ACE Examinations

Student Name: _____

2014

YEAR 11

HALF YEARLY EXAMINATION

Mathematics

General Instructions

- Reading time - 5 minutes
- Working time - 90 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Show all necessary working in Questions 6-8

Total marks - 50

Section I

5 marks

- Attempt Questions 1-5
- Allow about 8 minutes for this section

Section II

45 marks

- Attempt Questions 6-8
- Allow about 1 hour 22 minutes for this section

STANDARD INTEGRALS

$\int x^n dx$	$= \frac{1}{n+1}x^{n+1},\ n \neq -1; x \neq 0, \text{if } n < 0$
$\int \frac{1}{x} dx$	$= \ln x,\ x > 0$
$\int e^{ax} dx$	$= \frac{1}{a}e^{ax},\ a \neq 0$
$\int \cos ax\, dx$	$= \frac{1}{a}\sin ax,\ a \neq 0$
$\int \sin ax\, dx$	$= -\frac{1}{a}\cos ax,\ a \neq 0$
$\int \sec^2 ax\, dx$	$= \frac{1}{a}\tan ax,\ a \neq 0$
$\int \sec ax \tan ax\, dx$	$= \frac{1}{a}\sec ax,\ a \neq 0$
$\int \frac{1}{a^2+x^2} dx$	$= \frac{1}{a}\tan^{-1}\frac{x}{a},\ a \neq 0$
$\int \frac{1}{\sqrt{a^2-x^2}} dx$	$= \sin^{-1}\frac{x}{a},\ a > 0,\ -a < x < a$
$\int \frac{1}{\sqrt{x^2-a^2}} dx$	$= \ln\left(x+\sqrt{x^2-a^2}\right),\ x > a > 0$
$\int \frac{1}{\sqrt{x^2+a^2}} dx$	$= \ln\left(x+\sqrt{x^2+a^2}\right)$

NOTE: $\ln x = \log_e x,\ x > 0$

Section I

5 marks
Attempt Questions 1 - 5
Allow about 8 minutes for this section

Use the multiple-choice answer sheet for Questions 1-5

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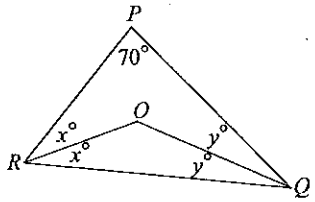
1 What is the solution to the inequality $|x-3| \leq 1$?

(A) $x \leq 2$ or $x \geq 4$
(B) $x \leq 2$ or $x \leq 4$
(C) $x \geq 2$ or $x \geq 4$
(D) $x \geq 2$ or $x \leq 4$

2 Evaluate $\sqrt[3]{\dfrac{46.12 \times 188.63}{(78.45)^2 - (35.50)^2}}$ correct to three significant figures.

(A) 1.21
(B) 1.211
(C) 1.33
(D) 1.333

3 In the diagram below, OR and OQ bisect $\angle PRQ$ and $\angle PQR$ respectively. $\angle RPQ = 70°$



Not to scale

What is the size of $\angle ROQ$?

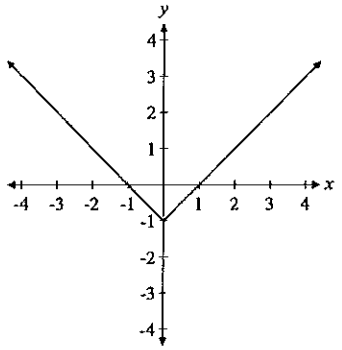
(A) 70°
(B) 100°
(C) 125°
(D) 140°

4 Simplify $\dfrac{(2ab^3)^3}{2a^2b^2}$

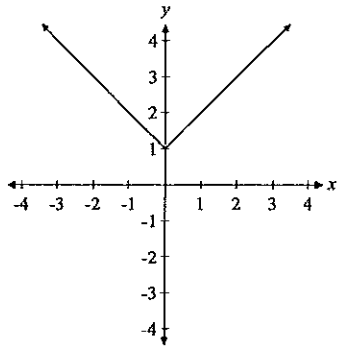
(A) ab^7
(B) $2ab^7$
(C) $4ab^4$
(D) $4ab^7$

5 Which graph best represents $y = |x| - 1$?

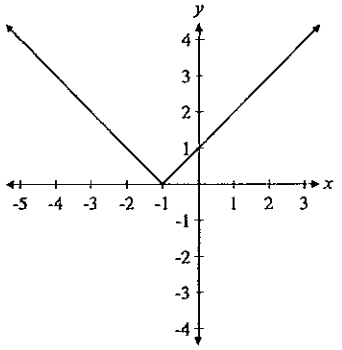
(A)



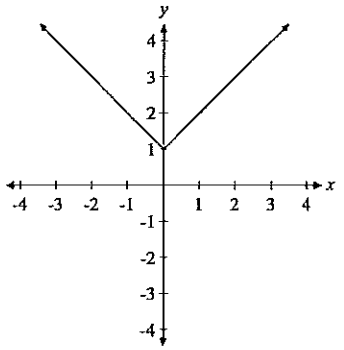
(B)



(C)



(D)



Section II

45 marks
Attempt Questions 6 – 8
Allow about 1 hours and 22 minutes for this section

Answer each question in the appropriate writing booklet.

All necessary working should be shown in every question.

Question 6 (15 marks) **Marks**

(a) Solve

(i) $y-3=\frac{3y}{2}+2$ **1**

(ii) $6(x+2)=5(x-6)$ **1**

(b) Express $\frac{1}{3-\sqrt{5}}$ in the form $a+b\sqrt{5}$, where a and b are rational numbers. **2**

(c) Factorise completely

(i) x^2-25 **1**

(ii) $27-8m^3$ **1**

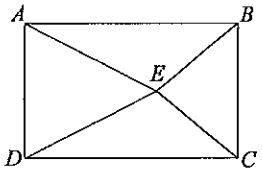
(iii) $xa+xb-3a-3b$ **1**

(d) Is the function $f(x)=x^3-6x$ even, odd or neither? **1**

(e) Express $(418.6)^2 \div 0.0179$ in scientific notation correct to four significant figures. **2**

(f) Solve the quadratic equation $x^2-6x+7=0$. **2**

(g) E is a point inside rectangle $ABCD$. $EC = EB$



(i) Prove that $\Delta DEC \equiv \Delta ABE$. **2**

(ii) Prove that ΔAED is isosceles. **1**

Question 7 (15 marks) **Marks**

(a) Simplify $\dfrac{x^2-3x}{y^2-6y+9} \div \dfrac{x}{y-3}$ **2**

(b) The function $y=f(x)$ is defined as follows:

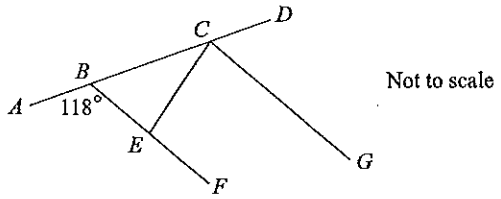
$$f(x)=\begin{cases} -2 \text{ for } x \le -1 \\ x+1 \text{ for } -1<x<2 \\ x^2+1 \text{ for } x \ge 2 \end{cases}$$

(i) Find the value of $f(-1)$. **1**

(ii) Write an expression for $f(a^2+3)$? **1**

(iii) Sketch the function $y=f(x)$. **2**

(c) BF is parallel to CG, $BC=EC$ and $\angle ABE=118^\circ$.



Not to scale

(i) Show that $\angle BEC=62^\circ$. **2**

(ii) Hence or otherwise, show that CG bisects $\angle DCE$. **2**

(d) Given that $f(x)=4x^2-7$, determine the values of x for which $f(x)=137$ **2**

(e) Make neat sketches of the following equations on separate sets of axes.
Mark clearly the essential features of each graph.

(i) $y=x^2+1$ **1**

(ii) $x^2+(y+3)^2=36$ **1**

(iii) $y=2^{-x}$ **1**

Question 8 (15 marks) **Marks**

(a) Draw a sketch showing the region on the number plane where the inequalities $x^2+y^2 \leq 4$ and $x \leq 1$ hold simultaneously. **3**

(b) Solve the simultaneous equations. **2**

$x-2y=4$

$2x+y=3$

(c) Expand then simplify each of the following.

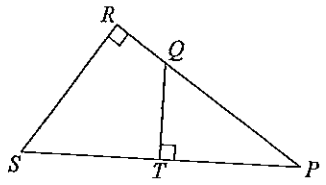
(i) $10-(y-4)^2$ **1**

(ii) $8p(2p+3q)-6p(3p-4q)$ **1**

(iii) $3(4a-7)(4a+7)-2(2a-5)^2$ **1**

(d) Solve $9^x-10(3^x)+9=0$ **2**

(e) In diagram, $QT \perp SP$, $\angle PRS=90^\circ$, $PT=4$, $RS=7$ and $PQ=5$.



(i) Prove that ΔPQT is similar to ΔPRS. **3**

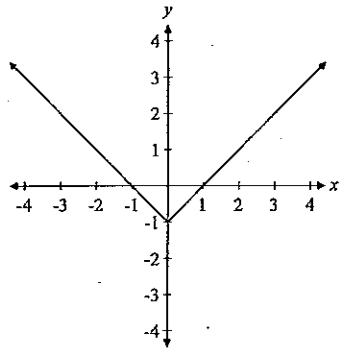
(ii) Find the length of QR. Give reasons for your answer. **2**

End of paper

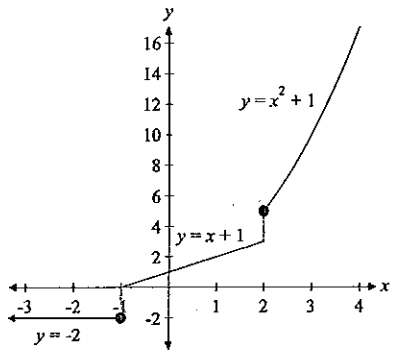
ACE Examination 2014

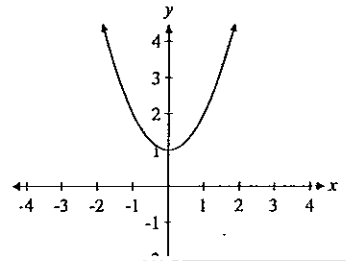
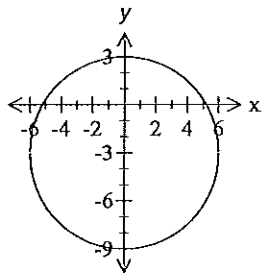
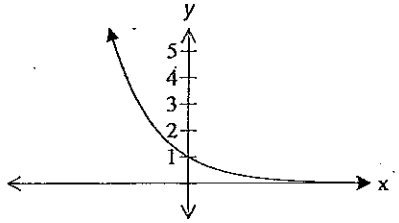
Year 11 Mathematics Half Yearly Examination

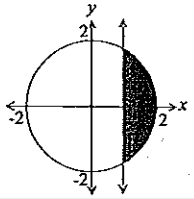
Worked solutions and marking guidelines

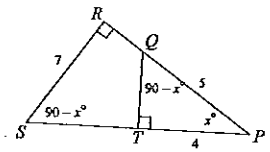
Section I		
	Solution	**Criteria**
1	$\lvert x-3\rvert \le 1$ $x-3 \le 1$ and $x-3 \ge -1$ $x \le 4$ $x \ge 2$	1 Mark: D
2	$\sqrt[3]{\dfrac{46.12\times 188.63}{(78.45)^2-(35.50)^2}} = 1.211362677\ldots \approx 1.21$	1 Mark: A
3	$x+x+y+y+70=180$ (angle sum of a ΔPQR is 180) $2x+2y=110$ $x+y=55$ $x+y+\angle ROQ=180$ (angle sum of a ΔROQ is 180) $\angle ROQ=180-(x+y)$ $\angle ROQ=180-55=125^\circ$	1 Mark: C
4	$\dfrac{(2ab^3)^3}{2a^2b^2}=\dfrac{8a^3b^9}{2a^2b^2}$ $=4ab^7$	1 Mark: D
5		1 Mark: A

Section II		
	Solution	**Criteria**
6(a)(i)	$2\times(y-3)=\left(\dfrac{3y}{2}+2\right)\times 2$ $2y-6=3y+4$ $y=-10$	1 Mark: Correct answer
6(a)(ii)	$6(x+2)=5(x-6)$ $6x+12=5x-30$ $x=-42$	1 Mark: Correct answer
6(b)	$\dfrac{1}{3-\sqrt{5}}=\dfrac{1}{3-\sqrt{5}}\times\dfrac{3+\sqrt{5}}{3+\sqrt{5}}$ $=\dfrac{3+\sqrt{5}}{4}$ $=\dfrac{3}{4}+\dfrac{1}{4}\sqrt{5}$	2 Marks: Correct answer. 1 Mark: Recognises the significance of the conjugate.
6(c)(i)	$x^2-25=(x+5)(x-5)$	1 Mark: Correct answer
6(c)(ii)	$27-8m^3=3^3-(2m)^3$ $=(3-2m)(9+6m+4m^2)$	1 Mark: Correct answer
6(c)(ii)	$xa+xb-3a-3b=x(a+b)-3(a+b)$ $=(a+b)(x-3)$	1 Mark: Correct answer
6(d)	$f(x)=x^3-6x$ $f(-x)=(-x)^3-6(-x)$ $=-\left(x^3-6x\right)$ $=-f(x)$ Function is odd	1 Mark: Correct answer
6(e)	$(418.6)^2\div 0.0179=9789159.777\ldots$ $\approx 9.789\times 10^6$	2 Marks: Correct answer. 1 Mark: Finds 9789159.77
6(f)	$x^2-6x+7=0$ $x=\dfrac{-(-6)\pm\sqrt{(-6)^2-4\times 1\times 7}}{2\times 1}$ $=\dfrac{6\pm\sqrt{8}}{2}$ $=3\pm\sqrt{2}\approx 4.41$ or 1.59	2 Marks: Correct answer. 1 Mark: Uses quadratic formula with one correct value.

6(g) (i)	ΔBCE is isosceles (two sides are equal, $EC = EB$) $\angle ECB = \angle EBC$ (base angles of an isosceles triangle are equal) $\angle DCB = \angle ABC = 90^\circ$ (all angles in a rectangle are equal) $\therefore \angle DCE = \angle ABE$ (adjacent angles) Consider ΔDEC and ΔABE $EC = EB$ (given) $DC = AB$ (opposite sides of a rectangle are equal) $\angle DCE = \angle ABE$ (shown above) $\Delta DEC \equiv \Delta ABE$ (SAS)	2 Marks: Correct answer. 1 Mark: One correct statement
6(g) (ii)	$DE = AE$ (matching sides in congruent triangles) $\therefore \Delta AED$ is isosceles (two sides are equal)	1 Mark: Correct answer
7(a)	$\frac{x^2-3x}{y^2-6y+9} \div \frac{x}{y-3} = \frac{x(x-3)}{(y-3)^2} \times \frac{(y-3)}{x}$ $= \frac{(x-3)}{(y-3)}$	2 Marks: Correct answer. 1 Mark: Factorises one term
7(b) (i)	$f(-1) = -2$	1 Mark: Correct answer
7(b) (ii)	$f(a^2+3) = (a^2+3)^2 + 1$ $(a^2+3 \geq 3$ for all values of $a)$ $= a^4 + 6a^2 + 10$	1 Mark: Correct answer
7(b) (iii)		2 Marks: Correct answer. 1 Mark: Sketches one correct graph.
7(c) (i)	$\angle ABE + \angle CBE = 180^\circ$ (Straight line measures 180°) $118^\circ + \angle CBE = 180^\circ$ $\angle CBE = 62^\circ$ ΔBCE is an isosceles triangle (two sides equal, $BC = EC$). $\therefore \angle CBE = \angle BEC$ (base angles in isosceles triangle are equal) Hence $\angle BEC = 62^\circ$	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.

7(c) (ii)	$\angle BEC = \angle ECG = 62^\circ$ (alternate angles, equal $BF//CG$) $\angle CBE = \angle DCG = 62^\circ$ (corresponding angles, equal $BF//CG$) $\therefore \angle ECG = \angle DCG = 62^\circ$ Hence CG bisects $\angle DCE$.	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.
7(d)	$f(x) = 4x^2 - 7 = 137$ $4x^2 = 144$ $x^2 = 36$ $x = \pm 6$	2 Marks: Correct answer. 1 Mark: Finds one solution or shows some understanding.
7(e) (i)		1 Mark: Correct answer
7(e) (ii)	 Circle centre $(0,-3)$ and radius 6	1 Mark: Correct answer
7(e) (iii)		1 Mark: Correct answer

8(a)	$x^2+y^2=4$ is a circle centre (0,0) and radius 2 $x=1$ is a vertical line passing through (1,0)  	3 Marks: Correct answer. 2 Marks: Graphs one region correctly 1 Mark: Shows some understanding
8(b)	$x-2y=4 \quad (1)$ $2x+y=3 \quad (2)$ Multiply equation (2) by 2 $4x+2y=6 \quad (3)$ Equation (1) + (3) $5x=10$ $x=2$ Substitute $x=2$ into equation (1) $2-2y=4$ $-2y=2$ $y=-1$ Solution is $x=2$ and $y=-1$.	2 Marks: Correct answer. 1 Mark: Finds one of the solutions.
8(c) (i)	$10-(y-4)^2=10-(y^2-8y+16)$ $=-y^2+8y-6$	1 Mark: Correct answer
8(c) (ii)	$8p(2p+3q)-6p(3p-4q)=16p^2+24pq-18p^2+24pq$ $=-2p^2+48pq$	1 Mark: Correct answer
8(c) (iii)	$3(4a-7)(4a+7)-4(2a-5)^2=3(16a^2-49)-4(4a^2-20a+25)$ $=48a^2-147-16a^2+80a-100$ $=32a^2+80a-247$	1 Mark: Correct answer
8(d)	$(3^x)^2-10(3^x)+9=0$ Let $m=3^x$ $m^2-10m+9=0$ $(m-9)(m-1)=0$ $m=9$ or $m=1$ $3^x=3^2 \quad 3^x=3^0$ $x=2 \quad x=0$	2 Marks: Correct answer. 1 Mark: Identifies a reduction to a quadratic.

8(e) (i)	In ΔPQT Let $x=\angle QPT$ $\angle QTP=90$ (given $QT \perp SP$) $\angle PQT+\angle QPS+\angle QTP=180°$ (angle sum of a triangle is 180) $\angle PQT+x+90=180°$ $\angle PQT=90-x$  In ΔPRS $\angle RSP+90+x=180$ (angle sum of a triangle is 180°) $\angle RSP=90-x$ $\therefore \Delta PQT$ is similar to ΔPRS (equiangular)	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: One relevant property stated or implied.
8(e) (ii)	Use Pythagoras theorem to find QT in ΔPQT $QR^2=QT^2+PT^2$ $5^2=QT^2+4^2$ $QT=3$ Corresponding sides are in proportion with similar triangles. $\frac{RP}{RS}=\frac{PT}{QT}$ or $\frac{RP}{7}=\frac{4}{3}$ $RP=\frac{28}{3}=9\frac{1}{3}$ Therefore $QR=9\frac{1}{3}-5$ $=4\frac{1}{3}$	2 Marks: Correct answer. 1 Mark: Uses corresponding sides in similar triangles with some correct values.