ACE Examinations

Student Name: ____________________

2015

YEAR 12

YEARLY EXAMINATION

Mathematics

General Instructions

- Reading time - 5 minutes
- Working time - 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Show relevant mathematical reasoning and/or calculations in Questions 11-16

Total marks - 100

Section I

10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II

90 marks

- Attempt Questions 11-16
- Allow about 2 hour 45 minutes for this section

STUDENT NUMBBR/NAME: ....

STANDARD INTEGRALS

$\int x^n dx$	$= \frac{1}{n+1}x^{n+1},\quad n \neq -1;\quad x \neq 0,\ \text{if } n < 0$
$\int \frac{1}{x} dx$	$= \ln x,\quad x > 0$
$\int e^{ax} dx$	$= \frac{1}{a}e^{ax},\quad a \neq 0$
$\int \cos ax\, dx$	$= \frac{1}{a}\sin ax,\quad a \neq 0$
$\int \sin ax\, dx$	$= -\frac{1}{a}\cos ax,\quad a \neq 0$
$\int \sec^2 ax\, dx$	$= \frac{1}{a}\tan ax,\quad a \neq 0$
$\int \sec ax \tan ax\, dx$	$= \frac{1}{a}\sec ax,\quad a \neq 0$
$\int \frac{1}{a^2+x^2} dx$	$= \frac{1}{a}\tan^{-1}\frac{x}{a},\quad a \neq 0$
$\int \frac{1}{\sqrt{a^2-x^2}} dx$	$= \sin^{-1}\frac{x}{a},\quad a > 0,\ -a < x < a$
$\int \frac{1}{\sqrt{x^2-a^2}} dx$	$= \ln\left(x+\sqrt{x^2-a^2}\right),\quad x > a > 0$
$\int \frac{1}{\sqrt{x^2+a^2}} dx$	$= \ln\left(x+\sqrt{x^2+a^2}\right)$

NOTE: $\ln x = \log_e x,\quad x > 0$

Section I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

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1 What is the value of $\lim_{x \to 3} \frac{x^2+2x-15}{x-3}$?

(A) 0
(B) 3
(C) 5
(D) 8

2 What is the solution to the equation $2\cos^2 x - 1 = 0$ in the domain $0 \le x \le 2\pi$?

(A) $x = \frac{\pi}{6}, \frac{11\pi}{6}$

(B) $x = \frac{\pi}{4}, \frac{7\pi}{4}$

(C) $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

(D) $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

3 An infinite geometric series has a first term of 3 and a limiting sum of 1.8.
What is the common ratio?

(A) $-0.\dot{3}$
(B) $-0.\dot{6}$
(C) -1.5
(D) -3.75

4 What is the value of $\int_0^1 (e^{3x}+1)\,dx$?

(A) $\frac{1}{3}e^3$ (B) e^3

(C) $\frac{1}{3}(e^3+1)$ (D) $\frac{1}{3}(e^3+2)$

5 What is the value of $f'(x)$ if $f(x) = \sqrt{(2x^2+1)^3}$?

(A) $f'(x) = \frac{3}{2}\sqrt{2x^2+1}$

(B) $f'(x) = 4x\sqrt{2x^2+1}$

(C) $f'(x) = 6x\sqrt{2x^2+1}$

(D) $f'(x) = 12x\sqrt{2x^2+1}$

6 What is the equation of the tangent to the curve $y = \cos x$ at the point $\left(\frac{\pi}{2}, 0\right)$?

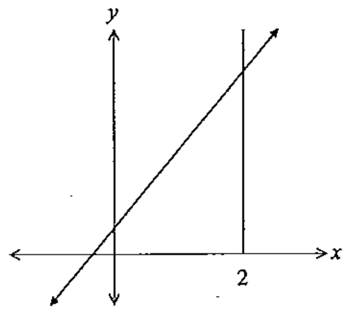
(A) $x - y - \frac{\pi}{2} = 0$

(B) $x + y - \frac{\pi}{2} = 0$

(C) $y = 0$

(D) $2x + y - \pi = 0$

7 A region in the first quadrant is bounded by the line $y = 3x+1$, the x-axis, the y-axis, and the line $x = 2$.



What is the volume of the solid of revolution formed when this region is rotated about the x-axis?

(A) 8 units^3
(B) 38 units^3
(C) 8π units^3
(D) 38π units^3

8 The acceleration of a particle moving in a straight line is given by the formula $a = 12t + 6$. Initially the particle is at $x = 5$ metres and the initial velocity of the particle is –36 m/s. When is the particle at rest?

(A) $t = 0$

(B) $t = 1$

(C) $t = 2$

(D) $t = 3$

9 What are the coordinates of the focus of the parabola $x^2 = 6y + 2x + 11$?

(A) $\left(-\frac{3}{2}, 1\right)$

(B) $\left(-\frac{1}{2}, 1\right)$

(C) $\left(1, -\frac{3}{2}\right)$

(D) $\left(1, -\frac{1}{2}\right)$

10 What is the value of $[f(x)]^2$ if $f(x) = 4 + 2^{-x}$?

(A) $16 + 2^{3-x} + 2^{-2x}$

(B) $16 + 2^{2-x} + 2^{-2x}$

(C) $17 + 2^{3-x}$

(D) $17 + 2^{2-x}$

Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **Marks**

(a) Simplify $\frac{y}{y^2 - 4} - \frac{2}{y - 2}$ **2**

(b) An insurance company has calculated that the probability of a woman being alive in 50 years time is 0.85 and the probability of her husband being alive in 50 years time is 0.75. What is the probability that in 50 years time:

(i) both will be alive; **1**

(ii) only one of them will be alive? **1**

(c) For the arithmetic sequence 4, 9, 14, 19, ….

(i) Write the rule to describe the nth term. **1**

(ii) What is the 25th term? **1**

(iii) Find the sum of the first 100 terms. **1**

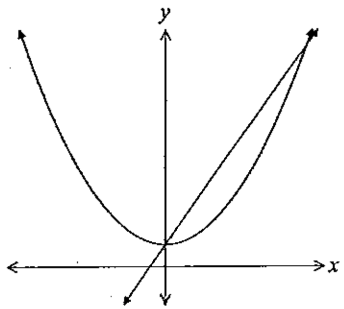
(d) Differentiate

(i) $\tan 5x$ **1**

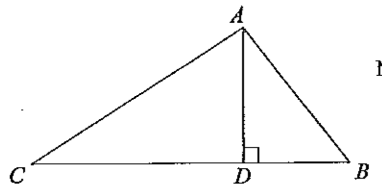
(ii) $\frac{\log_e x}{x}$ **1**

(iii) $x \cos x$ **1**

(e) What is the area enclosed between the curves $y = x^2 + 1$ and $y = 3x + 1$? **2**



(f) In the triangle ABC, $\angle ACB = 30^\circ$, $\angle ABC = 50^\circ$ and $BC = 10$ cm.
The foot of the perpendicular from A to BC is D.



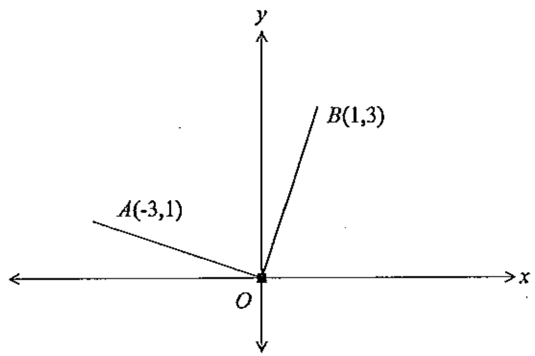
Not to scale

(i) Use the Sine Rule to find an expression for the length of AB. **2**

(ii) Hence or otherwise, find the length of AD. **1**
Answer correct to two decimal places.

Question 12 (15marks) **Marks**

(a) Points $A(-3,1)$ and $B(1,3)$ are on a number plane.
Copy the diagram into your writing booklet.



(i) Find the gradient of line OA. **1**

(ii) Show that OA is perpendicular to OB. **1**

(iii) $OACD$ is a quadrilateral in which BC is parallel to OA. **2**
Show that the equation of BC is $x + 3y - 10 = 0$.

(iv) The point C lies on the line $x = -2$. **1**
What are the coordinates of point C?

(v) Show that the length of the line BC is $\sqrt{10}$. **1**

(vi) Find the area of $OACD$. **1**

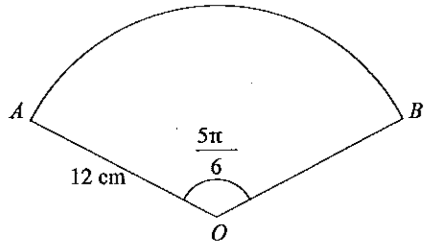
(b) The table shows the values of a function $f(x)$ for five values of x. **2**

x	1	1.5	2	2.5	3
$f(x)$	4	1.5	–2	2.5	8

Use Simpson's rule with these five vales to estimate $\int_1^3 f(x)\,dx$.

(c) Two-digit numbers are formed from the digits 2, 3, 4, 6 with no repetition of digits allowed. A two-digit number is then selected at random. What is the probability the number is prime? **2**

(d)



In the diagram, AB is an arc of a circle with centre O. **1**

The radius $OA = 12$ cm and the angle AOB is $\frac{5\pi}{6}$ radians.

Find the length of the arc AB.

(e) Consider the functions $y = x^2$ and $y = x^2 - 3x + 2$.

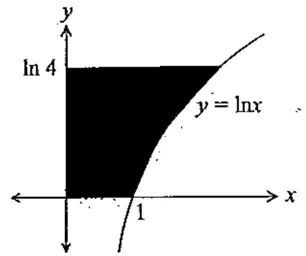
(i) Sketch the two functions on the same axes. **2**

(ii) Hence or otherwise find the values of x such that $x^2 > (x-1)(x-2)$. **1**

Question 13 (15 marks) **Marks**

(a) In the diagram, the shaded region bounded by the curve $y = \ln x$, x-axis, y-axis and the line $y = \ln 4$, is rotated about the y-axis. **3**

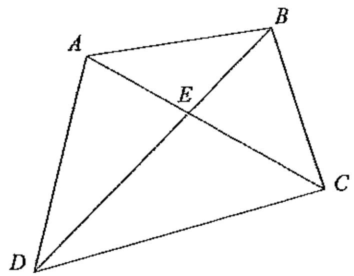
Find the exact volume of the solid of revolution.



(b) Evaluate $\int_0^{\frac{\pi}{6}} (x^2 + \sin 2x)\,dx$ **2**

(c) In quadrilateral $ABCD$ the diagonals AC and BD intersect at E.

Given $AE = 3$, $EC = 6$, $BE = 4$ and $ED = 8$.



Not to scale

(i) Show that $\Delta ABE \parallel\!| \Delta DEC$ **3**

(ii) What type of quadrilateral is $ABCD$? Justify your answer. **2**

(d) Find the shortest distance between the point $(0,5)$ and the line $3x - y + 1 = 0$. **2**

(e) The parabola $y = ax^2 + bx + c$ has a vertex at (3, 1) and passes through (0, 0).

(i) Find the other x-intercept of the parabola. **1**

(ii) Find a, b and c. **2**

Question 14 (15 marks) **Marks**

(a) A function $f(x)$ is defined by $f(x) = x^2(3-x)$.

(i) Find the stationary points for the curve $y = f(x)$ and determine their nature. **2**

(ii) Sketch the graph of $y = f(x)$ showing the stationary points and x-intercepts. **2**

(iii) Find the equation of the tangent to the curve at the point $P(1,2)$. **1**

(b) The quadratic equation $2m^2 - 3m + 6 = 0$ has roots α and β.

Find the value of:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\alpha^2 + \beta^2$ **1**

(c) The displacement of a object at time (t) seconds is given by: **3**

$$x = 3e^{-2t} + 10e^{-t} + 4t$$

Find the time the object comes to rest.

(d) The third and seventh terms of a geometric series are 1.25 and 20 respectively. What is the first term? **2**

(e) Find $\int_0^{\frac{\pi}{8}} \sec^2 2x\,dx$. **2**

Question 15 (15 marks) **Marks**

(a) (i) On the same set of axes, sketch the graphs of $y = \sin x$ and $y = 1 - \cos x$ over the domain $0 \le \theta \le \pi$. **2**

(ii) Write down the values of x for which $\sin x = 1 - \cos x$ in the domain $0 \le \theta \le \pi$ **1**

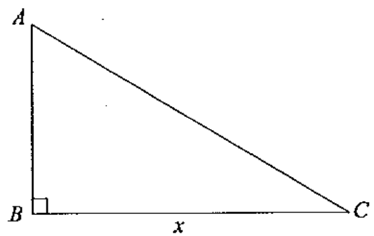
(iii) Find the area between of $y = \sin x$ and $y = 1 - \cos x$ over the domain $0 \le \theta \le \pi$. **2**

(b) Scarlett left \$1000 in her will for World Vision. Her instructions were that this money be invested at 5% interest, compounded annually.

(i) How much money would be given to World Vision after 100 years? **1**

(ii) Scarlett has requested her family invest a further \$1000 at the beginning of each subsequent year at the same interest rate. **2**
How much money would be given to World Vision after 100 years if her family followed Scarlett's instructions?

(c)



A cable of length 3 metres is to be bent to form the hypotenuse and base of a right-angled triangle ABC. Let the length of the base BC is x metres.

(i) What is the length of the hypotenuse AC in terms of x? **1**

(ii) Show that the area of the triangle ABC is $0.5x\sqrt{9-6x}$. **2**

(iii) What value of x gives the maximum possible area of the triangle? **3**

(iv) Find the maximum possible area of the triangle. **1**

Question 16 (15 marks) **Marks**

(a) An object is moving in a straight line and its velocity is given by;

$$v = 1 - 2\sin 2t \quad \text{for } t \geq 0$$

where v is measured in metres per second and t in seconds.
Initially the object is at the origin.

(i) Find the displacement x, as a function of t. **2**

(ii) What is the position of the object when $t = \frac{\pi}{3}$? **1**

(iii) Find the acceleration a, as a function of t. **1**

(iv) Sketch the graph of a, as a function of t, for $0 \leq t \leq \pi$. **1**

(v) What is the maximum acceleration of the object? **1**

(b) The radiation in a rock after a nuclear accident was 8,000 becquerel (bq). One year later, the radiation in the rock was 7,000 bq. It is known that the radiation in the rock is given by the formula:

$$R = R_0 e^{-kt}.$$

where R_0 and k are constants and t is the time measured in years.

(i) Evaluate the constants R_0 and k. **2**

(ii) What is the radiation of the rock after 10 years? **1**
Answer correct to the nearest whole number.

(iii) The region will become safe when the radiation of the rock reaches 50 bq. After how many years will the region become safe? **2**

(c) A triangle ABC is right-angled at C. D is the point on AB such that CD is perpendicular to AB. Let $\angle BAC = \theta$.
Draw a diagram showing this information.

(i) Given that $8AD + 2BC = 7AB$.
Show that $8\cos\theta + 2\tan\theta = 7\sec\theta$ **2**

(ii) Find θ **2**

End of paper

ACE Examination 2015

HSC Mathematics Yearly Examination

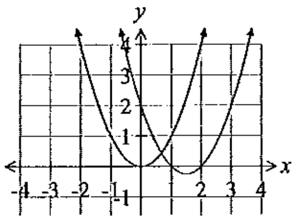
Worked solutions and marking guidelines

Section I		
	Solution	**Criteria**
1	$\lim_{x\to 3}\frac{x^2+2x-15}{x-3}=\lim_{x\to 3}\frac{(x+5)(x-3)}{(x-3)}$ $=\lim_{x\to 3}(x+5)$ $=8$	1 Mark: D
2	$2\cos^2 x-1=0$ $\cos^2 x=\frac{1}{2}$ or $\cos x=\pm\frac{1}{\sqrt{2}}$ $x=\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}$	1 Mark: D
3	$a=3$ and $S=1.8$ $S=\frac{a}{1-r}$ $1.8=\frac{3}{1-r}$ $1.8-1.8r=3$ $1.8r=-1.2$ $r=-0.\dot{6}$	1 Mark: B
4	$\int_0^1(e^{3x}+1)dx=\left[\frac{1}{3}e^{3x}+x\right]_0^1$ $=(\frac{1}{3}e^3+1)-(\frac{1}{3})$ $=\frac{1}{3}(e^3+2)$	1 Mark: D
5	$f(x)=\sqrt{(2x^2+1)^3}$ $=(2x^2+1)^{\frac{3}{2}}$ $f'(x)=\frac{3}{2}\times(2x^2+1)^{\frac{1}{2}}\times 4x$ $=6x\sqrt{(2x^2+1)}$	1 Mark: C
6	$y=\cos x$, $\frac{dy}{dx}=-\sin x$. At the point $\left(\frac{\pi}{2},0\right)$ $m=-\sin\frac{\pi}{2}=-1$ $y-y_1=m(x-x_1)$ $y-0=-1(x-\frac{\pi}{2})$ $x+y-\frac{\pi}{2}=0$	1 Mark: B
7	$V=\pi\int_a^b y^2dx$ $=\pi\int_0^2(3x+1)^2dx$ $=\pi\int_0^2(9x^2+6x+1)dx$ $=\pi\left[3x^3+3x^2+x\right]_0^2$ $=\pi\left[3\times 2^3+3\times 2^2+2\right]$ $=38\pi$ units^3	1 Mark: D
8	$a=12t+6$ $v=6t^2+6t+c$ When $t=0$ then $v=-36$ $-36=6\times 0^2+6\times 0+c$ or $c=-36$ $v=6t^2+6t-36$ $=6(t+3)(t-2)$ Particle at rest ($v=0$) when $t=2$	1 Mark: C
9	$x^2=6y+2x+11$ $x^2-2x=6y+11$ $(x-1)^2-1=6y+11$ $(x-1)^2=6(y+2)$ $(x-1)^2=4\times\frac{3}{2}\times(y+2)$ Vertex is $(1,-2)$ and focal length is $\frac{3}{2}$. Focus is $\left(1,-\frac{1}{2}\right)$	1 Mark: D
10	$[f(x)]^2=[4+2^{-x}]^2$ $=16+8\times 2^{-x}+2^{-2x}$ $=16+2^3\times 2^{-x}+2^{-2x}$ $=16+2^{3-x}+2^{-2x}$	1 Mark: A

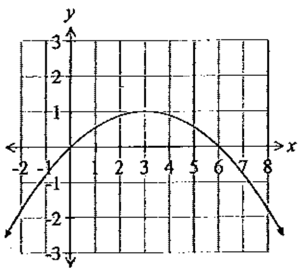
Section II

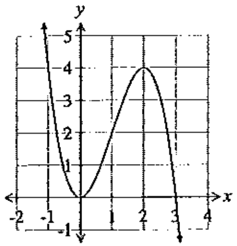
11(a)	$\frac{y}{y^2-4}-\frac{2}{y-2}=\frac{y}{(y+2)(y-2)}-\frac{2}{(y-2)}$ $=\frac{y-2(y+2)}{(y+2)(y-2)}$ $=\frac{-y-4}{y^2-4}$	2 Marks: Correct answer. 1 Mark: Finds a common denominator or shows some understanding.
11(b) (i)	$P(E)=0.85\times 0.75$ $=0.6375$	1 Mark: Correct answer.
11(b) (ii)	Woman alive, husband dead or woman dead, husband alive. $P(E)=0.85\times 0.25+0.15\times 0.75$ $=0.325$	1 Mark: Correct answer.
11(c) (i)	$a=4$ and $d=5$ for the sequence 4, 9, 14, 19, …. $T_n=a+(n-1)d$ $=4+(n-1)\times 5$ $=5n-1$	1 Mark: Correct answer.
11(c) (ii)	$T_{25}=5\times 25-1$ $=124$	1 Mark: Correct answer.
11(c) (iii)	$S_n=\frac{n}{2}[2a+(n-1)d]$ $S_{100}=\frac{100}{2}[2\times 4+(100-1)\times 5]$ $=25{,}150$	1 Mark: Correct answer.
11(d) (i)	$\frac{d}{dx}(\tan 5x)=\sec^2 5x\times\frac{d}{dx}(5x)$ $=5\sec^2 5x$	1 Mark: Correct answer.
11(d) (ii)	$\frac{d}{dx}\left(\frac{\log_e x}{x}\right)=\frac{x\times\frac{1}{x}-\log_e x\times 1}{x^2}$ $=\frac{1-\log_e x}{x^2}$	1 Mark: Correct answer.
11(d) (iii)	$\frac{d}{dx}(x\cos x)=-x\sin x+\cos x$	1 Mark: Correct answer.

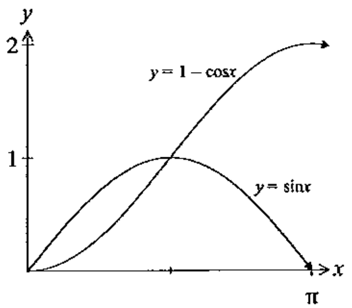
11(e)	$x^2+1=3x+1$ $x^2-3x=0$ $x(x-3)=0$ Point of intersection occurs when $x=0$ and $x=3$ $A=\int_0^3(3x+1)-(x^2+1)dx$ $=\int_0^3(3x-x^2)dx$ $=\left[\frac{3x^2}{2}-\frac{x^3}{3}\right]_0^3$ $=\left[\left(\frac{3\times 3^2}{2}-\frac{3^3}{3}\right)-\left(\frac{3\times 0^2}{2}-\frac{0^3}{3}\right)\right]=\frac{9}{2}$ square units	2 Marks: Correct answer. 1 Mark: Finds the points of intersection or shows some understanding of the problem.
11(f) (i)	$\angle BAC=180^\circ-30^\circ-50^\circ=100^\circ$ $\frac{AB}{\sin\angle ACB}=\frac{BC}{\sin\angle BAC}$ $\frac{AB}{\sin 30^\circ}=\frac{10}{\sin 100^\circ}$ $AB=\frac{10\sin 30^\circ}{\sin 100^\circ}$	2 Marks: Correct answer. 1 Mark: Finds angle *BAC* or uses the Sine Rule with two correct values.
11(f) (ii)	$\sin 50^\circ=\frac{AD}{AB}$ $AD=\frac{10\sin 30^\circ\sin 50^\circ}{\sin 100^\circ}$ $=3.8893095\ldots\approx 3.89$ cm	1 Mark: Correct answer.
12(a) (i)	Gradient of OA: $m=\frac{y_2-y_1}{x_2-x_1}=\frac{0-1}{0--3}=-\frac{1}{3}$	1 Mark: Correct answer.
12(a) (ii)	Gradient of OB: $m=\frac{y_2-y_1}{x_2-x_1}=\frac{0-3}{0-1}=3$ Perpendicular lines occur when $m_1m_2=-1$ $m_1m_2=-1$ $-\frac{1}{3}\times 3=-1$ True	1 Mark: Correct answer.
12(a) (iii)	If BC is parallel to OA then it has the same gradient or $m=-\frac{1}{3}$ $y-y_1=m(x-x_1)$ $y-3=-\frac{1}{3}(x-1)$ $3y-9=-x+1$ or $x+3y-10=0$	2 Marks: Correct answer. 1 Mark: Uses the gradient intercept form with at least 1 correct value.

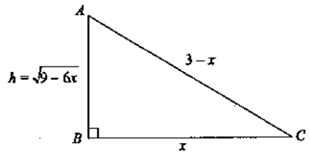
12(a) (iv)	The point C lies on the line $x=-2$ Substitute -2 for x into $x+3y-10=0$ $-2+3y-10=0$ $3y=12$ or $y=4$ Coordinates of C are $(-2,4)$.	1 Mark: Correct answer.
12(a) (v)	$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $BC=\sqrt{(-2-1)^2+(4-3)^2}=\sqrt{10}$	1 Mark: Correct answer.
12(a) (vi)	Quadrilateral $OACD$ is a square (Rectangle with all sides equal) $A=s^2=\left(\sqrt{10}\right)^2=10$ square units	1 Mark: Correct answer
12(b)	$\int_1^3 f(x)dx\approx\frac{h}{3}[y_0+y_4+4(y_1+y_3)+2y_2]$ $\approx\frac{0.5}{3}[4+8+4(1.5+2.5)+2\times-2]\approx 4$	2 Marks: Correct answer. 1 Mark: Uses Simpson's rule.
12(c)	$S=\{23,\ 24,\ 26,\ 32,\ 34,\ 36,\ 42,\ 43,\ 46,\ 62,\ 63,\ 64\}$ $P(\text{prime})=\frac{2}{12}=\frac{1}{6}$	2 Marks: Correct answer. 1 Mark: Finds sample space.
12(d)	$l=r\theta$ $=12\times\frac{5\pi}{6}=10\pi$ radians	1 Mark: Correct answer.
12(e) (i)	$y=x^2-3x+2=(x-1)(x-2)$ 	2 Marks: Correct answer. 1 Mark: One graph drawn correctly.
12(e) (ii)	Now $y=x^2>(x-1)(x-2)$ Point of intersection from the graph is $x=\frac{2}{3}$ Alternatively $x^2=(x-1)(x-2)$ $x^2=x^2-3x+2$ $3x=2$ or $x=\frac{2}{3}$ Therefore $x^2>(x-1)(x-2)$ when $x>\frac{2}{3}$	1 Mark: Correct answer.

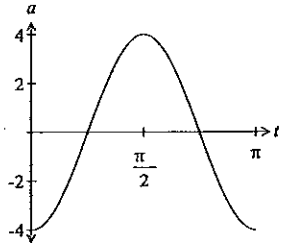
13(a)	$V=\pi\int_0^{\ln 4}\left(e^y\right)^2 dy$ $=\pi\left[\frac{1}{2}e^{2y}\right]_0^{\ln 4}$ $=\frac{\pi}{2}\left(e^{2\ln 4}-e^0\right)$ $=\frac{15\pi}{2}$ cubic units	3 Marks: Correct answer. 2 Mark: Makes significant progress. 1 Mark: Uses volume formula with at least one correct value
13(b)	$\int_0^{\frac{\pi}{6}}(x^2+\sin 2x)dx=\left[\frac{x^3}{3}-\frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{6}}$ $=\left[\frac{\left(\frac{\pi}{6}\right)^3}{3}-\frac{1}{2}\cos 2\times\frac{\pi}{6}\right]-\left[\frac{0^3}{3}-\frac{1}{2}\cos 2\times 0\right]$ $=\left[\frac{\pi^3}{648}-\frac{1}{4}\right]+\frac{1}{2}$ $=\frac{\pi^3}{648}+\frac{1}{4}=0.297849\ldots$	2 Marks: Correct answer. 1 Mark: Finds the primitive function or shows some understanding.
13(c) (i)	In ΔABE and ΔDEC $\angle AEB=\angle DEC$ (vertically opposite angles are equal) $\frac{AE}{EC}=\frac{BE}{ED}$ $\left(\frac{AE}{EC}=\frac{3}{6}=\frac{1}{2}\right.$ and $\left.\frac{BE}{ED}=\frac{4}{8}=\frac{1}{2}\right)$ $\Delta ABE\ \vert\vert\vert\ \Delta DEC$ (two pairs of corresponding sides are in proportion and the include angles are equal)	3 Marks: Correct answer. 2 Marks: Makes significant progress. 1 Mark: One relevant statement
13(c) (ii)	$\angle BAE=\angle DCE$ (matching angles in similar triangles are equal) Therefore $\angle BAE$ and $\angle DCE$ are alternate angles and equal. $\therefore AB\,\square\,CD$ (alternate angles are only equal if the lines are parallel) Therefore $ABCD$ is a trapezium (one pair of opposite sides parallel)	2 Marks: Correct answer. 1 Mark: Shows some understanding.
13(d)	$d=\left\vert\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right\vert$ $=\left\vert\frac{3\times 0-1\times 5+1}{\sqrt{3^2+(-1)^2}}\right\vert$ $=\left\vert\frac{-4}{\sqrt{10}}\right\vert=\frac{4}{\sqrt{10}}\times\frac{\sqrt{10}}{\sqrt{10}}$ $=\frac{2\sqrt{10}}{5}$	2 Marks: Correct answer. 1 Mark: Uses the perpendicular distance formula with one correct value.

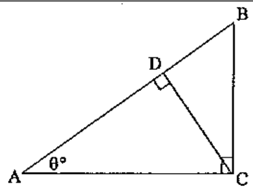
13(e) (i)	The parabola is symmetrical about the vertex of (3, 1). If the parabola passes through the origin it is concave down and the other x-intercept is (6, 0). 	1 Mark: Correct answer.
13(e) (ii)	The points (0, 0), (3, 1) and (6, 0) satisfy $y=ax^2+bx+c$ Sub (0, 0) into $y=ax^2+bx+c$ results in $c=0$ Sub (6, 0) into $y=ax^2+bx+c$ results in $0=36a+6b$ (1) Sub (3, 1) into $y=ax^2+bx+c$ results in $1=9a+3b$ (2) Multiply eqn (2) by 2 $2=18a+6b$ (3) Eqn (1) − (3) $-2=18a$ or $a=-\frac{1}{9}$ Sub $a=-\frac{1}{9}$ into eqn (2) $1=9\times-\frac{1}{9}+3b$ $3b=2$ or $b=\frac{2}{3}$ Therefore $a=-\frac{1}{9}$, $b=\frac{2}{3}$ and $c=0$	2 Marks: Correct answer. 1 Mark: Finds one correct value or shows some understanding.
14(a) (i)	$f(x)=x^2(3-x)=3x^2-x^3$ Stationary points $f'(x)=0$ $6x-3x^2=0$ $3x(2-x)=0$ $x=0,\ x=2$ Stationary points are $(0,0)$ and $(2,4)$. $f''(x)=6-6x$ At $(0,0)$, $f''(0)=6>0$, Minimum stationary point At $(2,4)$, $f''(1)=-6<0$, Maximum stationary point	2 Marks: Correct answer. 1 Mark: Finds one of the stationary points or recognises $6x-3x^2=0$.

14(a) (ii)	x-intercepts ($y=0$) $x^2(3-x)=0$ $x=0,\ x=3$  Max (2,4) Min (0,0)	2 Marks: Correct answer. 1 Mark: Makes some progress towards sketching the curve.
14(a) (iii)	$f'(x)=6x-3x^2$ At the point $P(1,2)$ $f'(1)=6\times1-3\times1^2=3$ Point slope formula $y-y_1=m(x-x_1)$ $y-2=3(x-1)$ $y=3x-1$ or $3x-y-1=0$	1 Mark: Correct answer
14(b) (i)	$\alpha+\beta=-\frac{b}{a}=-\frac{-3}{2}=\frac{3}{2}$	1 Mark: Correct answer
14(b) (ii)	$\alpha\beta=\frac{c}{a}=\frac{6}{2}=3$	1 Mark: Correct answer
14(b) (iii)	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ $=\left(\frac{3}{2}\right)^2-2\times3=-\frac{15}{4}$	1 Mark: Correct answer
14(c)	The object comes to rest when $\dot{x}=0$ $x=3e^{-2t}+10e^{-t}+4t$ $\dot{x}=-6e^{-2t}-10e^{-t}+4$ $=-2(3e^{-2t}+5e^{-t}-2)$ Let $m=e^{-t}$ $-2(3m^2+5m-2)=0$ $-2(3m-1)(m+2)=0$ Hence $3m-1=0$ or $m+2=0$ $m=\frac{1}{3}$ $m=-2$ $e^{-t}=\frac{1}{3}$ $e^{-t}=-2$ (no solution) Therefore $t=-\log_e\frac{1}{3}$ $=\log_e 3$ $\approx 1.0986...$	3 Marks: Correct answer. 2 Marks: Finds and factorises a a quadratic equation. 1 Mark: Correctly differentiates x.

14(d)	$T_3 = ar^2 = 1.25$ and $T_7 = ar^6 = 20$ Divide the two equations $\frac{ar^6}{ar^2} = \frac{20}{1.25}$ $r^4 = 16$ $r = \pm 2$ $T_7 = a \times (\pm 2)^6 = 20$ $a = \frac{20}{64} = \frac{5}{16}$	2 Marks: Correct answer. 1 Mark: Finds two equations using the nth term of a GP or shows some understanding.
14(e)	$\int_0^{\frac{\pi}{8}} \sec^2 2x dx = \left[\frac{1}{2}\tan 2x\right]_0^{\frac{\pi}{8}}$ $= \frac{1}{2}\left(\tan\frac{\pi}{4} - \tan 0\right) = \frac{1}{2}$	2 Marks: Correct answer. 1 Mark: Finds the primitive function or shows some understanding.
15(a) (i)		2 Marks: Correct answer. 1 Mark: Draws one of the curves.
15(a) (ii)	$x = 0$ or $x = \frac{\pi}{2}$ (from the graph)	1 Mark: Correct answer.
15(a) (iii)	$A = \int_0^{\frac{\pi}{2}} [\sin x - (1-\cos x)]dx + \int_{\frac{\pi}{2}}^{\pi} [1 - \cos x - \sin x]dx$ $= [-\cos x - x - \sin x]_0^{\frac{\pi}{2}} + [x - \sin x + \cos x]_{\frac{\pi}{2}}^{\pi}$ $= \left(0 - \frac{\pi}{2} + 1 - (-1)\right) + \left(\pi - 0 - 1 - (\frac{\pi}{2} - 1)\right)$ $= 2 - \frac{\pi}{2} + \frac{\pi}{2}$ $= 2$ square units	2 Marks: Correct answer. 1 Mark: Shows some understanding.

15(b) (i)	$A = P(1+r)^n$ $= 1000(1+0.05)^{100}$ $= \$131501.2578 \approx \$131,501.26$	1 Mark: Correct answer.
15(b) (ii)	$A_{100} = 1000(1.05)^{100} + 1000(1.05)^{99} + ... + 1000(1.05)^1$ G.P. with $a = 1000(1.05)$, $r = 1.05$ and $n = 100$ $A_{100} = \frac{1000(1.05)\left[1.05^{100} - 1\right]}{1.05 - 1}$ $= \$2740526.415$ $\approx \$2,740,526.42$	2 Marks: Correct answer. 1 Mark: Identifies a G.P. with 100 terms.
15(c) (i)	$AC = (3 - x)$ metres	1 Mark: Correct answer.
15(c) (ii)	$(3-x)^2 = h^2 + x^2$ $h^2 = 9 - 6x + x^2 - x^2$ $h = \sqrt{9-6x}$ ($h > 0$ as h is a height) $A = \frac{1}{2}bh$ $= 0.5x\sqrt{9-6x}$ m^2 	2 Marks: Correct answer. 1 Mark: Finds the height of the triangle or shows some understanding.
15(c) (iii)	Maximum occurs when $\frac{dA}{dx} = 0$ $A = 0.5x\sqrt{9-6x}$ $\frac{dA}{dx} = 0.5\left[x \times \frac{1}{2}(9-6x)^{-\frac{1}{2}} \times -6 + (9-6x)^{\frac{1}{2}} \times 1\right]$ $= \frac{1}{2}(9-6x)^{-\frac{1}{2}}\left[-3x + (9-6x)^1\right]$ $= \frac{-9x+9}{2\sqrt{9-6x}} = \frac{9(1-x)}{2\sqrt{9-6x}}$ Now $\frac{dA}{dx} = \frac{9(1-x)}{2\sqrt{9-6x}} = 0$ $\therefore x = 1$ Test whether $x = 1$ is a maximum $x = 0.9$, $\frac{dA}{dx} = \frac{9(1-0.9)}{2\sqrt{9-6\times 0.9}} > 0$ $x = 1.1$, $\frac{dA}{dx} = \frac{9(1-1.1)}{2\sqrt{9-6\times 1.1}} < 0$ Maximum value occurs when $x = 1$.	3 Marks: Correct answer. 2 Marks: Finds $x = 1$ 1 Mark: Calculates the first derivative or has some understanding of the problem.

15(c) (iv)	$A=0.5x\sqrt{9-6x}$ $=0.5\times1\sqrt{9-6\times1}=0.5\sqrt{3}\text{ m}^2$	1 Mark: Correct answer.
16(a) (i)	$x=\int(1-2\sin 2t)\,dt$ $=t+\cos 2t+C$ Initially $t=0$ and $x=0$ $0=0+\cos(2\times0)+C\rightarrow C=-1$ $x=t+\cos 2t-1$	2 Marks: Correct answer. 1 Mark: Integrates the velocity function.
16(a) (ii)	When $t=\frac{\pi}{3}$ then $x=\frac{\pi}{3}+\cos\left(2\times\frac{\pi}{3}\right)-1$ $=\frac{\pi}{3}-\frac{1}{2}-1=\frac{\pi}{3}-\frac{3}{2}$	1 Mark: Correct answer
16(a) (iii)	$a=\frac{d}{dt}(1-2\sin 2t)$ $=-4\cos 2t$	1 Mark: Correct answer
16(a) (iv)	$a=-4\cos 2t$ for $0\le t\le\pi$ 	1 Mark: Correct answer
16(a) (v)	From the graph or $-1\le\cos 2t\le1$ ($-4\le-4\cos 2t\le4$) Maximum acceleration is 4 m/s^2	1 Mark: Correct answer
16(b) (i)	Initially $t=0$ and $R=8000$ $R=R_0e^{-kt}$ $8000=R_0e^{-k\times0}$ $R_0=8000$ Also $t=1$ and $R=7000$ $7000=8000e^{-k\times1}$ $e^{-k}=\frac{7000}{8000}$ $-k=\log_e\frac{7}{8}$ $k=-\log_e\frac{7}{8}=0.13353139...$	2 Marks: Correct answer. 1 Mark: Finds the correct value for R_0 or k.

16(b) (ii)	We need to find R when $t=10$ $R=8000e^{\log_e\frac{7}{8}\times10}$ $=2104.604609...\approx2105$ bq	1 Mark: Correct answer.
16(b) (iii)	We need to find t when $R=50$. $50=8000e^{-k\times t}$ $e^{-kt}=\frac{1}{160}$ $-kt=\log_e\frac{1}{160}$ $t=-\frac{1}{k}\log_e\frac{1}{160}$ $=\log_e\frac{1}{160}\div\log_e\frac{7}{8}$ $=38.0073458..\approx38$ years	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.
16(c) (i)	 $\cos\theta=\frac{AD}{AC}$ $\quad\tan\theta=\frac{BC}{AC}$ $\quad\cos\theta=\frac{AC}{AB}$ $AD=AC\cos\theta$ $\quad BC=AC\tan\theta$ $\quad AB=AC\sec\theta$ Now $8AD+2BC=7AB$ $8AC\cos\theta+2AC\tan\theta=7AC\sec\theta$ $8\cos\theta+2\tan\theta=7\sec\theta$	2 Marks: Correct answer. 1 Mark: Draws the diagram and makes some progress towards the solution.
16(c) (ii)	$8\cos\theta+2\tan\theta=7\sec\theta$ $8\cos\theta+\frac{2\sin\theta}{\cos\theta}=\frac{7}{\cos\theta}$ $8\cos^2\theta+2\sin\theta=7$ $8(1-\sin^2\theta)+2\sin\theta=7$ $8\sin^2\theta-2\sin\theta-1=0$ $(2\sin\theta-1)(4\sin\theta+1)=0$ $2\sin\theta-1=0$ or $4\sin\theta+1=0$ $\sin\theta=\frac{1}{2}$ $\quad\sin\theta=-\frac{1}{4}$ $\theta=30^\circ$ $\quad\theta=165^\circ31'$ Now $0^\circ\le\theta\le90^\circ$ as θ is in a right-angled triangle. $\therefore\theta=30^\circ$	2 Marks: Correct answer. 1 Mark: Finds the quadratic equation in $\sin\theta$ or shows some understanding of the problem.