\frac{e^{2x}}{3x^2}$

(B) $\frac{2xe^{2x}-3e^{2x}-12}{x^4}$

(C) $\frac{2xe^{2x}-3e^{2x}-12}{x}$

(D) $\frac{2xe^{2x}+3e^{2x}+12}{x^4}$

7 Solve for x: $|2x+3|=-x+3$.

(A) There are no solutions.
(B) There is only one solution.
(C) There are two solutions.
(D) There are three solutions.

8 Two-digit numbers are formed from the digits 2, 3, 4, 6 with no repetition of digits allowed. A two-digit number is then selected at random. What is the probability that the number is prime?

(A) $\frac{1}{12}$

(B) $\frac{1}{8}$

(C) $\frac{1}{6}$

(D) $\frac{5}{12}$

9 The gradient function of a curve is $\frac{dy}{dx}=3-\frac{2}{x^2}$.

What is the equation of the curve if it passes through the point $(1,-2)$?

(A) $y=\frac{4}{x^3}$

(B) $y=\frac{2}{x}-4$

(C) $y=3x-\frac{2}{x}-3$

(D) $y=3x+\frac{2}{x}-7$

10 What is the domain and range of the function $y=\frac{1}{\sqrt{x-9}}$?

(A) $\{x:x\geq 9\}$ and $\{y:y>0\}$
(B) $\{x:x>9\}$ and $\{y:y>0\}$
(C) $\{x:-\infty\leq x\leq\infty\}$ and $\{y:-\infty\leq y\leq\infty\}$
(D) $\{x:-3\geq x\geq 3\}$ and $\{y:y<0\}$

Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet.

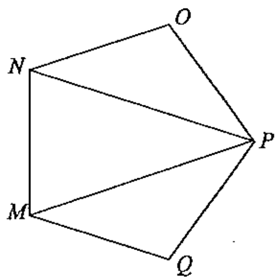
Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **Marks**

(a) Simplify $\frac{5}{x-2}-\frac{2}{x-3}$. **2**

(b) Express $\frac{1}{\sqrt{5}-2}$ with a rational denominator. **2**

(c)



The diagram shows a regular pentagon *MNOPQ*.
Each of the sides *MN*, *NO*, *OP*, *PQ* and *QM* is of length x metres.
Each of the angles $\angle MNO$, $\angle NOP$, $\angle OPQ$, $\angle PQM$ and $\angle QMN$ is 108°.

(i) Show that triangle *NOP* is isosceles and hence find $\angle ONP$. **2**

(ii) Show that triangles *NOP* and *PQM* are congruent. **2**

(iii) Find the size of $\angle MPN$. **1**

(d) Boxes are stacked in layers, where each layer contains one box less than the layer below. There are six boxes in the top layer, seven boxes in the next layer, and so on. There are n layers altogether.

(i) Write down the number of boxes in the bottom layer. **2**

(ii) Show that there are $\frac{1}{2}n(n+11)$ boxes. **2**

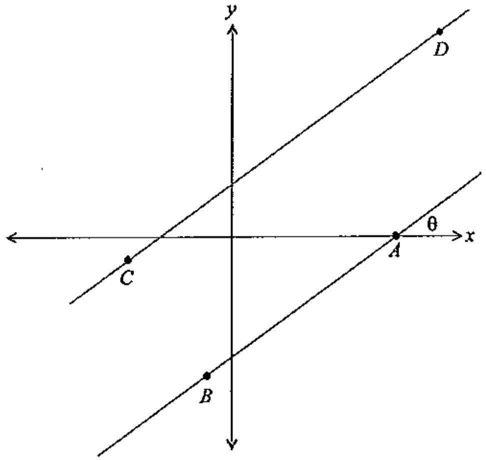
(e) A particle moves along the x-axis with acceleration $3t-2$. Initially it is 4 units to the right of the origin, with a velocity of 2 units per second. What is the position of the particle after 5 seconds? **2**

Question 12 (15marks) **Marks**

(a) (i) Find $\int \frac{x}{x^2+3}dx$. **2**

(ii) Evaluate $\int_0^{\frac{\pi}{3}} \cos 2x dx$. **2**

(b)



The diagram shows the points $B(-1,-4)$, $C(-3,-1)$ and $D(6,5)$ in the Cartesian plane. The point $A(a,0)$ is the point where the line AB intersects with the x-axis.

(i) Show that the gradient of CD is $\frac{2}{3}$. **1**

(ii) Find the equation of CD. **2**

(iii) If $AB \parallel CD$ find the value of a, the x-coordinate of the point A. **2**

(iv) Show the distance BC is $\sqrt{13}$. **1**

(v) Find the size of angle θ correct to the nearest minute. **1**

(vi) Find the perpendicular distance from B to the line CD. **1**

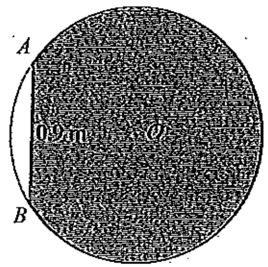
(c) A particle moves so that its displacement from the origin is $x = -t^2 + 6t + 7$ where x is displacement in metres and t is time in seconds

(i) What is the initial displacement of the particle? **1**

(ii) When will the particle be at the origin? **2**

Question 13 (15 marks) **Marks**

(a) A small segment of a circle has been removed as shown below.
The circle has a centre O and radius 0.9 metres.
The length of the straight edge AB is also 0.9 metres.



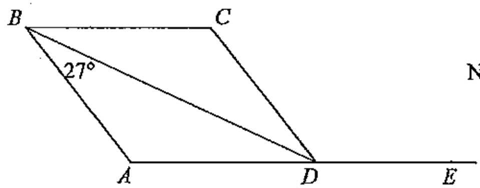
Not to scale

(i) Explain why $\angle AOB = \frac{\pi}{3}$. **1**

(ii) What is the shaded area? **3**

(b) Find the value of k if the sum of the roots of $x^2 - (k-1)x + 2k = 0$ is equal to the product of the roots. **2**

(c)



Not to scale

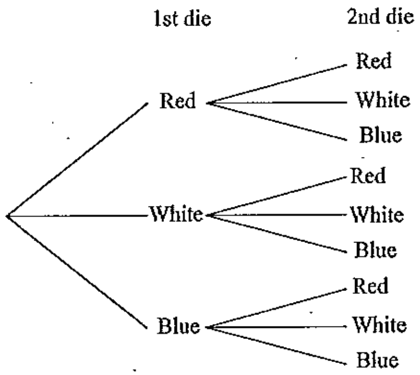
In the diagram, $ABCD$ is a rhombus where $\angle ABD = 27°$ and AD is produced to E. Copy the diagram onto your answer sheet.

(i) What is the value of $\angle ABC$? **1**

(ii) What is the value of $\angle CDE$? Give reasons. **2**

(d) Aiden plays a game where he rolls two dice. The first die has three red faces, two white faces and one blue face. The other die has two red faces, two white faces and two blue faces.

(i) Copy and complete the tree diagram below to show all the possible outcomes and their probabilities. **1**



(ii) Find the probability both dice show red. **1**

(iii) Find the probability one die shows red and one die shows white. **1**

(iv) Find the probability both dice do not show red or both dice do not show blue. **1**

(e) The second term of an arithmetic series is 37 and the sixth term is 17. What is the sum of the first ten terms? **2**

HSC Mathematics

Question 14 (15 marks) **Marks**

(a) Consider the function $f(x) = 4x^3 - 4x^2$.

(i) Find the stationary points and determine their nature. **3**

(ii) Sketch the graph of the function $y = f(x)$ showing these stationary points. **2**

(iii) Show the point(s) at which $y = f(x)$ cuts the x-axis. **1**

(iv) Determine the values of x for which $f(x)$ is positive. **1**

(b) Solve the equation $3^{2x} + 3^x - 12 = 0$. **2**

(c) Find the volume when $y = \log_e x$ is rotated about the y-axis between $y = 1$ and $y = 3$. Express your answer in exact form. **3**

(d) Differentiate with respect to x.

(i) $(e^x - 3)^4$ **1**

(ii) $x \tan x$ **1**

(iii) $\log_e(\cos x)$ **1**

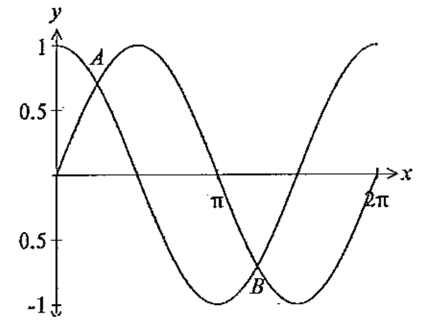
Question 15 (15 marks) **Marks**

(a) Twenty kilograms of sugar is placed in a container of water and begins to dissolve. After t hours the amount A kg of undissolved sugar is $A = 20e^{-kt}$.

(i) Calculate k, given that $A = 4.8$ when $t = 5$. **1**

(ii) After how many hours does 1 kg of sugar remain undissolved? **2**

(b)



The diagram shows the graphs $y = \sin x$ and $y = \cos x$ in the domain $0 \le x \le 2\pi$. The graphs intersect at points A and B.

(i) What are the coordinates of A and B? **2**

(ii) Find the area enclosed by the two graphs between A and B. **2**

(c) On 1 June 2006, Patrick invested \$20,000 in a bank account that paid interest at a fixed rate of 7% per annum, compounded annually

(i) How much is in the account after the payment of interest on 1 June 2016 if no additional deposits were made? **1**

(ii) Patrick decided to add \$2000 to his account on 1 June each year beginning on 1 June 2007. How much is in his account on 1 June 2016 after the payment of interest and his deposit? **3**

(iii) Patrick's friend Bella invested \$20,000 in an account at another bank on 1 June 2006 and made no further deposits. On 1 June 2016, the balance of Bella's account was \$49,565. What was the annual rate of compound interest paid on Bella's account? **2**

(d) Differentiate $\log_2 x^2$. **2**

Question 16 (15 marks) **Marks**

(a) Find the equation of the normal to the curve $y = x\log_e x$ at the point on the curve where $x = 1$. **3**

(b) The speed of a train was recorded at intervals of one minute. The times, in minutes, and the corresponding speeds v, in kilometres per hour, are listed in the table below.

Time (min)	0	1	2	3	4
Speed (km/h)	0	24	35	28	50

(i) Explain why the distance x, in km, travelled by the train in these four minutes is given by $x = \int_0^{\frac{1}{15}} v\,dt$. **1**

(ii) Estimate x by using Simpson's Rule with five function values. **2**

(c) A parabola has the equation $16y = x^2 - 4x - 12$

(i) Find the coordinates of the vertex. **2**

(ii) Find the coordinates of the focus. **1**

(iii) Find the equation of the directrix. **1**

(d) A can is the shape of a closed cylinder with a height h cm and a radius r cm. The volume of the can is 200 cm^3.

(i) Find an expression for h in terms of r. **1**

(ii) Show that the surface area SA cm^2 of the can is given by the formula: **2**

$$SA = 2\pi r^2 + \frac{400}{r}$$

(iii) If the area of the metal used to make the can is to be minimized, find the radius of the can. **2**

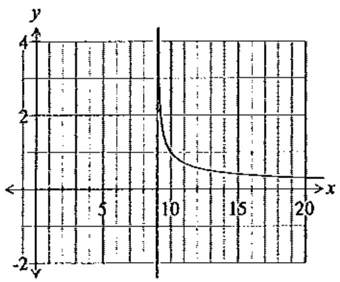
End of paper

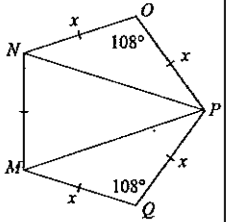
ACE Examination 2016

HSC Mathematics Yearly Examination

Worked solutions and marking guidelines

Section I		
	Solution	**Criteria**
1	$\lim\limits_{x\to7}\frac{x^2+5x-84}{x-7}=\lim\limits_{x\to7}\frac{(x+12)(x-7)}{(x-7)}$ $=\lim\limits_{x\to7}(x+12)=7+12$ $=19$	1 Mark: D
2	Positive definite (always positive – concave up) $a>0$ and $\Delta<0$	1 Mark: C
3	$\sum\limits_{r=1}^{4}2^{1-r}=2^0+2^{-1}+2^{-2}+2^{-3}$ $=1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$ $=\frac{15}{8}$	1 Mark: C
4	Range of values for $y=\cos x$ is $-1\le y\le1$ Greatest value for $4-2\cos x$ occurs when $\cos x=-1$ Therefore the greatest value is 6 $(4-2\cos x=4-2\times-1=6)$	1 Mark: C
5	$\log_e(\frac{x+2}{x})=\log_e 4$ $(\frac{x+2}{x})=4$ $x+2=4x$ $3x=2$ $x=\frac{2}{3}$	1 Mark: B
6	$y=\frac{e^{2x}+4}{x^3}$ $\frac{dy}{dx}=\frac{x^3\times2e^{2x}-(e^{2x}+4)\times3x^2}{x^6}$ $=\frac{x^2(2xe^{2x}-3e^{2x}-12)}{x^6}=\frac{2xe^{2x}-3e^{2x}-12}{x^4}$	1 Mark: B

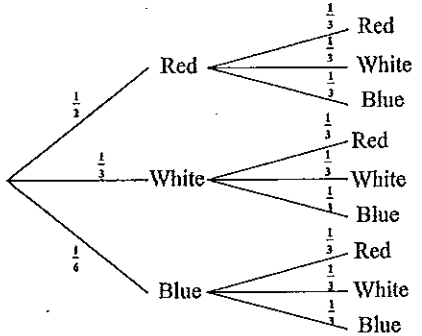
7	$2x+3=-x+3$ or $-(2x+3)=-x+3$ $3x=0$ $-2x-3=-x+3$ $x=0$ $x=-6$ Therefore there are two solutions.	1 Mark: C
8	$S=\{23, 24, 26, 32, 34, 36, 42, 43, 46, 62, 63, 64\}$ $P(\text{prime})=\frac{2}{12}$ $=\frac{1}{6}$	1 Mark: C
9	$\frac{dy}{dx}=3-\frac{2}{x^2}=3-2x^{-2}$ $y=3x+2x^{-1}+C$ Point $(1,-2)$ satisfies the equation. $-2=3\times1+2\times1^{-1}+C$ $C=-7$ $\therefore y=3x+\frac{2}{x}-7$	1 Mark: D
10	$\sqrt{x-9}\ne0$ or $x\ne9$ Also $x-9>0$ or $x>9$ Domain: $\{x:x>9\}$ $\lim\limits_{x\to\infty}\frac{1}{\sqrt{x-9}}\to0$ Range: $\{y:y>0\}$ 	1 Mark: B

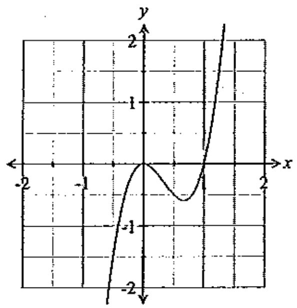
Section II		
11(a)	$\frac{5}{x-2}-\frac{2}{x-3}=\frac{5(x-3)}{(x-2)(x-3)}-\frac{2(x-2)}{(x-2)(x-3)}$ $=\frac{5x-15-2x+4}{(x-2)(x-3)}$ $=\frac{3x-11}{(x-2)(x-3)}$	2 Marks: Correct answer. 1 Mark: Finds a common denominator or shows some understanding.
11(b)	$\frac{1}{\sqrt{5}-2}=\frac{1}{\sqrt{5}-2}\times\frac{\sqrt{5}+2}{\sqrt{5}+2}$ $=\frac{\sqrt{5}+2}{5-4}$ $=\sqrt{5}+2$	2 Marks: Correct answer. 1 Mark: Multiplies by the conjugate.
11(c) (i)	Consider ΔNOP $NO=OP=x$ (given) ΔNOP is isosceles (two equal sides).  Now $\angle NOP=108^\circ$ (given) $\angle OPN=\angle ONP$ (base angles in an isosceles triangle are equal) $\angle NOP+\angle OPN+\angle ONP=180^\circ$ (Angle of a triangle is 180°) $108^\circ+\angle ONP+\angle ONP=180^\circ$ $2\times\angle ONP=180^\circ-108^\circ$ $\angle ONP=36^\circ$	2 Marks: Correct answer. 1 Mark: Shows ΔNOP is isosceles or finds $\angle ONP$.
11(c) (ii)	Consider ΔNOP and ΔPQM $OP=PQ$ (given) $\angle NOP=\angle PQM$ (given both angles are 108°) $NO=QM$ (given) $\Delta NOP\equiv\Delta PQM$ (SAS)	2 Marks: Correct answer. 1 Mark: One correct statement.
11(c) (iii)	$\angle OPQ=\angle QPM+\angle MPN+\angle OPN$ $108^\circ=36^\circ+\angle MPN+36^\circ$ $\angle MPN=36^\circ$	1 Mark: Correct answer.
11(d) (i)	Number of boxes in each layer from the top are an AP: 6,7,8... $T_n=a+(n-1)d$ $=6+(n-1)\times1$ $=6+n-1$ $=n+5$	2 Marks: Correct answer. 1 Mark: Recognises AP and uses nth term formula

11(d) (ii)	Sum the boxes in each layer. ($a=6$ and $l=n+5$) $S_n=\frac{n}{2}(a+l)$ $=\frac{n}{2}(6+n+5)$ $=\frac{1}{2}n(n+11)$	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution
11(e)	$a=3t-2$ $v=\frac{3t^2}{2}-2t+C$ When $t=0$ then $v=2$ $2=\frac{3\times0^2}{2}-2\times0+C$ or $C=2$ $v=\frac{3t^2}{2}-2t+2$ $x=\frac{t^3}{2}-t^2+2t+k$ When $t=0$ then $x=4$ $4=\frac{0^3}{2}-0^2+2\times0+k$ or $k=4$ $x=\frac{t^3}{2}-t^2+2t+4$ When $t=5$ $x=\frac{5^3}{2}-5^2+2\times5+4=51.5$ units The particle is 51.5 units to the right after 5 seconds.	2 Marks: Correct answer. 1 Mark:
12(a) (i)	$\int\frac{x}{x^2+3}dx=\frac{1}{2}\int\frac{2x}{x^2+3}dx$ $=\frac{1}{2}\log_e(x^2+3)+C$	2 Marks: Correct answer. 1 Mark: Recognises the log function as the primitive.
12(a) (ii)	$\int_0^{\frac{\pi}{3}}\cos 2x\,dx=\left[\frac{\sin 2x}{2}\right]_0^{\frac{\pi}{3}}$ $=\left[\frac{\sin\frac{2\pi}{3}}{2}\right]-\left[\frac{\sin 0}{2}\right]$ $=\frac{\sqrt{3}}{4}$	2 Marks: Correct answer. 1 Mark: Finds the primitive function or shows some understanding.

12(b) (i)	Gradient of CD: $m=\frac{y_2-y_1}{x_2-x_1}=\frac{5--1}{6--3}=\frac{6}{9}=\frac{2}{3}$	1 Mark: Correct answer.
12(b) (ii)	$y-y_1=m(x-x_1)$ $y--1=\frac{2}{3}(x--3)$ $3y+3=2x+6$ $2x-3y+3=0$	2 Marks: Correct answer. 1 Mark: Uses the point-slope formula with one correct value.
12(b) (iii)	If AB is parallel to CD then it has the same gradient or $m=\frac{2}{3}$ $m=\frac{y_2-y_1}{x_2-x_1}$ $\frac{2}{3}=\frac{-4-0}{-1-a}$ $-2-2a=-12$ $-2a=-10$ $a=5$	2 Marks: Correct answer. 1 Mark: Recognises that parallel lines have the same gradient.
12(b) (iv)	$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $BC=\sqrt{(-3--1)^2+(-1--4)^2}$ $=\sqrt{4+9}$ $=\sqrt{13}$	1 Mark: Correct answer.
12(b) (v)	$\tan\theta=\frac{2}{3}$ $\theta=\tan^{-1}\frac{2}{3}=33°41'$	1 Mark: Correct answer.
12(b) (vi)	$d=\left\|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right\|$ $=\left\|\frac{2\times(-1)-3\times(-4)+3}{\sqrt{2^2+(-3)^2}}\right\|=\left\|\frac{13}{\sqrt{13}}\right\|=\sqrt{13}$	1 Mark: Correct answer.
12(c) (i)	Initially $t=0$ $x=-t^2+6t+7$ $=-0^2+6\times0+7=7$ m Initial displacement is 7 metres	1 Mark: Correct answer.
12(c) (ii)	Particle is at the origin at $x=0$. $x=-t^2+6t+7$ $0=-(t-7)(t+1)$ $\therefore t=7$ or $t=-1$ (ignore as time is always positive) Particle is at the origin at 7 seconds.	2 Marks: Correct answer. 1 Mark: Uses $x=0$ and factorises the quadratic.

13(a) (i)	Consider ΔAOB $OA=OB=0.9$ m (radii) $AB=0.9$ m (given) ΔAOB is an equilateral triangle (three equal sides). $\angle AOB=\frac{\pi}{3}$ (all angles in an equilateral triangle are equal)	1 Mark: Correct answer.
13(a) (ii)	Area of the sector AOB $A=\frac{1}{2}r^2\theta$ $=\frac{1}{2}\times0.9^2\times\frac{\pi}{3}$ $=0.135\pi$ m^2 Area of ΔAOB $A=\frac{1}{2}ab\sin C$ $=\frac{1}{2}\times0.9\times0.9\times\sin\frac{\pi}{3}$ $=\frac{81\sqrt{3}}{400}$ m^2 Area of minor segment $A=\left(0.135\pi-\frac{81\sqrt{3}}{400}\right)$ m^2 Area of the circle $A=\pi r^2$ $=\pi\times0.9^2=0.81\pi$ m^2 Shaded area $=0.81\pi-\left(0.135\pi-\frac{81\sqrt{3}}{400}\right)$ $=0.675\pi+\frac{81\sqrt{3}}{400}$ $=2.47131533...$ ≈2.47 m^2	3 Marks: Correct answer. 2 Mark: Makes significant progress such as finding at least two of the areas 1 Mark: Finds one of the required areas.
13(b)	$\alpha+\beta=-\frac{b}{a}=-\frac{-(k-1)}{1}=k-1$ $\alpha\beta=\frac{c}{a}=\frac{2k}{1}=2k$ Therefore $k-1=2k$ $k=-1$	2 Marks: Correct answer. 1 Mark: Finds the sum or the product of the roots.
13(c) (i)	$\angle ABC=2\times27°=54°$ (diagonals of a rhombus bisect the angles through which they pass)	1 Mark: Correct answer.
13(c) (ii)	$\angle BAD+\angle ABC=180°$ $\angle BAD+54°=180°$ $\angle BAD=126°$ (co-interior angles are supplementary with $AD\parallel BC$) $\angle CDE=\angle BAD$ $=126°$ (corresponding angles are equal with $AB\parallel CD$)	2 Marks: Correct answer. 1 Mark: Correct answer with insufficient reasoning.

13(d) (i)	1st die 2nd die 	1 Mark: Correct answer.
13(d) (ii)	$P(RR)=\frac{1}{2}\times\frac{1}{3}$ $=\frac{1}{6}$	1 Mark: Correct answer.
13(d) (iii)	$P(R \text{ and } W)=P(RW)+P(WR)$ $=\frac{1}{2}\times\frac{1}{3}+\frac{1}{3}\times\frac{1}{3}$ $=\frac{5}{18}$	1 Mark: Correct answer.
13(d) (iv)	$P(E)=1-P(RR)-P(BB)$ $=1-\frac{1}{2}\times\frac{1}{3}-\frac{1}{6}\times\frac{1}{3}$ $=\frac{7}{9}$	1 Mark: Correct answer.
13(e)	$T_n=a+(n-1)d$ $T_2=a+d=37$........(1) $T_6=a+5d=17$.......(2) Equation (2) – (1) $4d=-20$ $d=-5$ Substitute $d=-5$ into Equation (1) $a-5=37$ or $a=42$ $S_n=\frac{n}{2}\left[2a+(n-1)d\right]$ $=\frac{10}{2}\left[2\times42+(10-1)\times-5\right]$ $=195$	2 Marks: Correct answer. 1 Mark: Finds the general formula for the second and sixth terms.

14(a) (i)	Stationary points $f'(x)=0$ $f'(x)=12x^2-8x$ $=4x(3x-2)$ $x=0,\ x=\frac{2}{3}$ Stationary points are $(0,0)$ and $(\frac{2}{3},-\frac{16}{27})$. $f''(x)=24x-8$ At $(0,0)$, $f''(0)=-8<0$, Maximum stationary point At $\left(\frac{2}{3},-\frac{16}{27}\right)$, $f''(\frac{2}{3})=8>0$, Minimum stationary point	3 Marks: Correct answer. 2 Marks: Finds the stationary points or makes significant progress. 1 Mark: Finds the first derivative.
14(a) (ii)	 Maxima (0,0) Minima $\left(\frac{2}{3},-\frac{16}{27}\right)$	2 Marks: Correct answer. 1 Mark: Makes some progress towards sketching the curve.
14(a) (iii)	The curve cuts the x-axis when $f(x)=0$ $f(x)=4x^3-4x^2=4x^2(x-1)=0$ The curve cuts the x-axis at $x=0$ and $x=1$	1 Mark: Correct answer.
14(a) (iv)	$f(x)>0$ when $x>1$ (see graph)	1 Mark: Correct answer.
14(b)	$3^{2x}+3^x-12=0$ $(3^x)^2+3^x-12=0$ Let $m=3^x$ $m^2+m-12=0$ $(m-3)(m+4)=0$ Therefore $m=3$ or $m=-4$ $3^x=3$ $\quad$ $3^x=-4$ $x=1$ $\quad$ No solution	2 Marks: Correct answer. 1 Mark: Recognises a quadratic equation.

14(c)	$y=\log_e x$ or $x=e^y$ $V=\pi\int_1^3 x^2 dy$ $=\pi\int_1^3 (e^y)^2 dy$ $=\frac{\pi}{2}\left[e^{2y}\right]_1^3$ $=\frac{\pi}{2}(e^6-e^2)$ cubic units	3 Marks: Correct answer. 2 Mark: Makes significant progress. 1 Mark: Uses the volume formula with at least one correct value.
14(d) (i)	$\frac{d}{dx}(e^x-3)^4=4(e^x-3)^3 e^x$ $=4e^x(e^x-3)^3$	1 Mark: Correct answer.
14(d) (ii)	$\frac{d}{dx}x\tan x=x\sec^2 x+\tan x$	1 Mark: Correct answer.
14(d) (iii)	$\frac{d}{dx}\log_e(\cos x)=\frac{1}{\cos x}\times(-\sin x)$ $=-\tan x$	1 Mark: Correct answer.
15(a) (i)	$4.8=20e^{-k\times 5}$ $e^{-5k}=0.24$ $-5k=\log_e 0.24$ $k=-\frac{\log_e 0.24}{5}\approx 0.2854232711...$	1 Mark: Correct answer.
15(a) (ii)	We need to find t when $A=1$. $1=20e^{-kt}$ $e^{-kt}=\frac{1}{20}$ $-kt=\log_e 0.05$ $t=\frac{5}{\log_e 0.24}\times\log_e 0.05=10.49575342..\approx 10$ years	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.
15(b) (i)	$y=\sin x$ (1) $y=\cos x$ (2) Equation (1) divided by equation (2) $\frac{\sin x}{\cos x}=1$ or $\tan x=1$ $x=\frac{\pi}{4},\frac{5\pi}{4},...$ Therefore $A\left(\frac{\pi}{4},\frac{1}{\sqrt{2}}\right)$ and $B\left(\frac{5\pi}{4},-\frac{1}{\sqrt{2}}\right)$	2 Marks: Correct answer. 1 Mark: Finds one value for x or shows some understanding.

15(b) (ii)	$A=\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}}[\sin x-\cos x]dx$ $=[-\cos x-\sin x]_{\frac{\pi}{4}}^{\frac{5\pi}{4}}$ $=\left(-\cos\frac{5\pi}{4}-\sin\frac{5\pi}{4}\right)-\left(-\cos\frac{\pi}{4}-\sin\frac{\pi}{4}\right)$ $=\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)-\left(-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}\right)$ $=\frac{4}{\sqrt{2}}=2\sqrt{2}$ square units	2 Marks: Correct answer. 1 Mark: Sets up the integral or shows some understanding.
15(c) (i)	$A=P(1+r)^n$ $=\$20,000(1+0.07)^{10}$ $=\$39,343.02715$ $\approx\$39,343.03$	1 Mark: Correct answer.
15(c) (ii)	$A_{10}=2,000(1.07)^{10}+2,000(1.07)^9+...+2,000$ $=2,000\times[(1.07)^{10}+(1.07)^9+...+1]$ G.P. with $a=1$, $r=1.07$ and $n=10$ $A_{10}=2,000\times\frac{1[1.07^{10}-1]}{1.07-1}$ $=\$27,632.89592...$ $\approx\$27,632.90$ Final amount $=\$39,343.03+\$27,632.90$ $=\$66,975.93$	3 Marks: Correct answer. 2 Mark: Finds the amount of the annuity or makes significant progress. 1 Mark: Identifies a G.P. with 10 terms.
15(c) (iii)	$A=P(1+r)^n$ $\$49,565=\$20,000\times(1+r)^{10}$ $(1+r)^{10}=2.47825$ $1+r=\sqrt[10]{2.47825}$ $r=\sqrt[10]{2.47825}-1$ $=0.095000989...$ $\approx 9.5\%$	2 Marks: Correct answer. 1 Mark: Uses the compound interest formula with one correct value.
15(d)	$\frac{d}{dx}\log_2 x^2=\frac{d}{dx}\left(\frac{\log_e x^2}{\log_e 2}\right)$ (change in the base rule) $=\frac{1}{\log_e 2}\times\frac{2x}{x^2}$ $=\frac{2}{x\log_e 2}$	2 Marks: Correct answer. 1 Mark: Uses the change in the base rule

Question	Solution	Marks
16(a)	$y = x\log_e x$ $\frac{dy}{dx} = x\times\frac{1}{x} + \log_e x \times 1 = 1 + \log_e x$ At $x = 1$ $y = x\log_e x = 1\times\log_e 1 = 0$ $\frac{dy}{dx} = 1+\log_e x = 1+\log_e 1 = 1$ (Gradient of the tangent) Gradient of the normal $m_1 m_2 = -1$ $m_1 \times 1 = -1$ or $m_1 = -1$ Equation of the normal $y - y_1 = m(x - x_1)$ $y - 0 = -1(x-1)$ $x + y - 1 = 0$	3 Marks: Correct answer. 2 Marks: Finds the gradient of the normal. 1 Mark: Finds the derivative of the function.
16(b) (i)	$v = \frac{dx}{dt}$ $x = \int v\,dt = \int_0^{\frac{1}{15}} v\,dt$ Since v is in km/h, the time needs to be converted from minutes to hours. Upper boundary of 4 minutes is $\frac{1}{15}$ hours.	1 Mark: Correct answer.

16(b) (ii)

Time (h)	0	$\frac{1}{60}$	$\frac{1}{30}$	$\frac{1}{20}$	$\frac{1}{15}$
Speed (km/h)	0	24	35	28	50

$\int_1^3 f(x)dx \approx \frac{h}{3}[y_0 + y_4 + 4(y_1 + y_3) + 2y_2]$

$= \frac{\frac{1}{60}}{3}[0 + 50 + 4(24+28) + 2\times 35]$

$= 1.8222222...$

≈ 1.82km

2 Marks: Correct answer.
1 Mark: Uses Simpson's rule.

Question	Solution	Marks
16(c) (i)	$16y = x^2 - 4x - 12$ $x^2 - 4x = 16y + 12$ $(x-2)^2 - 4 = 16y + 12$ $(x-2)^2 = 16(y+1)$ $(x-2)^2 = 4\times 4\times(y+1)$ Vertex is $(2,-1)$	2 Marks: Correct answer. 1 Mark: Completes the square or shows some understanding.
16(c) (ii)	The parabola is in the form $(x-a)^2 = 4a(y-k)$ (Concave up) Focal length is 4 and the focus is $(2,3)$	1 Mark: Correct answer.
16(c) (iii)	Directrix is 4 units below the vertex $y = -5$	1 Mark: Correct answer.

HSC Mathematics

Question	Solution	Marks
16(d) (i)	$V = \pi r^2 h$ $200 = \pi r^2 \times h$ $h = \frac{200}{\pi r^2}$	1 Mark: Correct answer.
16(d) (ii)	$SA = 2\pi r^2 + 2\pi r h$ $= 2\pi r^2 + 2\pi r \times \frac{200}{\pi r^2}$ $= 2\pi r^2 + \frac{400}{r}$	2 Marks: Correct answer. 1 Mark: Applies the formula for the *SA* of a cylinder
16(d) (iii)	$SA = 2\pi r^2 + \frac{400}{r}$ $\frac{dSA}{dr} = 4\pi r - 400r^{-2}$ Minimal *SA* occurs when $\frac{dSA}{dr} = 0$ $4\pi r - 400r^{-2} = 0$ $4r\left(\pi - \frac{100}{r^3}\right) = 0$ Hence $r = 0$ (no can) or $\pi - \frac{100}{r^3} = 0$ $r = \sqrt[3]{\frac{100}{\pi}}$ $= 3.169202884... \approx 3$ cm Check $\frac{d^2SA}{dr^2} = 4\pi + 800r^{-3}$ $= 4\pi + \frac{800}{r^3}$ At $r = 3$ $\frac{d^2SA}{dr^2} > 0$ and is a minima	2 Marks: Correct answer. 1 Mark: Differentiates the *SA* formula with respect to r