ACE Examinations

Student Name: ____________________

2015

YEAR 12

YEARLY EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time - 5 minutes
- Working time - 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Show relevant mathematical reasoning and/or calculations in Questions 11-14

Total marks - 70

Section I
10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II
60 marks

- Attempt Questions 11-14
- Allow about 1 hour 45 minutes for this section

STUDENT NUMBER/NAME: ....

STANDARD INTEGRALS

$\int x^n dx$	$= \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$
$\int \frac{1}{x} dx$	$= \ln x, \quad x > 0$
$\int e^{ax} dx$	$= \frac{1}{a}e^{ax}, \quad a \neq 0$
$\int \cos ax \, dx$	$= \frac{1}{a}\sin ax, \quad a \neq 0$
$\int \sin ax \, dx$	$= -\frac{1}{a}\cos ax, \quad a \neq 0$
$\int \sec^2 ax \, dx$	$= \frac{1}{a}\tan ax, \quad a \neq 0$
$\int \sec ax \tan ax \, dx$	$= \frac{1}{a}\sec ax, \quad a \neq 0$
$\int \frac{1}{a^2 + x^2} dx$	$= \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$
$\int \frac{1}{\sqrt{a^2 - x^2}} dx$	$= \sin^{-1}\frac{x}{a}, \quad a > 0, \ -a < x < a$
$\int \frac{1}{\sqrt{x^2 - a^2}} dx$	$= \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$
$\int \frac{1}{\sqrt{x^2 + a^2}} dx$	$= \ln\left(x + \sqrt{x^2 + a^2}\right)$

NOTE: $\ln x = \log_e x, \quad x > 0$

Section I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

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1 The polynomial $P(x)=x^3+2x+k$ has $(x-2)$ as a factor.
What is the value of k?

(A) -12
(B) -10
(C) 10
(D) 12

2 How many different ways can 4 people be chosen from a group of 20 people?

(A) 4,845
(B) 160,000
(C) 116,280
(D) 240,000

3 A stone is thrown at an angle of α to the horizontal. The position of the stone at time t seconds is given by $x=Vt\cos\alpha$ and $y=Vt\sin\alpha-\frac{1}{2}gt^2$ where g m/s² is the acceleration due to gravity and v m/s is the initial velocity of projection.
What is the maximum height reached by the stone?

(A) $\dfrac{V\sin\alpha}{g}$
(B) $\dfrac{g\sin\alpha}{V}$
(C) $\dfrac{V^2\sin^2\alpha}{2g}$
(D) $\dfrac{g\sin^2\alpha}{2V^2}$

HSC Mathematics Extension 1

4 The functions $y=x$ and $y=x^3$ meet at the point $(1,1)$.
What is the acute angle between the tangents to these functions at this point?

(A) 10°
(B) 27°
(C) 45°
(D) 63°

5 What is $\dfrac{d}{dx}\left(x\cos^{-1}x-\sqrt{1-x^2}\right)$?

(A) $\dfrac{-2}{\sqrt{1-x^2}}$
(B) $\dfrac{-1}{\sqrt{1-x^2}}$
(C) $\cos^{-1}x$
(D) $\sin^{-1}x$

6 The acceleration of a particle is defined in terms of its position by the equation $a=2x+4$. If $v=5$ when $x=2$, what is the velocity when $x=4$?

(A) 5 ms^{-1}
(B) 7 ms^{-1}
(C) $\sqrt{65}$ ms^{-1}
(D) $\sqrt{95}$ ms^{-1}

7 What is the domain and range of $y=2\cos^{-1}(x-1)$?

(A) Domain: $0\le x\le 2$. Range: $0\le y\le \pi$
(B) Domain: $-1\le x\le 1$. Range: $0\le y\le \pi$
(C) Domain: $0\le x\le 2$. Range: $0\le y\le 2\pi$
(D) Domain: $-1\le x\le 1$. Range: $0\le y\le 2\pi$

8 What is the value of $\int_e^{e^2}\frac{1}{x\log x}dx$? Use the substitution $u=\log_e x$.

(A) $\log_e 0.5$
(B) $\log_e 2$
(C) $\log_e 4$
(D) 1

9 At a checkpoint 6% of the vehicles are trucks. A random sample of 30 vehicles is photographed passing through the checkpoint. What is the probability that three of the 30 vehicles will be trucks?

(A) 0.100

(B) 0.165

(C) 0.195

(D) 0.216

10 What is the solution to the inequality $\frac{3}{x(2x-1)} \geq 1$?

(A) $x \leq -1,\ 0 < x < \frac{1}{2}$ and $x \geq \frac{3}{2}$

(B) $-1 \leq x < 0$ and $\frac{1}{2} < x \leq \frac{3}{2}$

(C) $-\frac{3}{2} \leq x \leq -\frac{1}{2}$ and $0 < x \leq 1$

(D) $x \leq -\frac{3}{2},\ -\frac{1}{2} \leq x < 0$ and $x \geq 1$

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hour and 45 minutes for this section

Answer each question in the appropriate writing booklet.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **Marks**

(a) Find the exact value of $\int_{-1}^{1} \sqrt{4-x^2}\,dx$, using the substitution $x = 2\sin\theta$. **3**

(b) Use one application of Newton's method with an initial approximation of $x = 1$ to find the next approximation to the root of the equation $\log_e x - \frac{1}{x}$. **2**

(c) In a class of 27 students, there are 14 boys and 13 girls.

(i) The class elects a captain and a vice captain. In how many ways is this possible? **1**

(ii) The class needs to elect two boys and two girls for the student council. How many different representatives are possible? **2**

(iii) The class elected Henry and Olivia for the student council. How many different representatives are now possible? **1**

(d) The polynomial $P(x) = x^3 + bx^2 + cx + d$ has roots 0, 3 and –3.

(i) What are the values of b, c and d? **2**

(ii) Without using calculus, sketch the graph of $y = P(x)$. **1**

(iii) Hence or otherwise, solve the inequality $\frac{x^2-9}{x} > 0$ **1**

(e) The function $f(x)$ is given by $f(x) = 4\tan^{-1} x$. Find the slope of the tangent to the curve where the function $y = f(x)$ cuts the y-axis. **2**

Question 12 (15 marks) **Marks**

(a) The point $C(-3,8)$ divides the interval AB externally in the ratio $k:1$. Find the value of k if A is the point $(6,-4)$ and B is the point $(0,4)$. **2**

(b) The point $P(2ap,ap^2)$ is on the parabola $x^2=4ay$. The normal at P cuts the x-axis at S and the y-axis at T.

(i) Prove that the equation of the normal to the parabola at P is given by $x+py=2ap+ap^3$. **2**

(ii) Hence show that S is the point $(ap(2+p^2),0)$ and that T is the point $(0,a(2+p^2))$. **1**

(iii) Find the values of p such that P is the midpoint of ST. **1**

(c) Use the substitution $x=u^2+1$ to evaluate $\int_2^{10}\frac{x}{\sqrt{x-1}}dx$. **2**

(d) (i) Show that $\sin(x+\frac{\pi}{4})=\frac{\sin x+\cos x}{\sqrt{2}}$ **2**

(ii) Hence or otherwise, solve $\frac{\sin x+\cos x}{\sqrt{2}}=\frac{\sqrt{3}}{2}$ for $0\le x\le 2\pi$. **2**

(e) After time t years the number N of animals in a national park decreases according to the equation:

$$\frac{dN}{dt}=-0.09(N-100)$$

The initial number of animals in the national park is 500.

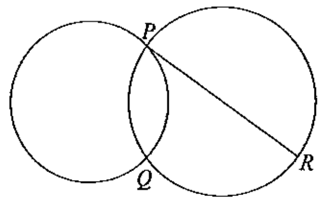
(i) Verify that $N=100+Ae^{-0.09t}$ is a solution of the above equation, where A is a constant. **1**

(ii) After one year the number of animals in the natural park is 400. Find the time taken for the number of animals to reach 200. Answer correct to three significant figures. **2**

Question 13 (15 marks) **Marks**

(a) Use mathematical induction to prove that 3^n-2n-1 is divisible by 4 for all positive integers greater than 1. **3**

(b) Two circles are intersecting at P and Q. The diameter of one circle is PR.



Copy this diagram into your writing booklet.

(i) Draw a straight line through P, parallel to QR to meet the second circle at S. Prove that QS is a diameter of the second circle. **2**

(ii) Prove that the circles have equal radii if TQ is parallel to PR. **2**

(c) Solve the equation $(n+2)!=72n!$ **2**

(d) James plays a game where three regular, six-sided dice are thrown once.

(i) What is the probability that all three dice show a 6? **1**

(ii) What is the probability that exactly two of the dice show a 6? **1**

(iii) What is the probability that exactly two of the dice show the same number? **1**

(e) A rock is projected horizontally from the top of a 25 metre high cliff. The rock is thrown with an initial velocity of 40 ms^{-1}. Assume $g=10$ ms^{-1}.

(i) Determine the parametric equations of the path. Take the origin at the base of the cliff. **2**

(ii) How far from the base of the cliff does the rock hit the sea? **1**

Question 14 (15 marks) **Marks**

(a) Find the term independent of x in the expansion of $(x^2-\frac{2}{x})^9$. **2**

(b) The velocity of a particle moving in a straight line is given by $v=10-x$ where x metres is the distance from fixed point O and v is the velocity in metres per second. Initially the particle is at O.

(i) Let a be the acceleration in metres per second squared. Find an expression for a in terms of x. **1**

(ii) Show that $x=10-10e^{-t}$ by integration. **3**

(iii) What is the limiting position of the particle? **1**

(c) A particle moves in a straight line under simple harmonic motion. Its displacement (x metres) from a fixed point O at any time (t seconds) is given by: $x=4\cos^2 t-1$.

(i) Find a expression for acceleration in terms of x. **2**

(ii) Sketch $x=4\cos^2 t-1$ for $0\le t\le\pi$. Clearly show the times when the particle passes through O. **2**

(iii) Find the time when the velocity of the particle is increasing most rapidly for $0\le t\le\pi$. **1**

(d) (i) Expand $(1-x)^{2n}$ using the binomial theorem. **1**

(ii) Hence prove the following identity. **2**

$${}^{2n}C_1+3\,{}^{2n}C_3+\ldots+(2n-1)\,{}^{2n}C_{2n-1}=2\,{}^{2n}C_2+4\,{}^{2n}C_4+\ldots+2n\,{}^{2n}C_{2n}$$

End of paper

ACE Examination 2015

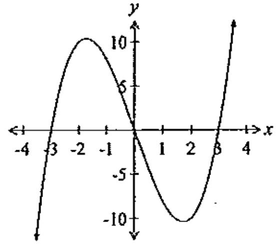
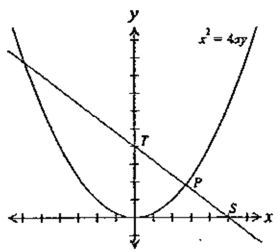
HSC Mathematics Extension 1 Yearly Examination

Worked solutions and marking guidelines

Section I

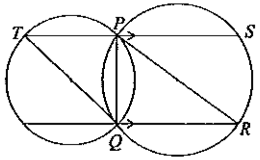
	Solution	Criteria
1	$P(x)=x^3+2x+k$ $P(2)=2^3+2\times 2+k=0$ ($x-2$ is a factor of $P(x)$) $8+4+k=0$ $k=-12$	1 Mark: A
2	${}^{20}C_4=4,845$	1 Mark: A
3	$y=Vt\sin\alpha-\frac{1}{2}gt^2$ $\dot{y}=V\sin\alpha-gt$ Maximum height when $\dot{y}=0$ $0=V\sin\alpha-gt$ $t=\frac{V\sin\alpha}{g}$ Maximum height $h=V\sin\alpha\times\frac{V\sin\alpha}{g}-\frac{1}{2}g\times\left(\frac{V\sin\alpha}{g}\right)^2$ $=\frac{V^2\sin^2\alpha}{2g}$	1 Mark: C
4	$y=x$ $m_1=1$ $y=x^3$, $\frac{dy}{dx}=3x^2$ At (1,1) $m_2=3$ $\tan\theta=\left\|\frac{m_1-m_2}{1+m_1m_2}\right\|$ $=\left\|\frac{1-3}{1+1\times 3}\right\|$ $=\frac{2}{4}=\frac{1}{2}$ $\theta=26.56505118...\approx 27^\circ$	1 Mark: B
5	$\frac{d}{dx}\left(x\cos^{-1}x-\sqrt{1-x^2}\right)=x\times\frac{-1}{\sqrt{1-x^2}}+\cos^{-1}x-\frac{1}{2\sqrt{1-x^2}}\times -2x$ $=\cos^{-1}x$	1 Mark: C
6	$a=2x+4$ $v^2=2\int(2x+4)\,dx$ $=2x^2+8x+c$ When $x=2$, $v=5$ then $c=1$ $v^2=2x^2+8x+1$ $v=\sqrt{2x^2+8x+1}$ (conditions indicate positive solution) When $x=4$ $v=\sqrt{2\times 4^2+8\times 4+1}$ $=\sqrt{65}$	1 Mark: C
7	Domain: $-1\le(x-1)\le 1$ or $0\le x\le 2$. Range: $0\le\cos^{-1}(x-1)\le\pi$ or $0\le y\le 2\pi$	1 Mark: C
8	$u=\log_e x$ and $du=\frac{1}{x}dx$ $\quad u=\log_e e=1$ $\quad u=\log_e e^2=2$ $\int_e^{e^2}\frac{1}{x\log x}dx=\int_1^2\frac{1}{u}du$ $=\left[\log_e u\right]_1^2$ $=\log_e 2-\log_e 1$ $=\log_e 2$	1 Mark: B
9	P(3 heavy) $={}^{30}C_3(0.94)^{27}(0.06)^3$ $=0.164980...$ ≈ 0.165	1 Mark: B
10	$\frac{3}{x(2x-1)}\ge 1$ $\quad x\ne 0$ or $x\ne\frac{1}{2}$ $x^2(2x-1)^2\times\frac{3}{x(2x-1)}\ge 1\times x^2(2x-1)^2$ $3x(2x-1)\ge x^2(2x-1)^2$ $\quad x\ne 0$ and $x\ne 2$ $3x(2x-1)-x^2(2x-1)^2\ge 0$ $x(2x-1)[3-x(2x-1)]\ge 0$ $x(2x-1)(2x^2-x-3)]\le 0$ $x(2x-1)(2x-3)(x+1)\le 0$ Critical points are -1, 0, $\frac{1}{2}$ and $\frac{3}{2}$ Test values in each region $-1\le x<0$ and $\frac{1}{2}<x\le\frac{3}{2}$	1 Mark: B

Section II		
	Solution	**Criteria**
11(a)	$x=2\sin\theta,\quad \frac{dx}{d\theta}=2\cos\theta,\quad dx=2\cos\theta\, d\theta$ When $x=1$ $1=2\sin\theta,\ \sin\theta=\frac{1}{2}$ or $\theta=\frac{\pi}{6}$ When $x=-1$ $-1=2\sin\theta,\ \sin\theta=-\frac{1}{2}$ or $\theta=-\frac{\pi}{6}$ $\int_{-1}^{1}\sqrt{4-x^2}\,dx=\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\sqrt{4-4\sin^2\theta}\times 2\cos\theta\, d\theta$ $=4\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\cos^2\theta\, d\theta$ $=8\left[\frac{1}{2}\left(\theta+\frac{1}{2}\sin 2\theta\right)\right]_0^{\frac{\pi}{6}}=\frac{2\pi}{3}+\sqrt{3}$	3 Marks: Correct answer. 2 Marks: Uses the substitution and simplifies the integral. 1 Mark: Adjusts the limits and finds dx.
11(b)	$f(x)=\log_e x-x^{-1}$ $\quad f'(x)=\frac{1}{x}+\frac{1}{x^2}$ $f(1)=\log_e 1-\frac{1}{1}=-1$ $\quad f'(1)=\frac{1}{1}+\frac{1}{1^2}=2$ $x_1=x_0-\frac{f(x_0)}{f'(x_0)}$ $=1-\left(\frac{-1}{2}\right)=1.5$	2 Marks: Correct answer. 1 Mark: Finds $f(1)$, $f'(1)$ or shows some understanding of Newton's method.
11(c) (i)	Order is important. (Select the captain then vice-captain) ${}^{27}P_2=702$	1 Mark: Correct answer.
11(c) (ii)	Order is not important ${}^{14}C_2\times{}^{13}C_2=91\times 78$ $=7098$	2 Marks: Correct answer. 1 Mark: Shows understanding
11(c) (iii)	Order is not important ${}^{13}C_1\times{}^{12}C_1=13\times 12$ $=156$	1 Mark: Correct answer.
11(d) (i)	$P(x)=x^3+bx^2+cx+d$ $P(0)=0^3+b\times 0^2+c\times 0+d=0,\ \therefore d=0$ $P(3)=3^3+b\times 3^2+c\times 3=0,\ \therefore 27+9b+3c=0$ $P(-3)=(-3)^3+b\times(-3)^2+c\times(-3)=0,\ \therefore -27+9b-3c=0$ By inspection $b=0$ and $c=-9$ Therefore $b=0$, $c=-9$ and $d=0$	2 Marks: Correct answer. 1 Mark: Uses the factor theorem or shows some understanding.

11(d) (ii)	$y=x^3-9x=x(x^2-9)$ 	1 Mark: Correct answer.
11(d) (iii)	$x^2\times\left(\frac{x^2-9}{x}\right)>0\times x^2$ $x(x^2-9)>0$ From the graph the solution is $-3<x<0$ or $x>3$	1 Mark: Correct answer.
11(e)	$f(x)=4\tan^{-1}x$ $f'(x)=\frac{4}{1+x^2}$ The curve cuts the y-axis when $x=0$ $f'(0)=\frac{4}{1+0^2}=4$ Slope of the tangent is 4.	2 Marks: Correct answer. 1 Mark: Differentiates the inverse function.
12(a)	$x=\frac{mx_2+nx_1}{m+n}$ or $y=\frac{my_2+ny_1}{m+n}$ $-3=\frac{k\times 0+1\times 6}{k+1}$ $\quad 8=\frac{k\times 4+1\times -4}{k+1}$ $-3(k+1)=6$ $\quad 8(k+1)=4k-4$ $k=-3$ $\quad k=-3$	2 Marks: Correct answer. 1 Mark: Uses the ratio formula with one correct value.
12(b) (i)		2 Marks: Correct answer. 1 Mark: Finds the gradient of the tangent or shows some understanding of the problem.

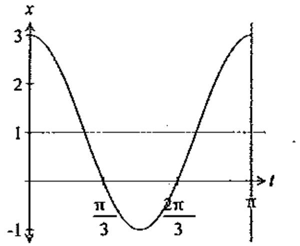
	To find the gradient of the normal $y=\frac{1}{4a}x^2$ and $\frac{dy}{dx}=\frac{1}{2a}x$ Gradient of the tangent at $P(2ap,ap^2)$ $\frac{dy}{dx}=\frac{1}{2a}\times 2ap=p$, Gradient of normal at $P(2ap,ap^2)$ $m=-\frac{1}{p}$ $(m_1m_2=-1)$ Equation of the normal at $P(2ap,ap^2)$ $y-y_1=m(x-x_1)$ $y-ap^2=-\frac{1}{p}(x-2ap)$ $py-ap^3=-x+2ap$ $x+py=2ap+ap^3$	
12(b) (ii)	S is on the x-axis ($y=0$) $x+p\times 0=2ap+ap^3$ $x=ap(2+p^2)$ S is the point $(ap(2+p^2),0)$ T is on the y-axis ($x=0$) $0+py=2ap+ap^3$ $y=a(2+p^2)$ T is the point $(0,a(2+p^2))$	1 Mark: Correct answer.
12(b) (iii)	P is the midpoint of ST $x=\frac{x_1+x_2}{2}$ $2ap=\frac{ap(2+p^2)+0}{2}$ $4ap=2ap+ap^3$ $ap(p^2-2)=0$ $p=0,\ p=\pm\sqrt{2}$ $\therefore p=\pm\sqrt{2}$ or $y=\frac{y_1+y_2}{2}$ $ap^2=\frac{0+a(2+p^2)}{2}$ $2ap^2=2a+ap^2$ $a(p^2-2)=0$ $\therefore p=\pm\sqrt{2}$.	1 Mark: Correct answer.
12(c)	$x=u^2+1$ or $u=\sqrt{x-1}$ when $x=10$ then $u=3$ $x=2$ then $u=1$ $\frac{dx}{du}=2u$ or $dx=2udu$ $\int_2^{10}\frac{x}{\sqrt{x-1}}dx=\int_1^3\frac{(u^2+1)\times 2u}{u}du$ $=\int_1^3 2u^2+2du$ $=\left[\frac{2}{3}u^3+2u\right]_1^3$ $=\left(\frac{2}{3}\times 27+6\right)-\left(\frac{2}{3}+2\right)=\frac{64}{3}=21\frac{1}{3}$	2 Marks: Correct answer. 1 Mark: Sets up the integration using the substitution

12(d) (i)	$\text{LHS}=\sin(x+\frac{\pi}{4})$ $=\sin x\cos\frac{\pi}{4}+\cos x\sin\frac{\pi}{4}$ $=\sin x\times\frac{1}{\sqrt{2}}+\cos x\times\frac{1}{\sqrt{2}}$ $=\frac{\sin x+\cos x}{\sqrt{2}}$ $=\text{RHS}$	2 Marks: Correct answer. 1 Mark: Uses the sum of angles formula or exact values.
12(d) (ii)	$\sin(x+\frac{\pi}{4})=\frac{\sqrt{3}}{2}$ $x+\frac{\pi}{4}=\frac{\pi}{3}$ or $\frac{2\pi}{3}$ $x=\frac{\pi}{12}$ or $\frac{5\pi}{12}$	2 Marks: Correct answer. 1 Mark: Finds one solution or shows some understanding.
12(e) (i)	$N=100+Ae^{-0.09t}$ $\frac{dN}{dt}=-0.09\times Ae^{-0.09t}$ $=-0.09(N-100)$	1 Mark: Correct answer.
12(e) (ii)	When $t=1$ then $N=400$ $400=100+Ae^{-0.09\times 1}$ $Ae^{-0.09\times 1}=300$ $A=\frac{300}{e^{-0.09}}=328.252285...$ We need to find t when $N=200$ $200=100+328.25...e^{-0.09t}$ $e^{-0.09t}=\frac{100}{328.25...}=0.3046...$ $t=\frac{\log_e 0.3064...}{-0.09}$ $=13.206803...$ ≈ 13.2 years	2 Marks: Correct answer. 1 Mark: Finds the value of A or shows similar understanding of the problem.

13(a)	Step 1: To prove the statement true for $n=2$ $3^n-2n-1=3^2-2\times2-1=4$ Divisible by 4 Result is true for $n=2$ Step 2: Assume the result true for $n=k$ $3^k-2k-1=4P$ (1) where P is an integer. To prove the result is true for $n=k+1$ $3^{k+1}-2(k+1)-1=4Q$ where Q is an integer. $\text{LHS}=3^{k+1}-2(k+1)-1$ $=3\times3^k-2k-2-1$ $=3\times(3^k-2k-1)+4k$ $=3(4P)+4k$ from (1) $=4(3P+k)$ $=4Q$ $=\text{RHS}$ Q is an integer as P and k are integers. Result is true for $n=k+1$ if true for $n=k$ Step 3: Result true by principle of mathematical induction.	3 Marks: Correct answer. 2 Marks: Proves the result true for $n=2$ and attempts to use the result of $n=k$ to prove the result for $n=k+1$. 1 Mark: Proves the result true for $n=2$.
13(b) (i)	 $\angle PQR=90^\circ$ (angle in a semicircle is 90°) $\angle SPQ+\angle PQR=180^\circ$ (cointerior angles, $PS//QR$) $\therefore\angle SPQ=90^\circ$ QS is a diameter ($\angle SPQ$ is an angle in a semicircle equal to 90°)	2 Marks: Correct answer. 1 Mark: States the angle in a semicircle is 90° or makes some progress towards the solution.
13(b) (ii)	$\angle PQR=\angle TPQ=90^\circ$ (alternate angles are equal, $PS//QR$) QT is a diameter ($\angle TPQ$ is an angle in a semicircle equal to 90°) If $TQ//PR$ then $PRQT$ is a parallelogram. $PR=QT$ (opposite sides of a parallelogram are equal) QT and PR are equal diameters. Therefore the two circles have equal radii.	2 Marks: Correct answer. 1 Mark: Shows some understanding.

13(c)	$(n+2)!=72n!$ $(n+2)(n+1)n!=72n!$ $n^2+3n+2=72$ $n^2+3n-70=0$ $(n-7)(n+10)=0$ $\therefore n=7\ (n\ge0)$	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.
13(d) (i)	$P(666)=\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}=\frac{1}{216}$	1 Mark: Correct answer.
13(d) (ii)	$P(E)=P(66X)+P(6X6)+P(X66)$ $=3\times\left(\frac{1}{6}\times\frac{1}{6}\times\frac{5}{6}\right)$ $=\frac{5}{72}$	1 Mark: Correct answer.
13(d) (iii)	Probability that exactly two of the dice show a 6 is the same as exactly two of the dice show a 5,.... $P(E)=6\times\frac{5}{72}=\frac{5}{12}$	1 Mark: Correct answer.
13(e) (i)	Horizontal $\ddot{x}=0$ $\dot{x}=V\cos\theta$ When $t=0$, $V=40$, $\theta=0$ implies $V\cos\theta=40$ $\dot{x}=40$ $x=40t+C$ When $t=0$, $x=0$ implies $C=0$ $x=40t$ Vertical $\ddot{y}=-10$ $\dot{y}=-10t+V\sin\theta$ When $t=0$, $V=0$, $\theta=0$ implies $V\sin\theta=0$ $\dot{y}=-10t$ $y=-5t^2+C$ When $t=0$, $y=25$ implies $C=25$ $y=-5t^2+25$	2 Marks: Correct answer. 1 Mark: Finds horizontal or vertical parametric equations or shows some understanding of the problem.
13(e) (ii)	Particle reaches the sea when $y=0$ $0=-5t^2+25$ or $t=\sqrt{5}$ $(t>0)$ Horizontal distance from the base of the cliff. $x=40t$ $=40\sqrt{5}$ m	1 Mark: Correct answer.

14(a)	$T_{r+1} = {}^9C_r(x^2)^{9-r}\left(-\frac{2}{x}\right)^r$ $= {}^9C_r x^{18-2r}(-2)^r x^{-r}$ $= {}^9C_r(-2)^r x^{18-3r}$ Term independent of x $18-3r=0$ $r=6$ $T_7 = {}^9C_6(-2)^6 x^{18-3\times6}$ $= {}^9C_6(-2)^6$ $=5376$	2 Marks: Correct answer. 1 Mark: Correctly uses the general term.
14(b) (i)	$v=10-x$ $v^2=100-20x+x^2$ $\frac{1}{2}v^2=50-10x+\frac{1}{2}x^2$ $a=\frac{d}{dx}\left(50-10x+\frac{1}{2}x^2\right)$ $=x-10$	1 Mark: Correct answer.
14(b) (ii)	$\frac{dx}{dt}=10-x$ $\frac{dt}{dx}=\frac{1}{10-x}$ $t=-\log_e(10-x)+C$ Initially $t=0$ and $x=0$ $0=-\log_e(10-0)+C$ or $C=\log_e 10$ $t=-\log_e(10-x)+\log_e 10$ $=\log_e\left(\frac{10}{10-x}\right)$ $e^t=\frac{10}{10-x}$ $e^{-t}=\frac{10-x}{10}$ $x=10-10e^{-t}$	3 Marks: Correct answer. 2 Marks: Finds the constant or makes significant progress. 1 Mark: Finds t in terms of x.
14(b) (iii)	$\lim_{t\to\infty}(10-10e^{-t})=10-10\times0$ $=10$ metres to the right	1 Mark: Correct answer.

14(c) (i)	$x=4\cos^2 t-1$ $x=2(\cos 2t+1)-1$ $=2\cos 2t+1$ $\dot{x}=-4\sin 2t$ $\ddot{x}=-4\times 2\cos 2t$ $=-4(x-1)$	2 Marks: Correct answer. 1 Mark: Shows some understanding.
14(c) (ii)	$x=4\cos^2 t-1$ or $x=2\cos 2t+1$ Stationary points $\dot{x}=0$, $\therefore -4\sin 2t=0$ or $t=0,\frac{\pi}{2},\pi,\ldots$ When $t=0$ $x=2\cos 2\times0+1=3$ When $t=\pi$ $x=2\cos 2\times\pi+1=3$ When $t=\frac{\pi}{2}$ $x=2\cos 2\times\frac{\pi}{2}+1=-1$ 	2 Marks: Correct answer. 1 Mark: Correct shape of the curve or shows some understanding.
14(c) (iii)	Velocity is increasing most rapidly when $\ddot{x}$ has the greatest positive value (or x takes the least value). Greatest value: $\ddot{x}=-8\cos 2t=8$ when $t=\frac{\pi}{2}$ Velocity is increasing most rapidly at $\frac{\pi}{2}$ seconds.	1 Mark: Correct answer.
14(d) (i)	$(1-x)^{2n}=1-{}^{2n}C_1x+{}^{2n}C_2x^2+\ldots+(-1)^r\,{}^{2n}C_rx^r\ldots+{}^{2n}C_{2n}x^{2n}$	1 Mark: Correct answer.
14(d) (ii)	Differentiating both sides. $-2n(1-x)^{2n-1}=-{}^{2n}C_1+2\,{}^{2n}C_2x+..+(-1)^r r\,{}^{2n}C_rx^{r-1}..+2n\,{}^{2n}C_{2n}x^{2n-1}$ Substitute $x=1$ $0=-{}^{2n}C_1+2\,{}^{2n}C_2+\ldots+(-1)^r r\,{}^{2n}C_r\ldots+2n\,{}^{2n}C_{2n}$ $\therefore {}^{2n}C_1+3\,{}^{2n}C_3+\ldots+(2n-1)\,{}^{2n}C_{2n-1}=2\,{}^{2n}C_2+4\,{}^{2n}C_4+\ldots+2n\,{}^{2n}C_{2n}$	2 Marks: Correct answer. 1 Mark: Differentiates both sides.