Alexandria Park Community School

2015 Preliminary Assessment Task 3

Mathematics Extension 1

General Instructions
- Reading time – 3 minutes
- Working time – 57 minutes
- Write using blue or black pen
  Black pen is preferred
- Board-approved calculators may be used
- In Questions 5-6, show relevant mathematical reasoning and/or calculations

Total Marks – 34

Section I

4 marks
- Attempt questions 1-4
- Allow about 7 minutes for this section

Section II

30 marks
- Attempt questions 5-6
- Allow about 50 minutes for this section

SECTION I: Objective Response Questions (4 marks)

1. If a curve has a point of inflexion then it must

(A) Change from a positive to a negative gradient

(B) Have a gradient of zero at the point of inflexion

(C) Change concavity

(D) All of the above

2. If α and β are the roots of the quadratic equation $x^2 - 5x + 2 = 0$, evaluate $\alpha\beta^2 + \alpha^2\beta$

(A) 10

(B) -10

(C) 5

(D) -5

3. For what values of k does the quadratic equation $x^2 + (k-3)x + k = 0$ have no real roots

(A) $k \leq 1, k \geq 9$

(B) $k < 1, k > 9$

(C) $1 \leq k \leq 9$

(D) $1 < k < 9$

4. If the minimum value of $3x^2 - 6x + k$ is 8, find the value of k

(A) 8

(B) 11

(C) $\frac{8}{3}$

(D) $\frac{11}{3}$

SECTION II

Marks

Question 5 (15 marks)

a) Differentiate

(i) $f(x) = 6x^4 + 3x^2 + 5$ **1**

(ii) $f(x) = (x+3)^3(x^2+1)$ **2**

b) Consider the curve given by $y = 7 + 4x^3 - 3x^4$

i) Find the coordinates of the stationary points **1**

ii) Find all values of x for which; $\frac{d^2y}{dx^2} = 0$ **2**

iii) Determine the nature of the stationary points **2**

iv) Sketch the curve for the domain $-1 \leq x \leq 2$ **2**

c) Sketch the following graph showing any stationary points, inflexions, asymptotes or any other features **5**

$$f(x) = 1 + \frac{1}{x^2 - 1}$$

End of Question 5

2015 Preliminary Assessment Task 3 *Extension 1 Mathematics*

Question 6 (15 marks)

Marks

a) Differentiate $f(x) = 4x^2 + 3x - 8$ from first principles **2**

b) Find the equation of the normal to the curve $f(x) = \frac{1}{\sqrt{x}}$ at the point x = 2 **3**

c) (i) Show that the curves $y = (3x-4)^3$ and $y = \frac{x-18}{4-3x}$ intersect at (2, 8) **1**

(ii) Find the acute angle to the nearest degree between the curves at this point **3**

d) Find the values of a, b and c for which $m^2 = a(m-1)^2 + b\,(m-2)^2 + c(m-3)^2$ **3**

e) Solve $2\sin^2 x = 3(\cos x + 1)$ for all real values of x **3**

End of Question 6

Student Name ______ Teacher______________

2015 Preliminary 3U Mathematics Task 3
Multiple Choice Answer Sheet

Completely fill the response oval representing the most correct answer.

1.	A ○	B ○	C ●	D ○
2.	A ●	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ●
4.	A ○	B ●	C ○	D ○

MC	Q5	Q6
4	14	8

$\frac{26}{32}$

Name	Class

Question

Section 2

a) i) $\frac{dy}{dx} = 24x^3 + 6x$ (1)

iii) $\frac{dy}{dx}$: $u(x+3)^3$ $v(x^2+1)$

$= 3(x+3)^2 \times (x^2+1) + 2x(x+3)^3$

(1) $(x+3)^2(3(x^2+1)+2x)$

$\frac{dy}{dx} = (x+3)^2(3x^2+5x)$

b) i) $y = 7+4x^3-3x^4$

$\frac{dy}{dx} = 12x^2 - 12x^3$

~~$12x^2(1-x^2)$~~

$12x^2(1-x^2)$

(1) $12x^2(1+x)(1-x)$

$\therefore (0,7)\ (1,8)$ (-1

ii) $\frac{d^2y}{dx^2} = 24x - 36x^2$

$12x(2-3x)$

(2) $x = 0, \frac{2}{3}$ $2 = 3x$ $x = \frac{2}{3}$

ii) $f''(0) = 0$ $\therefore$ Horizontal point at inflection

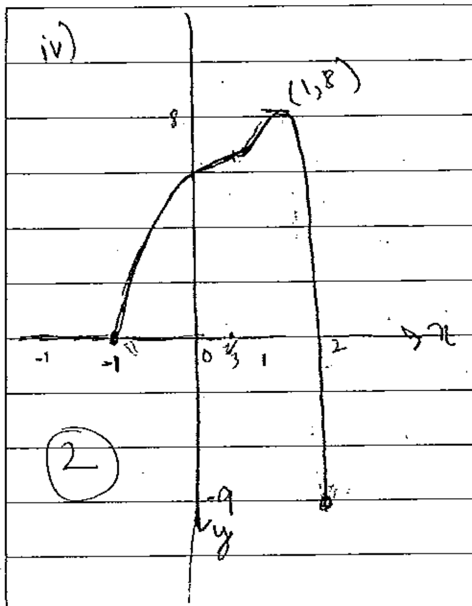
$f''(1) = -12 < 0$ $\therefore$ max. t.p

(2)

Name	Class

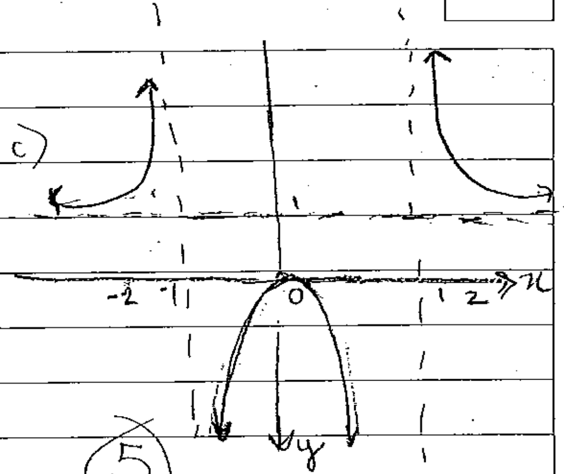
Question

iv)



2

c)



5

1.125

c) $f(x) = 1 + \frac{1}{x^2-1}$ $\quad \frac{dy}{dx} = (x^2-1)^{-1}$

$-1(x^2-1)^{-2} \cdot x \cdot 2x$

$\frac{x^2-1+1}{x^2-1} = \frac{x^2}{x^2-1}$ $\quad \frac{-2x}{(x^2-1)^2}$ stat. points $(0,0)$

$\therefore$ horizontal asymptote at 1

$x \neq \pm 1$ vertical asymptotes

$-2x(x^2-1)^{-2}$

$f''(1/2) < 0$ $\quad -2x(x^2-1)^{-2}$

$f''(-1/2) < 0 \therefore$ max t.p.p $\quad -4x(x^2-1)^{-3} \times 2x$

$-8x^2(x^2-1)^{-3}$

$f''(0) = 0$

and $f''(1/2) = f''(1/2)$

$\therefore$ no inflexion (sign change)

Name	Class

Question

Question 6

a) $\frac{f(x+h) - f(x)}{h}$

$\cancel{4(x+h)^2 + 3(x+h) - 8 - 4x^2 - 3x + 8}$

$\cancel{4x^2 + 8xh + 4h^2 + 3x + 3h - 8 - 4x^2 - 3x + 8}$

$8xh + 4h^2 + \cancel{3x} - x + 3h$

$\frac{\cancel{h}(8x + 4h + 3)}{\cancel{h}}$

$\therefore \frac{dy}{dx} = 8x + 3$ $\quad$ 2

b) $\frac{dy}{dx} = x^{-1/2}$ $\quad x^{-1/2}$

$-\frac{1}{2}x^{-1 1/2}$ $\quad -\frac{1}{2}x$

$\frac{-1}{2x\sqrt{x}} = \frac{dy}{dx}$

$f'(2) = \frac{-1}{4\sqrt{2}}$ $\quad$ normal $4\sqrt{2}$

$y - \frac{1}{\sqrt{2}} = 4\sqrt{2}(x-2)$ $\quad 8x - \sqrt{2}y - 15 = 0$

$y - \frac{1}{\sqrt{2}} = 4\sqrt{2}x - 8\sqrt{2}$ $\quad$ 2

$\sqrt{2}y - 1 = 8x - 16$

Name	Class	Question

c) (i) LHS $= 8$ RHS $= (6-4)^3$

$= 2^3 = 8$

$\therefore$ true for $y = (3x-4)^3$

LHS $= 8$ RHS $= \dfrac{2-18}{4-3(2)} = \dfrac{-16}{-2}$

$= 8$

$\therefore$ LHS = RHS

true for $y = \dfrac{x-18}{4-3x}$

2

$x-4)$

ii) $\tan\theta \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$ dx

$\dfrac{1(4-3x) + -3(x-18)}{(4-3x)^2}$

$f'(2)_1 = 36$

$f'(2)_2 = \cancel{\ } \; 2.5$ 33.5

$4 - 3x - 3x + 18$

$\therefore \left|\dfrac{\cancel{\ }}{\cancel{\ }\; 1 + 90}\right|$

$4 - 6x + 18$

$\dfrac{dy}{dx} = \dfrac{-6x+22}{(4-3x)^2}$

2

$\tan\theta = \cancel{\ } \; 0.368$

$\theta = \cancel{\ } \; 20°13'$

d) m^2 1 = ~~...~~

2 = ~~...~~ (3) $4 = a + C$

1 (1) $1 = b + 4C$

(2) $9 = 4a + b$

e) $2\sin^2 x = 3(\cos x + 1)$

$\sin^2 x = \dfrac{3(\cos x + 1)}{2}$

(d) $m^2 = a(m-1)^2 + b(m-2)^2 + c(m-3)^2$

Let $m = 1 \Rightarrow 1 = b + 4c \quad \ldots (1)$

Let $m = 2 \Rightarrow 4 = a + c \quad \ldots (2)$

Let $m = 3 \Rightarrow 9 = 4a + b \quad \ldots (3)$

(1) $-$ 4(2) $\quad -15 = b - 4a$

$\Rightarrow 4a - b = 15 \quad \ldots (4)$

(3) + (4) $\quad \Rightarrow 8a = 24$

$a = 3, \quad b = -3, \quad c = 1$

(e) $2\sin^2 x = 3\cos x + 3$

$2(1 - \cos^2 x) = 3\cos x + 3$

$\therefore 2\cos^2 x + 3\cos x + 1 = 0$

$(2\cos x + 1)(\cos x + 1) = 0$

$\cos x = -\frac{1}{2}$ or -1

$x = 120°, 240°, 180°$