ACE Examinations

Student Name: ____________________

2014

YEAR 11

HALF YEARLY EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time - 5 minutes
- Working time - 1 hour
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in Questions 6-8

Total marks - 35

Section 1
5 marks
Attempt Questions 1-5
Allow 8 minutes for this section

Section 2
30 marks
Attempt Questions 6-7
Allow 52 minutes for this section

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\ x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\ a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\ a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\ a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\ a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\ a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\ a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x,\ x > 0$

Section I

5 marks
Attempt Questions 1 - 5
Allow about 8 minutes for this section

Use the multiple-choice answer sheet for Questions 1-5

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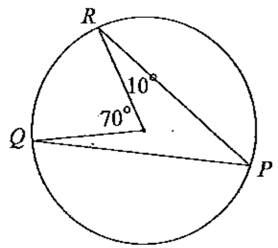
1 What is the value of $\angle RQP$?



(A) 55°
(B) 65°
(C) 70°
(D) 80°

2 What is the solution to the inequality $\frac{x-3}{x} > 0$?

(A) $x < 0$ and $x < 3$
(B) $x < 0$ and $x > 3$
(C) $x > 0$ and $x < 3$
(D) $x > 0$ and $x > 3$

3 Two boats A and B are out to sea. The cliff OT is 75 metres high where O is the point at the base of the cliff below T. The angle of elevation from A to T is 25° and the angle of elevation from B to T is 34°. The angle $\angle AOB = 120°$. Which of the following are the correct expressions for OA and OB?

(A) $OA = \frac{\tan 25°}{75}$ and $OB = \frac{\tan 34°}{75}$

(B) $OA = \frac{75}{\tan 25°}$ and $OB = \frac{75}{\tan 34°}$

(C) $OA = 75\tan 25°$ and $OB = 75\tan 34°$

(D) $OA = 120\tan 25°$ and $OB = 120\tan 34°$

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4 What is the acute angle between the lines $3x+4y-1=0$ and $2x+3y-2=0$?

(A) 3°11'
(B) 9°28'
(C) 70°36'
(D) 86°49'

5 What is the maximum value of $\cos\theta + 2\cos(\theta + 120°)$?

(A) $2\sqrt{3}$

(B) $\sqrt{3}$

(C) $\frac{\sqrt{3}}{2}$

(D) $\frac{\sqrt{3}}{4}$

Section II

30 marks
Attempt Questions 6–7
Allow about 52 minutes for this section

Answer each question in the appropriate writing booklet.

All necessary working should be shown in every question.

Question 6 (15 marks) **Marks**

(a) (i) Find the acute angle between the lines $x-\sqrt{3}y+3=0$ and $x-y+6=0$. **2**

(ii) Show that the two lines above and the line $\sqrt{3}x-y+6=0$ form an isosceles triangle. **2**

(b) (i) Sketch the graph of $y=|2x-1|$ **1**

(ii) Hence or otherwise, solve $|2x-1|\le|x-2|$ **2**

(c) (i) Show that $\dfrac{1-\cos 2\theta}{1+\cos 2\theta}=\tan^2\theta$ **2**

(ii) Hence show that the exact value of $\tan 22.5^\circ$ is $\sqrt{2}-1$ **1**

(d) X is an external point to a circle and FX is a tangent touching the circle at F. Points D and E are on the circle such that DE produced meets FX at X. FX is 8 cm and DE is 12 cm

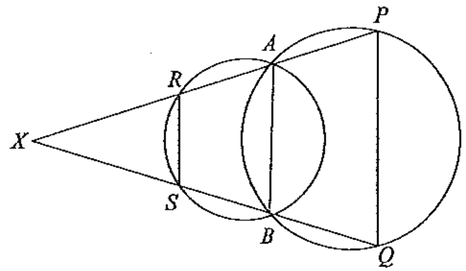
(i) Draw a diagram to show this information. **1**

(ii) Calculate the length XE. **2**

(e) Solve $\dfrac{4-x}{x^2+4}\ge 1$ **2**

Question 7 (15 marks) **Marks**

(a) Two circles intersect at A and B. The straight lines PAR and QBS intersect at X. Let $\angle SRA=\theta^\circ$.



(i) Prove that $PQ \parallel RS$. **2**

(ii) Find BS if $RX=5$, $AR=3$ and $SX=4$. **1**

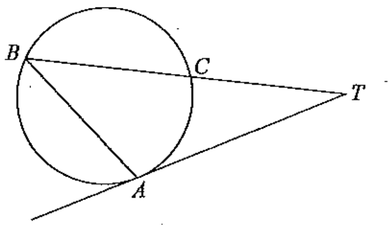
(b) (i) Let the acute angle between the lines $y=3x$ and $y=5x$ be θ. Show that $\tan\theta=\dfrac{1}{8}$. **1**

(ii) What is the value of m if acute angle between the lines $y=3x$ and $y=mx$ ($m<3$) is also θ? **2**

(c) Show that $\operatorname{cosec}\theta+\cot\theta=\cot\dfrac{\theta}{2}$ by making the substitution $t=\tan\dfrac{\theta}{2}$. **2**

(d) Solve the inequality $\dfrac{4}{x-3}\le x$. **3**

(e) A, B and C are three points on a circle. Chord BC produced meets the tangent at A in T. Prove $\angle ACT = \angle BAT$ **2**



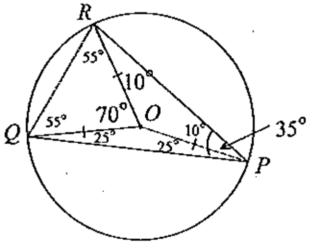
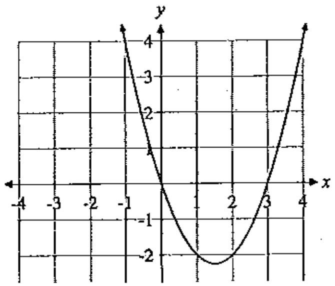
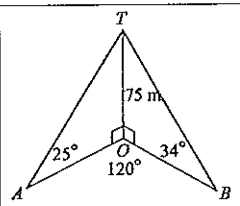
(f) Solve $\sec^2\theta + \tan\theta = 1$, where $180^\circ \le \theta \le 360^\circ$ **2**

End of paper

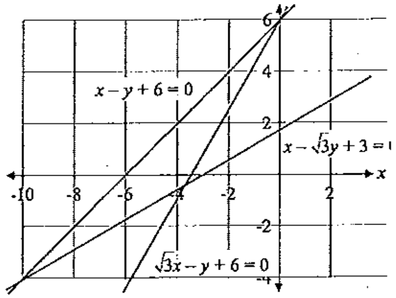
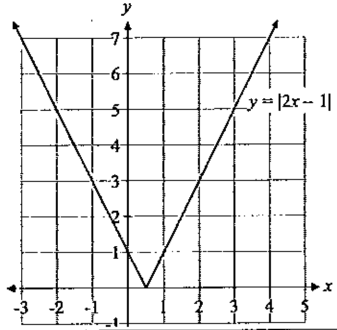
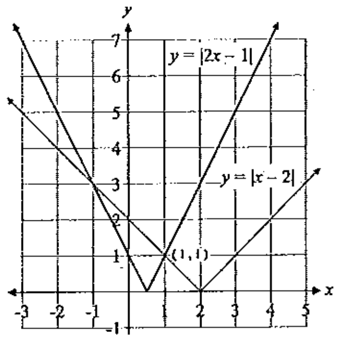
ACE Examination 2014

Year 11 Mathematics Extension 1 Half Yearly Examination

Worked solutions and marking guidelines

Section I		
	Solution	**Criteria**
1	 $\angle RQP = 55^\circ + 25^\circ = 80^\circ$	1 Mark: D
2	$x^2 \times \frac{x-3}{x} > 0 \times x^2 \qquad x \neq 0$ $x(x-3) > 0$  From the graph $x < 0$ and $x > 3$	1 Mark: B
3	 In ΔTOA In ΔTOB $\tan 25^\circ = \frac{75}{OA}$ $\tan 34^\circ = \frac{75}{OB}$ $OA = \frac{75}{\tan 25^\circ}$ $OB = \frac{75}{\tan 34^\circ}$	1 Mark: B

4	For $3x+4y=8$ then $m_1 = -\frac{3}{4}$ For $2x+3y=5$ then $m_2 = -\frac{2}{3}$ $\tan\theta = \left\lvert\frac{m_1 - m_2}{1+m_1 m_2}\right\rvert$ $= \left\lvert\frac{-\frac{3}{4} - -\frac{2}{3}}{1 + -\frac{3}{4} \times -\frac{2}{3}}\right\rvert = \frac{1}{18}$ $\theta = 3.17983012...$ $\approx 3^\circ 11'$	1 Mark: A
5	$\cos\theta + 2\cos(\theta+120) = \cos\theta + 2[\cos\theta\cos 120 - \sin\theta\sin 120)]$ $= \cos\theta + 2[\cos\theta \times -\frac{1}{2} - \sin\theta \times \frac{\sqrt{3}}{2})]$ $= -\sqrt{3}\sin\theta$ Smallest value of $\sin\theta$ is -1. Maximum value of $\cos\theta + 2\cos(\theta + 120^\circ)$ is $\sqrt{3}$	1 Mark: B
Section II		
6(a) (i)	For $x - y + 6 = 0$ then $m_1 = 1$. For $x - \sqrt{3}y + 3 = 0$ then $m_2 = \frac{1}{\sqrt{3}}$. $\tan\theta = \left\lvert\frac{m_1 - m_2}{1+m_1 m_2}\right\rvert$ $= \left\lvert\frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \times \frac{1}{\sqrt{3}}}\right\rvert$ $= \left\lvert\frac{\sqrt{3}-1}{\sqrt{3}+1}\right\rvert$ $\theta = 15^\circ$	2 Marks: Correct answer. 1 Mark: Finds the gradient of the two lines.

6(a) (ii)	 For $\sqrt{3}x-y+6=0$ then $m_1=\sqrt{3}$. For $x-y+6=0$ then $m_2=1$. $\tan\theta=\left\|\frac{m_1-m_2}{1+m_1m_2}\right\|=\left\|\frac{\sqrt{3}-1}{1+\sqrt{3}\times 1}\right\|$ or $\theta=15^\circ$ Isosceles triangle as two angles are equal (15°).	2 Marks: Correct answer. 1 Mark: Finds a gradient and uses angle between two lines formula.
6(b) (i)		1 Mark: Correct answer.
6(b) (ii)	Solve $\|2x-1\|=\|x-2\|$ $2x-1=x-2$ or $2x-1=-(x-2)$ $x=-1$ $x=1$  From the graph $\|2x-1\|\le\|x-2\|$ for $-1\le x\le 1$	2 Marks: Correct answer. 1 Mark: Shows some understanding of the problem.

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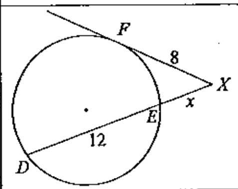
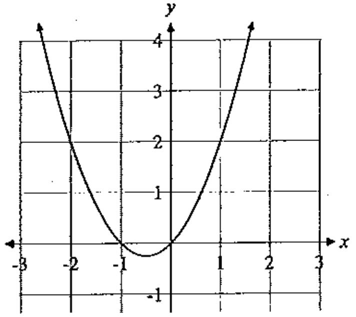
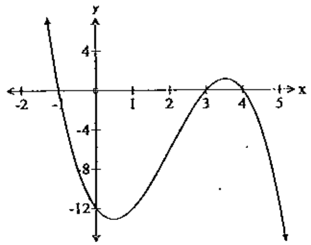
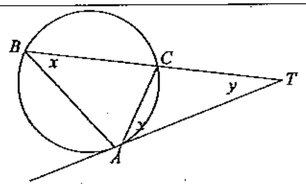
6(c) (i)	$\text{LHS}=\frac{1-(\cos^2\theta-\sin^2\theta)}{1+(\cos^2\theta-\sin^2\theta)}$ $=\frac{(1-\cos^2\theta)+\sin^2\theta}{(1-\sin^2\theta)+\cos^2\theta}$ $=\frac{2\sin^2\theta}{2\cos^2\theta}$ $=2\tan^2\theta$ $=\text{RHS}$	2 Marks: Correct answer. 1 Mark: Uses an appropriate trigonometric identity.
6(c) (ii)	$\tan\theta=\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}$ from part (i) $\tan 22.5^\circ=\sqrt{\frac{1-\cos 45^\circ}{1+\cos 45^\circ}}$ $=\sqrt{\left(1-\frac{1}{\sqrt{2}}\right)\div\left(1+\frac{1}{\sqrt{2}}\right)}$ $=\sqrt{\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)\times\left(\frac{\sqrt{2}}{\sqrt{2}+1}\right)}$ $=\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}\times\frac{\sqrt{2}-1}{\sqrt{2}-1}}$ $=\sqrt{(\sqrt{2}-1)^2}=\sqrt{2}-1$	1 Mark: Correct answer.
6(d) (i)		1 Mark: Correct answer.
6(d) (ii)	$FX^2=DX\times XE$ (Square of the length of the tangent from an external point is equal to the product of the intercepts of the secant passing through this point) $8^2=(x+12)\times x$ $64=x^2+12x$ $x^2+12x-64=0$ $(x+16)(x-4)=0$ $\therefore\ x=-16$ or $x=4$ Ignore $x=-16$ as x is a length. $\therefore\ x=4$ cm	2 Marks: Correct answer. 1 Mark: Shows some understanding of the problem.

6(e)	$(x^2+4)\times\dfrac{4-x}{(x^2+4)}\geq 1\times(x^2+4)$ (x^2+4) is always positive $4-x\geq x^2+4$ $x^2+x\leq 0$ $x(x+1)\leq 0$  $\therefore -1\leq x\leq 0$	2 Marks: Correct answer. 1 Mark: Multiplies both sides of the inequality by (x^2+4) or finds a critical point.
7(a) (i)	$\angle SRA+\angle SRX=180^\circ$ (straight line equals 180°) $\angle SRX=180-\theta$ $\angle SRX=\angle ABS$ (exterior angle of a cyclic quadrilateral equals the interior opposite angle) $\angle ABS=180-\theta$ $\angle ABS=\angle QPA$ (exterior angle of a cyclic quadrilateral equals the interior opposite angle) $\angle QPA=180-\theta$ $\angle SRX=\angle QPA$ (both $180-\theta$) $\therefore PQ\parallel RS$ (corresponding angles are equal only when the two lines are parallel)	2 Marks: Correct answer. 1 Mark: States an appropriate theorem used in the solution.
7(a) (ii)	$AB\parallel RS$ $\dfrac{BS}{SX}=\dfrac{AR}{RX}$ (family of parallel lines cuts the intercepts in proportion) $\dfrac{BS}{4}=\dfrac{3}{5}$ $BS=\dfrac{12}{5}$	1 Mark: Correct answer.
7(b) (i)	For $y=5x$ then $m_1=5$, For $y=3x$ then $m_2=3$. $\tan\theta=\left\|\dfrac{m_1-m_2}{1+m_1m_2}\right\|$ $=\left\|\dfrac{5-3}{1+5\times3}\right\|=\left\|\dfrac{2}{16}\right\|=\dfrac{1}{8}$	1 Mark: Correct answer.

7(b) (ii)	For $y=3x$ then $m_1=3$. For $y=mx$ then $m_2=m$ $\tan\theta=\left\|\dfrac{m_1-m_2}{1+m_1m_2}\right\|$ $\dfrac{1}{8}=\left\|\dfrac{3-m}{1+3\times m}\right\|$ $1+3m=24-8m$ $11m=23$ $m=\dfrac{23}{11}$	2 Marks: Correct answer. 1 Mark: Finds an equation for m
7(c)	If $\sin\theta=\dfrac{2t}{1+t^2}$ then $\operatorname{cosec}\theta=\dfrac{1+t^2}{2t}$ If $\tan\theta=\dfrac{2t}{1-t^2}$ then $\cot\theta=\dfrac{1-t^2}{2t}$ $\text{LHS}=\operatorname{cosec}\theta+\cot\theta$ $=\dfrac{1+t^2}{2t}+\dfrac{1-t^2}{2t}$ $=\dfrac{2}{2t}=\dfrac{1}{t}$ $=\dfrac{1}{\tan\frac{\theta}{2}}$ $=\cot\dfrac{\theta}{2}=\text{RHS}$	2 Marks: Correct answer. 1 Mark: Uses at least one correct t formula.
7(d)	$(x-3)^2\times\dfrac{4}{x-3}\leq x\times(x-3)^2$ $(x-3)\times4\leq x(x-3)^2\quad x\neq3$ $(x-3)[4-x(x-3)]\leq 0$ $(x-3)(4+3x-x^2)\leq 0$ $(x-3)(4-x)(1+x)\leq 0$  From the graph $-1\leq x<3$ and $x\geq 4$	3 Marks: Correct answer. 2 Marks: Finds one correct region or makes significant progress. 1 Mark: Multiplies both sides of the inequality by $(x-3)^2$ or finds a critical point.

7(e)	 Let $\angle CAT = x^\circ$ and $\angle CTA = y^\circ$ $\angle CAT = \angle ABC$ (angle between a tangent and a chord at the point of contact is equal to the opposite angle in the alternate segment) In ΔBTA $\angle BAT + \angle ABT + \angle BTA = 180^\circ$ (angle sum of a triangle) $\angle BAT + x + y = 180^\circ$ $\angle BAT = 180 - x - y$ In ΔCAT $\angle ACT + \angle CAT + \angle CTA = 180^\circ$ (angle sum of a triangle) $\angle ACT + x + y = 180^\circ$ $\angle ACT = 180 - x - y$ $\therefore \angle ACT = \angle BAT$ (both $180 - x - y$)	2 Marks: Correct answer. 1 Mark: Uses the alternate angle theorem
7(f)	$\sec^2\theta + \tan\theta = 1$ $(1 + \tan^2\theta) + \tan\theta = 1$ $\tan^2\theta + \tan\theta = 0$ $\tan\theta(\tan\theta + 1) = 0$ $\therefore \tan\theta = 0$ or $\tan\theta = -1$ $\therefore \theta = 180^\circ, 315^\circ$ or 360°	2 Marks: Correct answer. 1 Mark: Uses $\sec^2\theta = 1 + \tan^2\theta$ or find at least one solution.