1. Simplify each of the following completely, rationalizing any denominators:
   (a) $3\sqrt{8} + 5\sqrt{18}$,
   (b) $\dfrac{4+\sqrt{7}}{4-\sqrt{7}}$.

2. Simplify
   (a) $\dfrac{81a^4 - 16b^4}{3a - 2b}$,
   (b) $\dfrac{125a^3 + 27b^3}{5a + 3b}$.

3. Solve for x the inequalities:
   (a) $3 - 2x < 17$,
   (b) $4 \geq x^2$,
   (c) $3 + 2x \leq 5$.

4. (a) Express $5\frac{7}{11}$ as a recurring decimal.
   (b) Express $3{\cdot}7\dot{4}\dot{5}$ as a fraction.

5. If $x = 7 - 4\sqrt{3}$, show that $x + 1/x$ is rational, and find its value.

6. If $f(x) = x(x+4)$, find the values of x for which $f(x) = 5$.

7. Draw sketches of the following curves. Be careful to show any x-intercepts, y-intercepts and other important points.
   (a) Sketch $3x - 4y = 12$.
   (b) Sketch on one set of axes, $y = x^2$ and $y = x^2 - 4$.
   (c) Sketch on one set of axes, $y = 2^x$ and $y = 2^{-x}$.
   (d) Sketch $(x-3)^2 + y^2 = 25$.
   (e) Sketch on one set of axes, $y = \sqrt{x}$ and $y = \sqrt{x-2}$.
   (f) Sketch $y = -\sqrt{9 - x^2}$.State the domain and range of this function.
   (g) Sketch $y = 4/x$.
   (h) Sketch the function defined by $y = \begin{cases} x^2, & \text{for } x < 0, \\ 2x, & \text{for } x \geq 0. \end{cases}$

8. Explain why the inverse of $y = -\sqrt{9 - x^2}$ is not a function. You may refer to your sketch in 7 (f).

9. Find the inverse functions $f^{-1}(x)$ of these functions:

   (a) $f(x) = 9 - 7x$,

   (b) $f(x) = \dfrac{4-x}{3-x}$.

10. (a) Sketch the graph of the function $f(x) = \dfrac{1}{x} + 2$ and its inverse on the one axis.

    (b) Find the equation of the inverse of $f(x)$.

11. (a) Show that $x^2 + \dfrac{1}{x^2} = \left(x + \dfrac{1}{x}\right)^2 - 2$.

    (b) Hence if $x = \dfrac{3-\sqrt{2}}{3+\sqrt{2}}$, find with rational denominator $x + \dfrac{1}{x}$ and $x^2 + \dfrac{1}{x^2}$.

mls SGS February 2003

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1. (a) $3\sqrt{8} + 5\sqrt{18}$
3
$= 6\sqrt{2} + 15\sqrt{2}$
$= 21\sqrt{2}$ ✓

(b) $\dfrac{4+\sqrt{7}}{4-\sqrt{7}} = \dfrac{4+\sqrt{7}}{4-\sqrt{7}} \times \dfrac{4+\sqrt{7}}{4+\sqrt{7}}$ ✓

$= \dfrac{16 + 8\sqrt{7} + 7}{16 - 7}$

$= \dfrac{23 + 8\sqrt{7}}{9}$ ✓

2. (a) $\dfrac{81a^4 - 16b^4}{3a - 2b} = \dfrac{(9a^2 - 4b^2)(9a^2 + 4b^2)}{3a - 2b}$ ✓
5.

$= \dfrac{\cancel{(3a-2b)}(3a+2b)(9a^2+4b^2)}{\cancel{(3a-2b)}}$ ✓

$= (3a+2b)(9a^2+4b^2)$ ✓

(b) $\dfrac{125a^3 + 27b^3}{5a + 3b} = \dfrac{\cancel{(5a+3b)}(25a^2 - 15ab + 9b^2)}{\cancel{(5a+3b)}}$ ✓

$= 25a^2 - 15ab + 9b^2$ ✓

3. (a) $3 - 2x < 17$
5
$-2x < 14$ ✓
$x > -7$ ✓

(b) $4 \geq x^2$
$-2 \leq x \leq 2$ ✓✓

(c) $3 + 2x \leq 5$
$2x \leq 2$
$x \leq 1$ ✓

4. (a) $5\frac{7}{11} = 5.\dot{6}\dot{3}$ ✓
3

4(b) let $x = 3.7\dot{4}\dot{5}$
$100x = 374.545454\ldots$ ✓
$x = 3.745454\ldots$
$99x = 370.8$
$x = \dfrac{370.8}{99}$ or you can work with just the $0.7\dot{4}\dot{5}$ part.
$= \dfrac{3708}{990}$ ✓ $\left(\dfrac{206}{55}\right)$
$= 3\frac{41}{55}$

5. $x = 7 - 4\sqrt{3}$
2
$\dfrac{1}{x} = \dfrac{1}{7-4\sqrt{3}} \cdot \dfrac{7+4\sqrt{3}}{7+4\sqrt{3}}$

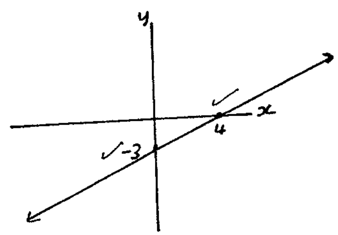
$= \dfrac{7+4\sqrt{3}}{49-48}$ ✓

$= 7 + 4\sqrt{3}$

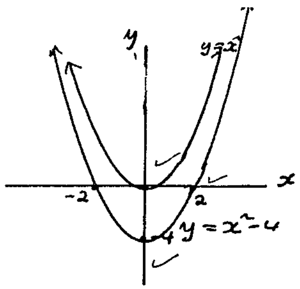
so, $x + \frac{1}{x} = 7 - 4\sqrt{3} + 7 + 4\sqrt{3}$
$= 14$ which is rational. ✓

6. $f(x) = x(x+4) = 5$ ✓
2
$x^2 + 4x - 5 = 0$
$(x+5)(x-1) = 0$
$x = -5, 1$ ✓

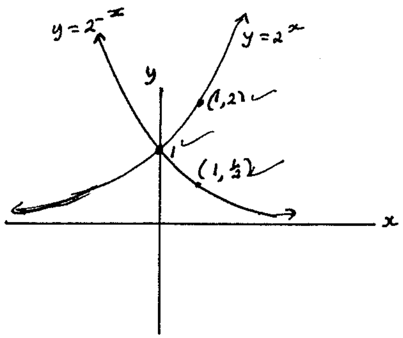
7. (a)
23



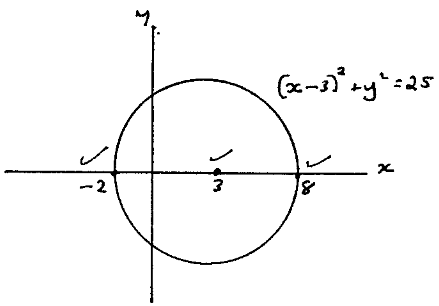
7 (b)



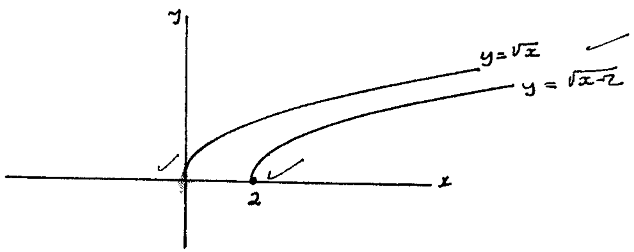
(c)



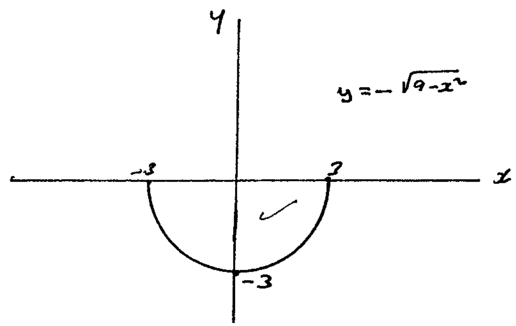
(d)



(e)



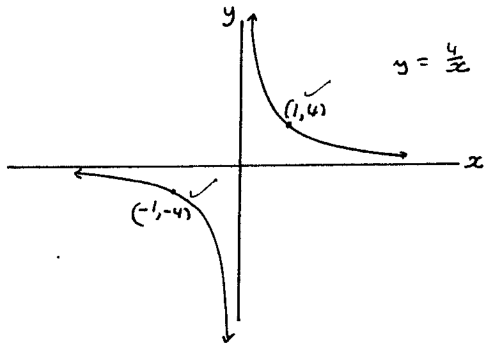
7. (f)



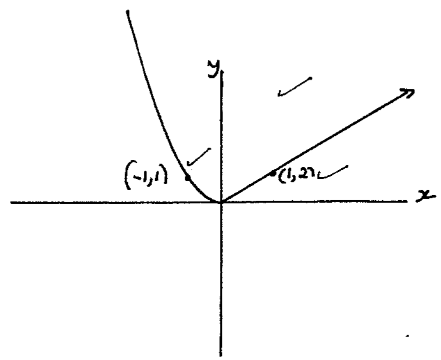
Domain: $-3 \leq x \leq 3$

Range: $-3 \leq y \leq 0$

(g)



(h)



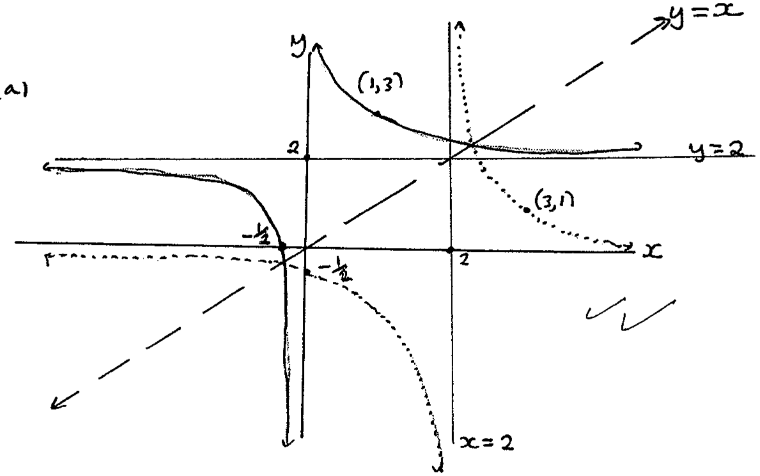
8. A horizontal line crosses the graph of
1 $y = -\sqrt{9-x^2}$ more than once, so a vertical line crosses its inverse more than once. ✓
So the inverse is not a function.

9. (a) $y = 9-7x$
5
so, $x = 9-7y$ ✓
$7y = 9-x$
$y = \dfrac{9-x}{7}$ ✓

(b) $y = \dfrac{4-x}{3-x}$

so, $x = \dfrac{4-y}{3-y}$ ✓
$x(3-y) = 4-y$ ✓
$3x - xy = 4-y$
$y - xy = 4-3x$
$y(1-x) = 4-3x$
$y = \dfrac{4-3x}{1-x}$ ✓

10. (a)
4



✓✓

10 (b) $y = \dfrac{1}{x} + 2$
$x = \dfrac{1}{y} + 2$ ✓
$\dfrac{1}{y} = x - 2$
$y = \dfrac{1}{x-2}$ ✓

11. (a) $\left(x + \frac{1}{x}\right)^2 = x^2 + 2.x.\frac{1}{x} + \frac{1}{x^2}$
4.
$= x^2 + 2 + \frac{1}{x^2}$
so $\left(x + \frac{1}{x}\right)^2 - 2 = x^2 + 2 + \frac{1}{x^2} - 2$ ✓
$= x^2 + \frac{1}{x^2}$

(b) $x = \dfrac{3-\sqrt{2}}{3+\sqrt{2}}$

$\dfrac{1}{x} = \dfrac{3+\sqrt{2}}{3-\sqrt{2}}$

so $x + \dfrac{1}{x} = \dfrac{3-\sqrt{2}}{3+\sqrt{2}} + \dfrac{3+\sqrt{2}}{3-\sqrt{2}}$

$= \dfrac{(3-\sqrt{2})^2 + (3+\sqrt{2})^2}{(3+\sqrt{2})(3-\sqrt{2})}$

$= \dfrac{9 - 6\sqrt{2} + 2 + 9 + 6\sqrt{2} + 2}{9-}$

$= \dfrac{22}{7}$ ✓

now $x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$ (from above)
$= \left(\frac{22}{7}\right)^2 - 2$ ✓
$= \frac{484}{49} - 2$ ✓
$= \cancel{\frac{484}{49}} = 7\frac{43}{49}$.