ACE Examinations

Student Name: _____

2014

YEAR 11

YEARLY EXAMINATION

Mathematics

General Instructions

- Reading time - 5 minutes
- Working time - 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Show all necessary working in Questions 11-14

Total marks - 70

Section I

10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II

60 marks

- Attempt Questions 11-14
- Allow about 1 hour 45 minutes for this section

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\ x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\ a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\ a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\ a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\ a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\ a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\ a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x,\ x > 0$

Section I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

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1 What is the value of $\dfrac{(2.34)^2-5.21}{\sqrt{19.75+7.08\times1.92}}$ correct to two significant figures?

(A) 0.04
(B) 0.05
(C) 0.045
(D) 0.046

2 What is the value of $\tan\theta$ given that $\cos\theta=-\dfrac{1}{2}$ for $180^\circ\le\theta\le360^\circ$?

(A) $-\sqrt{3}$
(B) $-\dfrac{1}{\sqrt{3}}$
(C) $\dfrac{1}{\sqrt{3}}$
(D) $\sqrt{3}$

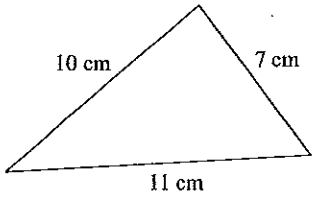
3 What is the solution to the inequality $|2x-3|\ge5$?

(A) $x\ge4$ or $x\le-1$
(B) $x\ge4$ or $x\ge-1$
(C) $x\le4$ or $x\le-1$
(D) $x\le4$ or $x\ge-1$

4 Evaluate $\lim\limits_{x\to4}\dfrac{x^2-3x-4}{x-4}$.

(A) Undefined
(B) 0
(C) 1
(D) 5

5 The following triangle has sides 7 cm, 10 cm and 11 cm.



Not to scale

Angle A is the smallest angle. Which of the following expressions is correct for angle A?

(A) $\cos A=\dfrac{7^2+11^2-10^2}{2\times7\times11}$
(B) $\cos A=\dfrac{10^2+7^2-11^2}{2\times7\times10}$
(C) $\cos A=\dfrac{10^2+11^2-7^2}{2\times10\times11}$
(D) $\cos A=\dfrac{10^2+7^2-11^2}{2\times10\times11}$

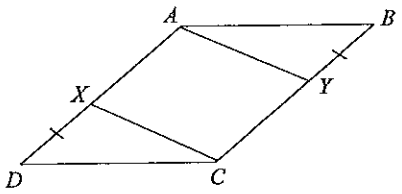
6 What is $\dfrac{4\sqrt{3}}{\sqrt{3}-\sqrt{2}}+\dfrac{4\sqrt{3}}{\sqrt{3}+\sqrt{2}}$ expressed with a rational denominator?

(A) 24
(B) 48
(C) 72
(D) 144

7 Which of the following is the correct expression for differentiating $f(x)=x^2-2x$ from first principles?

(A) $f'(x)=\lim\limits_{h\to0}\dfrac{(x-h)^2-2(x-h)-(x^2-2x)}{h}$
(B) $f'(x)=\lim\limits_{h\to0}\dfrac{(x-h)^2-2(x-h)-(x^2-2x)}{h}$
(C) $f'(x)=\lim\limits_{h\to0}\dfrac{(x+h)^2-2(x+h)+(x^2-2x)}{h}$
(D) $f'(x)=\lim\limits_{h\to0}\dfrac{(x+h)^2-2(x+h)-(x^2-2x)}{h}$

8 In the diagram below $ABCD$ is parallelogram and $BY = XD$.



Which test proves $\Delta ABY \equiv \Delta XCD$?

(A) AAA (B) AAS
(C) SAS (D) RHS

9 What is the equation of the line though the points $(6,0)$ and $(0,4)$?

(A) $3x+2y+12=0$
(B) $2x+3y+12=0$
(C) $3x+2y-12=0$
(D) $2x+3y-12=0$

10 What is the domain and range of the function $y=\sqrt{4-x}$?

(A) $x \le 4$; $y \ge 0$
(B) $0 \le x \le 4$; $y \ge 0$
(C) $x \ge 0$; $y \ge 0$
(D) All real x, all real y.

Section II

60 marks
Attempt Questions 11' 14
Allow about 1 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet.

All necessary working should be shown in every question.

Question 11 (15 marks) **Marks**

(a) Solve.

(i) $\frac{5y}{2}-\frac{2y}{3}=3$ **1**

(ii) $8(x+2)=14(9-x)$ **1**

(b) Factorise completely:

(i) a^2-4a+4 **1**

(ii) $ab-2bc+5a-10c$ **1**

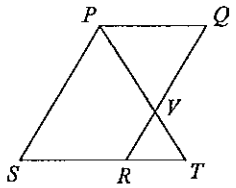
(c) Simplify $\sqrt{32}-\sqrt{18}+\sqrt{2}$. **2**

(d) Simplify $\frac{2^n \times 4^{n+1}}{8^n}$. **1**

(e) The base length x, of a square pyramid of volume V and perpendicular height h, is given by the formula $x=\sqrt{\frac{3V}{h}}$. Find the value of x, if $V=750$ and $h=6.95$. Answer correct to 2 decimal places **2**

(f) Find the equation of the normal to the curve $f(x)=x^2-4x+1$ at the point where $x=1$. **2**

(g) $PQRS$ is a parallelogram with SR produced to T.
Given $PS=110$ cm, $RT=40$ cm and $QV=60$ cm.



(i) Show that ΔPQV is similar to ΔTRV. **2**

(ii) Find the length of PQ. **2**

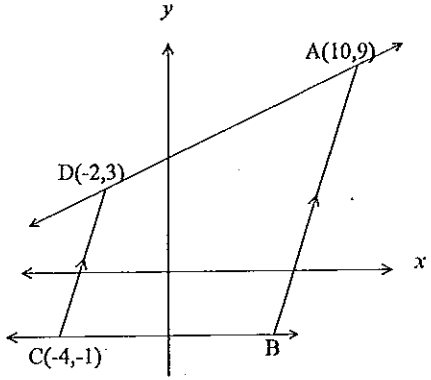
Question 12 (15 marks) **Marks**

(a) If $f(x)=3^x+3^{-x}$

(i) Evaluate $f(0)$ $f(1)$ and $f(-1)$. **2**

(ii) State the domain and range of $f(x)$. **1**

(b) Points $A(10,9)$, B, $C(-4,-1)$ and $D(-2,3)$ form a trapezium. Lines AB and CD are parallel. Line BC is parallel to the x-axis.



(i) Find the gradient of line AB? **1**

(ii) What is the equation of the line AB? **1**

(iii) Show that the coordinates of B are $(5,-1)$. **2**

(iv) Find the distance AD. **1**

(v) Find the equation of the circle centred A with radius AD. **1**

(vi) What are the coordinates of the midpoint of BD? **1**

(c) Molly is standing at the top of a vertical cliff. The cliff is 52 metres above sea level and she is 1.8 metres tall. Molly observes a ship out to sea with an angle of depression of 27°.

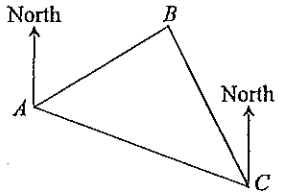
(i) Draw a neat sketch showing this information. **1**

(ii) How far is the ship from the base of the cliff? **2**

(d) Solve $4^x-5(2^x)+4=0$ **2**

Question 13 (15 marks) **Marks**

(a) Two ships leave port A at the same time. Ship X travels on a bearing of 070° and ship Y travels for 15.7 km on a bearing of 98° until the bearing of ship X from ship Y is 300°.



Not to scale

(i) Show that $\angle ABC = 130°$ 1

(ii) What is the bearing of port A from ship X? Answer the nearest degree 1

(iii) How far has ship X travelled? Answer correct to one decimal place. 2

(b) Find the value of k for which $x^2-(k-1)x-k=0$ has equal roots. 2

(c) Differentiate with respect to x.

(i) $3x^4+7x^2+1$ 1

(ii) $\sqrt{x^2+3}$ 1

(iii) $\dfrac{x+1}{x-1}$ 1

(d) Make neat sketches of the following equations on separate sets of axes. Mark clearly the essential features of each graph.

(i) $y=|x-2|$ (ii) $y=1+x^2$ 2

(iii) $xy=-2$ (iv) $y=-3^x$ 2

(e) For the function $f(x)=4x-3$, find:

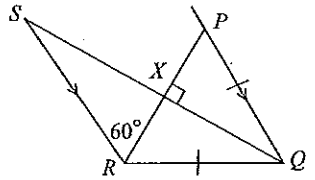
(i) the value of $f(3a-1)$. 1

(ii) the value of x at which $f(x)=0$. 1

Question 14 (15 marks) **Marks**

(a) Prove $\sin\theta\cos\theta\tan\theta=1-\cos^2\theta$. 2

(b) In the diagram below, PQ is parallel to SR, $PQ=QR$, $\angle XRS=60°$ and lines PR and SQ are perpendicular.



(i) Find the size of $\angle RQS$. Give reasons. 2

(ii) Prove that ΔQRS is an isosceles triangle. 2

(c) The quadratic equation $x^2-3x+7=0$ has roots α and β.
Find the value of:

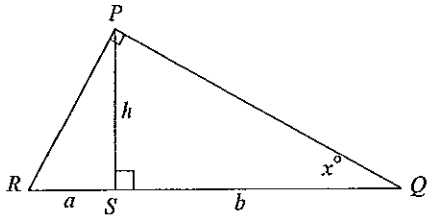
(i) $\alpha+\beta$ 1

(ii) $\alpha\beta$ 1

(iii) $\alpha^2+\beta^2$ 1

(iv) $\dfrac{3}{\alpha}+\dfrac{3}{\beta}$ 1

(d)



(i) Show that $\angle RPS=x°$ 2

(ii) Show that $h^2=ab$ 2

(iii) Hence find the area of ΔPQR in terms of a and b. 1

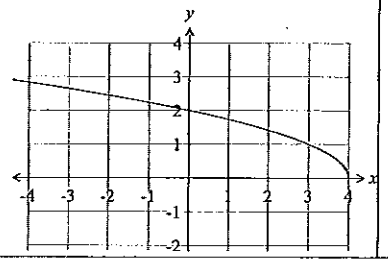
End of paper

ACE Examination 2014

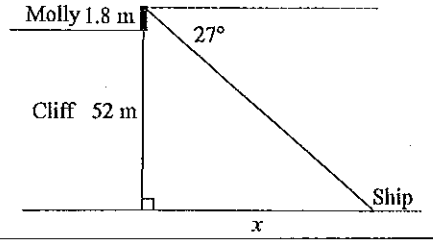
Year 11 Mathematics Yearly Examination

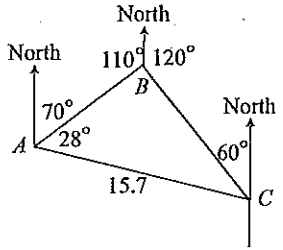
Worked solutions and marking guidelines

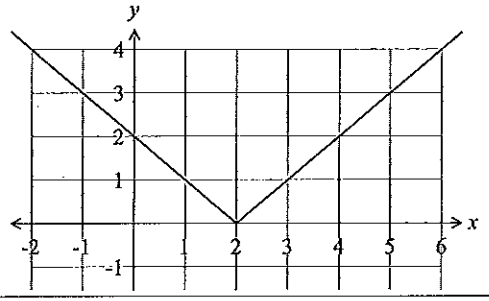
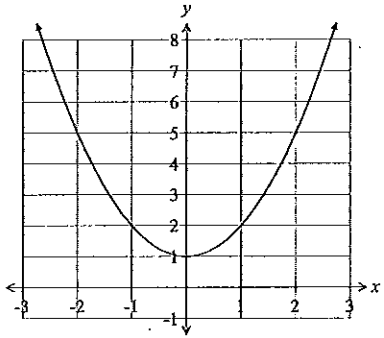
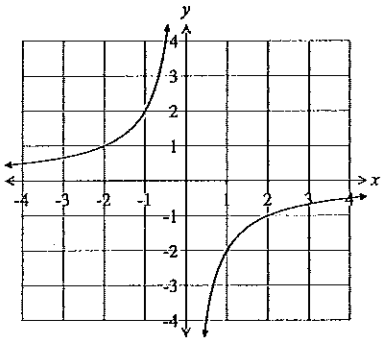
Section I		
	Solution	**Criteria**
1	$\dfrac{(2.34)^2-5.21}{\sqrt{19.75+7.08\times1.92}}=0.04599618658...$ ≈ 0.046 (2 signficant figures)	1 Mark: D
2	$\cos\theta=-\dfrac{1}{2}$ or $\theta=240^\circ$ $\tan 240^\circ=\sqrt{3}$	1 Mark: D
3	$\lvert 2x-3\rvert\geq 5$ $2x-3\geq 5$ and $2x-3\leq -5$ $2x\geq 8$ $\quad$ $2x\leq -2$ $x\geq 4$ $\quad$ $x\leq -1$	1 Mark: A
4	$\lim\limits_{x\to 4}\dfrac{x^2-3x-4}{x-4}=\lim\limits_{x\to 4}\dfrac{(x-4)(x+1)}{(x-4)}$ $=\lim\limits_{x\to 4}(x+1)=5$	1 Mark: D
5	Smallest angle is opposite the smallest side. $\cos A=\dfrac{10^2+11^2-7^2}{2\times10\times11}$	1 Mark: C
6	$\dfrac{4\sqrt{3}}{\sqrt{3}-\sqrt{2}}+\dfrac{4\sqrt{3}}{\sqrt{3}+\sqrt{2}}=\dfrac{4\sqrt{3}\left(\sqrt{3}+\sqrt{2}\right)+4\sqrt{3}\left(\sqrt{3}-\sqrt{2}\right)}{\left(\sqrt{3}-\sqrt{2}\right)\left(\sqrt{3}+\sqrt{2}\right)}$ $=\dfrac{12+4\sqrt{6}+12-4\sqrt{6}}{3-2}=24$	1 Mark: A
7	$f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $=\lim\limits_{h\to 0}\dfrac{(x+h)^2-2(x+h)-(x^2-2x)}{h}$	1 Mark: D
8	$AB=CD$ (opposite sides of a parallelogram are equal) $BY=XD$ (given data) $\angle ABY=\angle XDC$ (opposite angles of a parallelogram are equal) $\Delta ABY\equiv\Delta XCD$ (SAS)	1 Mark: C

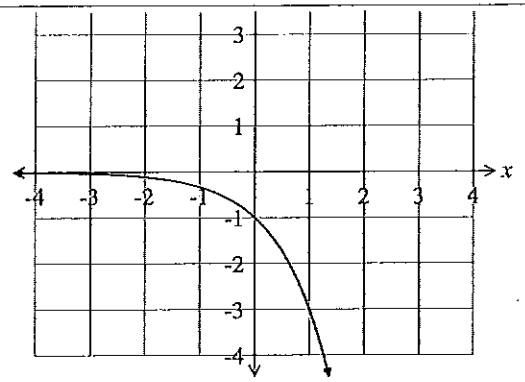
9	Gradient of line $M=\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{4-0}{0-6}=-\dfrac{4}{6}=-\dfrac{2}{3}$ $y-y_1=m(x-x_1)$ $\quad$ $y-0=-\dfrac{2}{3}(x-6)$ $3y=-2(x-6)$ $2x+3y-12=0$	1 Mark: D
10	$y=\sqrt{4-x}$ $y^2=4-x$ Top half of a parabola. Passes through (0,2) (4,0) Domain: $x\leq 4$ Range: $y\geq 0$ 	1 Mark: A
Section II		
11(a) (i)	$6\times\left(\dfrac{5y}{2}-\dfrac{2y}{3}\right)=3\times6$ $15y-4y=18$ $11y=18$ $y=\dfrac{18}{11}=1\dfrac{7}{11}$	1 Marks: Correct answer
11(a) (ii)	$8(x+2)=14(9-x)$ $8x+16=126-14x$ $22x=110$ $x=5$	1 Marks: Correct answer
11(b) (i)	$a^2-4a+4=(a-2)^2$	1 Marks: Correct answer
11(b) (ii)	$ab-2bc+5a-10c=b(a-2c)+5(a-2c)$ $=(a-2c)(b+5)$	1 Marks: Correct answer
11(c)	$\sqrt{32}-\sqrt{18}+\sqrt{2}=4\sqrt{2}-3\sqrt{2}+\sqrt{2}$ $=2\sqrt{2}$	2 Marks: Correct answer. 1 Mark: Shows some understanding.
11(d)	$\dfrac{2^n\times4^{n+1}}{8^n}=\dfrac{2^n\times2^{2n+2}}{2^{3n}}=2^2=4$	1 Marks: Correct answer

11(e)	$x=\sqrt{\frac{3V}{h}}$ $=\sqrt{\frac{3\times750}{6.95}}$ $=17.99280432...\approx17.99$	2 Marks: Correct answer. 1 Mark: Substitutes values into the formula.
11(f)	$f(x)=x^2-4x+1$ At $x=1$ $f(1)=1^2-4\times1+1=-2$ $f'(x)=2x-4$ At $x=1$ $f'(1)=2\times1-4=-2$ Normal is perpendicular to the gradient of the tangent $m_1m_2=-1,\quad m_1\times-2=-1,\quad m_1=\frac{1}{2}$ Equation of the normal at $(1,-2)$ $y-y_1=m(x-x_1)$ $y-(-2)=\frac{1}{2}(x-1)$ $2y+4=x-1$ $x-2y-5=0$	2 Marks: Correct answer. 1 Mark: Finds the gradient of the tangent or shows some understanding.
11(g) (i)	In ΔPQV and ΔTRV $\angle PVQ=\angle RVT$ (vertically opposite angles are equal) $\angle VPQ=\angle VTR$ (alternate angles are equal, parallel lines) $\angle PQV=\angle TRV$ (alternate angles are equal, parallel lines) ΔPQV is similar to ΔTRV (equiangular)	2 Marks: Correct answer. 1 Mark: Shows some understanding
11(g) (ii)	$\frac{PQ}{40}=\frac{60}{50}$ (corresponding sides in similar triangles) $PQ=\frac{60}{50}\times40$ $=48$ cm	2 Marks: Correct answer. 1 Mark: Uses corresponding sides in similar triangles
12(a) (i)	$f(0)=3^0+3^0=2$ $f(1)=3^{-1}+3^1=3\frac{1}{3},\qquad f(-1)=3^1+3^{-1}=3\frac{1}{3}$	2 Marks: Correct answer 1 Marks: Finds one of the values
12(a) (ii)	Domain: $\{x$: All real $x\}$ Range: $\{y$: $y\geq2\}$	1 Marks: Correct answer
12(b) (i)	Gradient of CD is equal to the gradient of AB (parallel) $M=\frac{y_2-y_1}{x_2-x_1}=\frac{3--1}{-2--4}=\frac{4}{2}=2$	1 Mark: Correct answer.

12(b) (ii)	$y-y_1=m(x-x_1)$ $y-9=2(x-10)$ $y-9=2x-20$ $2x-y-11=0$ (1)	1 Mark: Correct answer.
12(b) (iii)	The point B lies on the line BC $(y=-1)$ Substitute -1 for y into eqn (1) $2x-(-1)-11=0$ $2x=10$ or $x=5$ Coordinates of B are $(5,-1)$.	2 Marks: Correct answer. 1 Mark: Finds one of the coordinates.
12(b) (iv)	$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $=\sqrt{(10--2)^2+(9-3)^2}$ $=\sqrt{180}=6\sqrt{5}$ Distance AD is $\sqrt{180}$ or $6\sqrt{5}$	1 Mark: Correct answer.
12(b) (v)	Equation of a circle $(x-h)^2+(y-k)^2=r^2$ $(x-10)^2+(y-9)^2=\left(\sqrt{180}\right)^2=180$	1 Mark: Correct answer
12(b) (vi)	Mid-point formula $x=\frac{x_1+x_2}{2}=\frac{-2+5}{2}=\frac{3}{2}\qquad y=\frac{y_1+y_2}{2}=\frac{3+-1}{2}=1$ Midpoint is $\left(\frac{3}{2},1\right)$	1 Mark: Correct answer.
12(c) (i)		1 Mark: Correct answer.
12(c) (ii)	Complement of 27° is 63° $\tan63^\circ=\frac{x}{52+1.8}$ $x=53.8\tan63^\circ$ $=105.5884452...\approx106$ m Ship is 106 metres from the base of the cliff.	2 Marks: Correct answer. 1 Mark: Uses appropriate trig ratio.

12(d)	$4^x - 5(2^x) + 4 = 0$ Let $m = 2^x$ then $m^2 - 5m + 4 = 0$ $(m-4)(m-1) = 0$ $m = 4$ $\quad$ $m = 1$ $2^x = 4$ $\quad$ $2^x = 1$ $x = 2$ $\quad$ $x = 0$	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.
13(a) (i)	$\angle ABC + 110^\circ + 120^\circ = 360^\circ$ $\angle ABC = 130^\circ$ 	1 Marks: Correct answer
13(a) (ii)	Bearing $= 120^\circ + 130^\circ$ $= 250^\circ$ The bearing of B from A is 250°	1 Marks: Correct answer
13(a) (iii)	To find the distance AB we require $\angle BCA$ $\angle BCA + 28^\circ + 130^\circ = 180^\circ$ $\angle BCA = 22^\circ$ $\frac{AB}{\sin 22^\circ} = \frac{15.7}{\sin 130^\circ}$ $AB = \frac{15.7 \sin 22^\circ}{\sin 130^\circ}$ $= 7.67752259\ldots \approx 7.7$ km Ship X has travelled 7.7 km	2 Marks: Correct answer. 1 Mark: Uses sine rule with at least 2 correct values.
13(b)	Equal roots $\Delta = 0$ $\Delta = b^2 - 4ac$ $= (-(k-1))^2 - 4 \times 1 \times (-k)$ $= k^2 - 2k + 1 + 4k$ $= k^2 + 2k + 1$ $= (k+1)^2$ Therefore $k = -1$	2 Marks: Correct answer. 1 Mark: Recognises $\Delta = 0$ and substitutes at least one correct value.

13(c) (i)	$\frac{d}{dx}(3x^4 + 7x^2 + 1) = 12x^3 + 14x$	1 Marks: Correct answer
13(c) (ii)	$\frac{d}{dx}\sqrt{x^2+3} = \frac{d}{dx}(x^2+3)^{\frac{1}{2}} = \frac{1}{2}(x^2+3)^{\frac{1}{2}} \times 2x = \frac{x}{\sqrt{x^2+3}}$	1 Marks: Correct answer
13(c) (iii)	$\frac{d}{dx}\left(\frac{x+1}{x-1}\right) = \frac{(x-1)\times 1 - (x+1)\times 1}{(x-1)^2}$ $= \frac{-2}{(x-1)^2}$	1 Marks: Correct answer
13(d) (i)		1 Marks: Correct answer
13(d) (ii)		1 Marks: Correct answer
13(d) (iii)		1 Marks: Correct answer

13(d) (iv)		1 Marks: Correct answer
13(e) (i)	$f(x)=4x-3$ $f(3a-1)=4\times(3a-1)-3$ $=12a-4-2$ $=12a-6$	1 Marks: Correct answer
13(e) (ii)	$f(x)=4x-3$ $0=4x-3$ $4x=3$ $x=\frac{3}{4}$	1 Marks: Correct answer
14(a)	$\text{LHS}=\sin\theta\cos\theta\tan\theta$ $=\sin\theta\cos\theta\times\frac{\sin\theta}{\cos\theta}$ $=\sin^2\theta$ $=1-\cos^2\theta$ $=\text{RHS}$	2 Marks: Correct answer. 1 Mark: Uses a relevant trig identity.
!4(b) (i)	$\angle QPX=\angle XRS=60^\circ$ (alternate angles are equal, $PQ \parallel SR$) ΔPQR is an isosceles triangle (two equal sides, $PQ=QR$) $\angle QPX=\angle QRX=60^\circ$ (base angles are equal, isosceles triangle) $\angle RXQ+\angle PXQ=180^\circ$ (adjacent angles on a straight line) $\angle RXQ+90^\circ=180^\circ$ $\angle RXQ=90^\circ$ In ΔQXR $\angle RQX+\angle RXQ+\angle QRX=180^\circ$ (Angle sum of a triangle is 180°) $\angle RQX+90^\circ+60^\circ=180^\circ$ $\angle RQX=30^\circ$ or $\angle RQS=30^\circ$	2 Marks: Correct answer. 1 Mark: Finds $\angle RQS$ without appropriate reasons. Alternatively makes significant towards the solution.

!4(b) (ii)	In ΔQRS $\angle QSR+\angle QRS+\angle RQS=180^\circ$ (Angle sum of a triangle is 180°) $\angle QSR+(60^\circ+60^\circ)+30^\circ=180^\circ$ $\angle QSR=30^\circ$ $\therefore\angle RQS=\angle QSR=30^\circ$ ΔQRS is an isosceles triangle (two base angles are equal).	2 Marks: Correct answer. 1 Mark: Shows some understanding of the problem.
!4(c) (i)	$\alpha+\beta=-\frac{b}{a}=-\frac{-3}{1}=3$	1 Marks: Correct answer
!4(c) (ii)	$\alpha\beta=\frac{c}{a}=\frac{7}{1}=7$	1 Marks: Correct answer
!4(c) (iii)	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ $=(3)^2-2\times7=-5$	1 Marks: Correct answer
!4(c) (iv)	$\frac{3}{\alpha}+\frac{3}{\beta}=\frac{3\beta+3\alpha}{\alpha\beta}=\frac{3(\alpha+\beta)}{\alpha\beta}=\frac{3\times3}{7}=\frac{6}{7}$	1 Marks: Correct answer
!4(d) (i)	In ΔPQR $\angle PQR+\angle RPQ+\angle PRQ=180^\circ$ (Angle sum of triangle is 180°) $x+90^\circ+\angle PRQ=180^\circ$ $\angle PRQ=90^\circ-x$ In ΔPSR $\angle RPS+\angle PSR+\angle PRS=180^\circ$ (Angle sum of triangle is 180°) $\angle RPS+90^\circ+(90^\circ-x^\circ)=180^\circ$ $\angle RPS=x^\circ$	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.
14(d) (ii)	In ΔPSR $\tan x^\circ=\frac{a}{h}$ In ΔPSQ $\tan x^\circ=\frac{h}{b}$ Hence $\frac{a}{h}=\frac{h}{b}$ (both $\tan x^\circ$) $h^2=ab$	2 Marks: Correct answer. 1 Mark: Finds one trig equation.
14(d) (iii)	$h^2=ab$ $h=\sqrt{ab}$ $A=\frac{1}{2}bh$ $=\frac{1}{2}\times(a+b)\times\sqrt{ab}=\frac{(a+b)\sqrt{ab}}{2}$	1 Marks: Correct answer