Student Name / Number ____________

Advanced Mathematical Publications

SPECIMEN PAPER

2001 MATHEMATICS EXTENSION 1

Instructions :

Time Allowed: *2 periods* *(Plus 5 mins reading time)*

- Attempt ALL questions.
- ALL questions are of equal value.
- All necessary working should be shown.
- Marks may be deducted for poorly arranged or missing working.
- Use a SEPARATE Writing Booklet for each question.
- Write your Name / Student Number on every page.
- Board-approved calculators may be used.

Question 1 (12 marks) **Marks**

(a) Find the primitive function of the following: 4

(i) $\frac{1+x}{x^4}$ (ii) $(3x^2-1)^2$

(b) Evaluate the following definite integrals 4

(i) $\int_{-1}^{1}(x+1)^2(x-1)\,dx$

(ii) $\int_{-2}^{0}\frac{1}{\sqrt{9-8x}}\,dx$

(c) Prove that $\int_{\frac{1}{2}}^{1}\left(x^2+\frac{6}{x^3}\right)dx=\int_{1}^{2}\left(6x+\frac{1}{x^4}\right)dx$ 4

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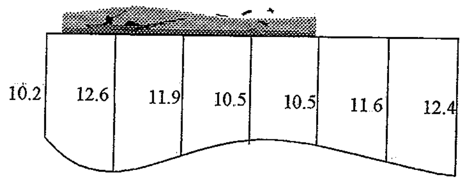
Question 2 (12 marks)

Marks

(a) 6



The diagram above shows the cross-section of a river bed taken by firing sonar waves at equal intervals into the water. The width of the river bed is 240 metres long and the different depths (h), in metres, are as shown. Copy and complete the table of values below into your Writing Booklet.

x	0	40					240
h	10.2						12.4

(i) Using (α) Trapezoidal rule and (β) Simpson's rule, find the approximate cross-sectional area of the river bed (to 1 decimal place).

(ii) Use the answer in (β) for this part. If the river current is flowing at the rate of 1.5 metres per second, how much water would flow through in 1 minute? (Express the answer in Megalitres, you may assume $1ML = 10^6$ cubic metres).

(b) (i) Sketch the graph of $y=\sqrt{x+2}$ and shade in the areas enclosed by the curve, the x and y axes and the line $y=3$. 4

(ii) Show that the shaded area is $\left(\frac{8\sqrt{2}}{3}+3\right)$ units^2.

(c) If $\frac{dy}{dx}=3\sqrt{x}$, find the equation of the curve $y=f(x)$ if the curve passes through the point $(4,15)$. 2

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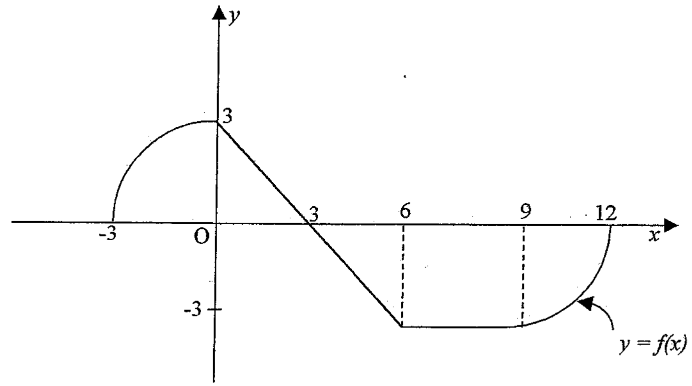
Question 3 (12 marks)

Marks

(a)



The diagram above shows a function $y=f(x)$ consisting of two quadrants, two triangles and a square.
Find

(i) the exact area enclosed by the function and the x-axis. 2

(ii) $\int_{-3}^{12} f(x)dx$ 1

(b) Find the locus of $P(x, y)$ which is 2 units away from the line $3x-4y+1=0$. 2

(c) (i) Find the locus of $P(x, y)$ whose gradient from the point $A(-1, 3)$ is twice the gradient from the point $B(2, 3)$. 3

(ii) Explain why the equation of the locus is ***not*** $y=3$.

(d) $P(x,y)$ is a point which moves so that its distance from $A(-1,5)$ is always equal to its distance from the line $y=1$. 4

(i) Show by derivation that the equation of the locus of P is
$$(x+1)^2=8y-24.$$

(ii) Sketch the locus of P clearly labeling the vertex, focus and the directrix.

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Question 4 (12 marks)

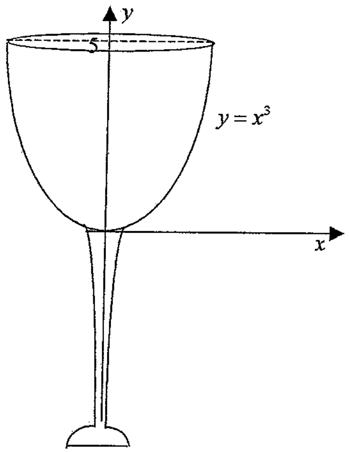
Marks

(a) The curves $y^2 = 4x$ and $y = 2x^2$ meet at the origin O and the point A forming a loop. **5**

(i) Show that the coordinates of A is (1, 2).

(ii) Show that the line OA divides the loop into two equal parts.

(b) The bowl of a wine glass is formed by rotating that part of the curve $y = x^3$, for the values of y from 0 to 5, through one complete revolution about the y-axis. **5**



Show that the amount of wine in the glass when the depth is y is $\frac{3}{5}\pi\sqrt[3]{y^5}$ cm³.

Find the capacity of the glass if it is 5 cm deep (to the nearest cu. cm)

(c) The latus rectum of a parabola measures 12 units and its extremities are (-6, 3) and (6, 3), find the equation of this parabola. **2**

End of Assessment task 2

(1) (a) (i) $-\frac{1}{3x^3} - \frac{1}{2x^2} + c$

(ii) $\frac{9x^5}{5} - 2x^3 + x + c$

(b) (i) $-1\frac{1}{3}$

(ii) $\frac{1}{2}$

(c) Show that LHS = RHS = $9\frac{7}{24}$.

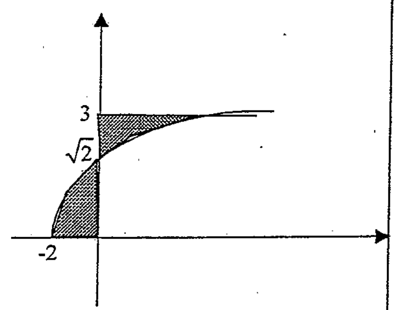
(2)

40	80	120	160	200
12.6	11.9	10.5	10.5	11.6

(a) (i) (α) 2736.0 (β) 2749.3

(ii) 0.2474 ML

(b) (i)



(ii) Proof

(c) $y = 2\sqrt{x^3} - 1$

(3) (a) (i) $18 + \frac{9\pi}{2}$ (ii) −9

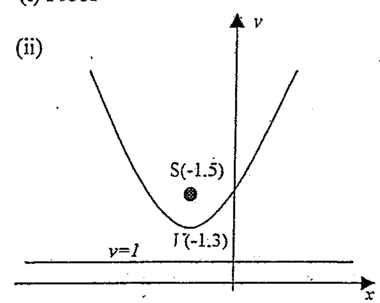
(b) $3x - 4y - 9 = 0$ or $3x - 4y + 11 = 0$

(c) (i) $x = -4;\ y \neq 3$

(ii) $y = 3$ has no gradient.

(d) (i) Proof

(ii)



(4) (a) (i) Proof (ii) Proof

(b) Proof; 28 cm³

(c) $x^2 = 12y$