Student name/number: ____________________

Advanced Mathematical Publications

(Place your crest here)

2001
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

Total marks **(120)**

- Attempt Questions 1 – 10
- All questions are of equal value

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided on page 12
- All necessary working should be shown in every question

Question 1(12 marks) **Start a NEW page.** **Marks**

(a) Express $\dfrac{1}{3+2\sqrt{5}}$ in the form of $a+b\sqrt{5}$ **2**
where a and b are rational numbers.

(b) In the diagram PQ is parallel to RS. Find the value of θ, giving reasons. **2**

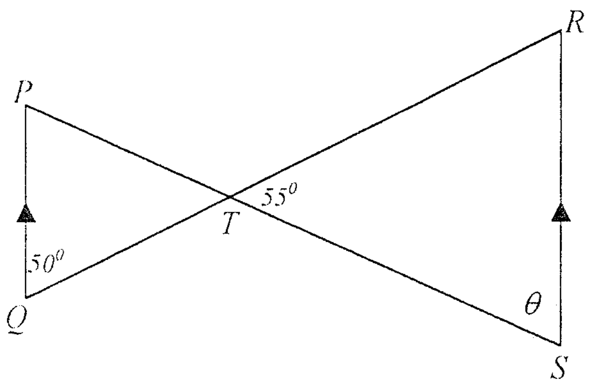


Figure not to scale

(c) Solve the quadratic equation $5x^2 = 3x + 2$ **2**

(d) The Goods and Services Tax has increased the quotes of tradespeople by 10%. **2**
If the quote for a job is \$8 250, how much of this amount is tax?

(e) Factorise fully $3m^3 - 24$. **2**

(f) Solve $\dfrac{2t}{t+5} = \dfrac{3}{8}$. **2**

Question 2(12 marks) **Start a NEW page.** **Marks**

(a) Kaz, the basketballer, has a probability of 0.9 of scoring a two pointer **2**
Find the probability of Kaz, from two shots

(i) scoring only once.

(ii) missing both times.

(b) Differentiate with respect to x

(i) $(4-3x)^8$ **2**

(ii) $3x\sin 4x$ **2**

(iii) $\dfrac{\log_e 3x}{x}$ **2**

(c) Find the gradient of the tangent to the curve $y=\sqrt{x}$ at the point where $x=25$. **2**

(d) A parabola has equation $y=x^2-8x+10$. **2**
Find the coordinates of the vertex and the focal length

Question 3 (12 marks) **Start a NEW page.** **Marks**

(a) (i) Draw a neat sketch of the line $3x + 2y - 9 = 0$ showing the main features **2**

(ii) Find the equation of the normal to the line $3x + 2y - 9 = 0$ passing through the point $(1, 3)$. **2**

(iii) Find where the normal cuts the x-axis and draw a neat sketch of the line on the same number plane as part (i). **2**

(iv) Find the area of the triangle enclosed by the two lines and the x-axis. **2**

(b) Find the point of intersection of the lines **2**

$2x - y = 12$ and $3x + y = 13$.

(c) Find the perpendicular distance from the point $(2, 4)$ to the line $3x - 4y = 1$. 2

Question 4 (12 marks) **Start a NEW page.** **Marks**

(a) An arc 9 cm long subtends an angle of 120° at the centre of the circle. Find the radius of the circle correct to 1 decimal place. **2**

(b) In a geometric series, the product of the first and second terms is 32 and the product of the third and fourth terms is 2. Find the first term, a and the common ratio, r. **3**

(c) Consider the curve $y = x^3 + 6x^2$. **5**

(i) Find the coordinates of the stationary points and determine their nature.

(ii) Find the coordinates of the point of inflexion.

(iii) Sketch the graph for the domain $-6 \le x \le 2$, clearly showing the main features.

(d) Let α and β be the roots of the equation $2x^2 - 4x + 6 = 0$ **2**
Find the value of $\left(\alpha^2 + 1\right)\left(\beta^2 + 1\right)$

Question 5 (12 marks) **Start a NEW page.** **Marks**

(a) 5

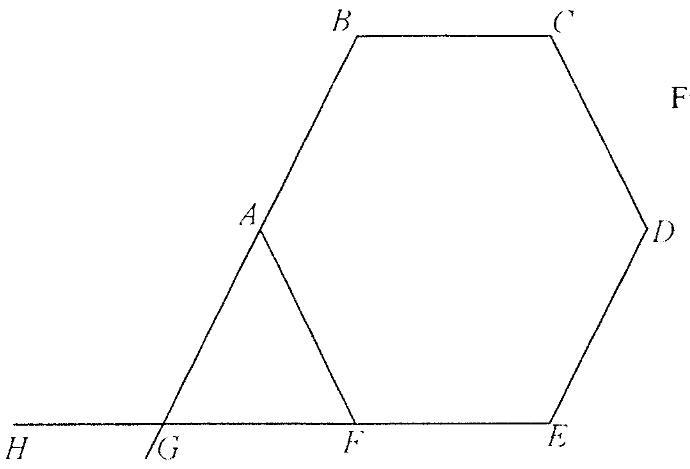


Figure not to scale

In the figure $ABCDEF$ is a regular hexagon with BA and EF produced to meet at G. The point H lies on EFG produced.

(i) Copy the diagram onto your writing booklet and find the size of $\angle BAF$.

(ii) Find the size of $\angle AGH$ giving reasons

(b) During a game of golf, Karrie hit the ball from the tee, T, 7° off line (as shown in the figure below) It travelled 213 metres to land at R which was 136 metres from the hole, H. 5
Find the distance from T to H to the nearest metre

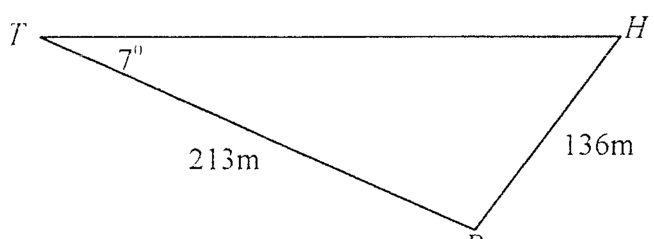


Figure not to scale

(c) Find the values of x which satisfy the inequality $7 - 4x < 13$. 2

Question 6 (12 marks) **Start a NEW page.** **Marks**

(a) A particle is projected vertically upwards from a point 30 metres above the ground. The path of the particle is given by **5**

$$h = 6\left(5 + 9t - 3t^2\right)$$

where h is the height in metres above the ground at time t seconds after projection. Find

(i) the time taken to reach the greatest height

(ii) the greatest height reached.

(iii) the magnitude and direction of the velocity after $2\frac{1}{2}$ seconds.

(iv) the magnitude and direction of the acceleration.

(b) Solve the equation **3**

$$x^6 = 2\left(5x^3 - 8\right).$$

(c) Consider the function given by $y = 2 + \cos^2 x$ **4**

(i) Copy and complete the following table onto your writing booklet. (Note that x is in radians)

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	3				

(ii) Apply Simpson's rule with five function values to find an approximation to

$$\int_0^{\pi} 2 + \cos^2 x \, dx .$$

Question 7 (12 marks) **Start a NEW page.** **Marks**

(a) (i) If $f(x) = 5 + 4\cos 3x$, find $f'(x)$ **1**

(ii) Hence show that $\int_{\frac{\pi}{3}}^{0} \frac{-12\sin 3x}{5 + 4\cos 3x}\,dx = 2.197$ **3**

correct to 3 decimal places.

(b) Two hundred tickets numbered 1 to 200 inclusive are sold in a raffle.
The raffle has three prizes. One ticket is drawn for the first prize and discarded
Then a second ticket is drawn for the second prize. This is discarded and then
the third prize ticket is drawn. What is the probability that

(i) all three prizes are won by tickets numbered 1 to 50 inclusive. **2**

(ii) at least one ticket numbered 1 to 50 inclusive wins a prize. **2**

(c) Evaluate:

(i) $\int_{0}^{1} 2e^{4x}\,dx$ **2**

(ii) $\int_{0}^{\frac{\pi}{2}} 2\sec^2 \frac{1}{2}\theta \; d\theta$ **2**

Question 8 (12 marks) **Start a NEW page.** **Marks**

(a) Find the centre and radius of the circle with equation **4**

$$x^2 + y^2 - 6x + 4y + 4 = 0$$

(b) Find the points of intersection between the curves $y = x^2 + 1$ and $y = 2 + x - x^2$ **5**
Calculate the area between the two curves.

(c) The present temperature of a star is 8500^0C and it is losing heat continuously in a way that in t million years, its temperature T^0C may be calculated from the equation **3**

$$T = T_0 e^{-0.06t}$$

(i) Find the temperature of the star in 4 million years (to the nearest degree).

(ii) After how many years from now will the temperature of the star be **halved.**

Question 9 (12 marks) **Start a NEW page.** **Marks**

(a) The seventh term in an arithmetic sequence is 14 and the thirteenth term is 32 **6**

(i) Find the value of the common difference and the value of the first term.

(ii) Find the sum of the first 70 terms.

(b)

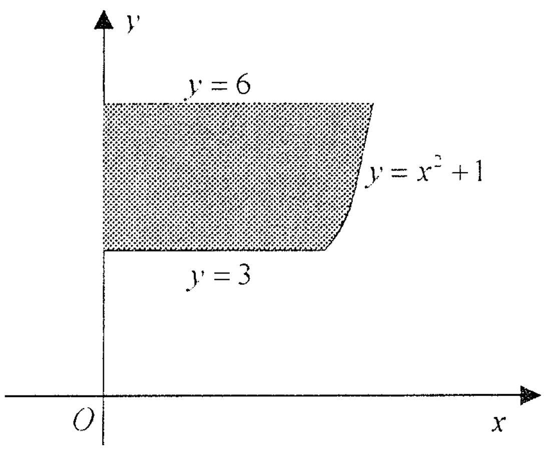


Figure not to scale

The shaded region in the diagram above is bounded by the curve $y = x^2 + 1$, the y-axis, and the lines $y = 3$ and $y = 6$. **3**

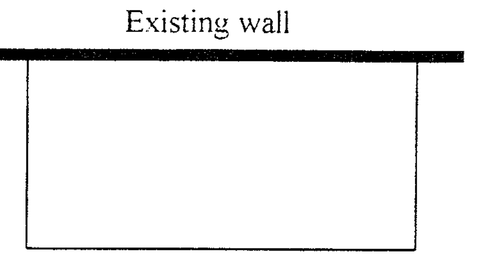
Calculate the volume of the solid of revolution formed when this region is rotated about the y-axis.

(c) \$15 000 is placed in a bank account and earns 7% p.a. interest compounded every six months. How much money to the nearest dollar is in the account at the end of 5 years, after the final interest has been paid. **3**

Question 10 (12 marks) **Start a NEW page.** **Marks**

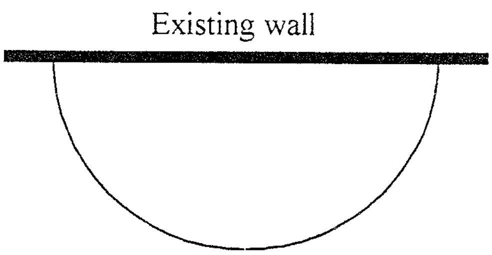
(a) Given that $\sum_{n=1}^{\infty} ar^n = \frac{a}{1-r}$, find the value of r such that the ratio of the second term to the limiting sum is 2.9. **4**

(b) **6**



200 metres of fencing is to be used with an existing wall to make a rectangular enclosure.

(i) Find the area of the largest possible rectangle.



(ii) Find the area enclosed had the fencing been made into a semi-circle with the existing wall.

(c) Solve $3\log_4 2 = \log_4 2x - \log_4 5$. **2**

End of examination

STANDARD INTEGRALS

$\int x^n \, dx$	$= \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$
$\int \frac{1}{x} \, dx$	$= \ln x, \quad x > 0$
$\int e^{ax} \, dx$	$= \frac{1}{a} e^{ax}, \quad a \neq 0$
$\int \cos ax \, dx$	$= \frac{1}{a} \sin ax, \quad a \neq 0$
$\int \sin ax \, dx$	$= -\frac{1}{a} \cos ax, \quad a \neq 0$
$\int \sec^2 ax \, dx$	$= \frac{1}{a} \tan ax, \quad a \neq 0$
$\int \sec ax \tan ax \, dx$	$= \frac{1}{a} \sec ax, \quad a \neq 0$
$\int \frac{1}{a^2 + x^2} \, dx$	$= \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$
$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx$	$= \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$
$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx$	$= \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$
$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx$	$= \ln\left(x + \sqrt{x^2 + a^2}\right)$

NOTE: $\ln x = \log_e x, \qquad x > 0$

Question 1 (12 Marks)

(a) $\dfrac{1}{\left(3+2\sqrt{5}\right)}\times\dfrac{\left(3-2\sqrt{5}\right)}{\left(3-2\sqrt{5}\right)}$

$=\dfrac{3-2\sqrt{5}}{-11}$ ✓

$=-\dfrac{3}{11}+\dfrac{2\sqrt{5}}{11}$ ✓

(b) $\angle TRS=50^0$ (Alternate angles $PQ\|RS$) ✓

$\theta=75^0$ (Angle sum of $\triangle$) ✓

(c) $5x^2-3x-2=0$

$(5x+2)(x-1)=0$ ✓

$x=-\dfrac{2}{5}$ or 1. ✓

(d) 110% is \$8250

1% is \$75 ✓

Tax 10% is \$750. ✓

(e) $3\left(m^3-2^3\right)$ ✓

$=3(m-2)\left(m^2+2m+4\right)$ ✓

(f) $16t=3t+15$

$13t=15$ ✓

$t=\dfrac{15}{13}=1\dfrac{2}{13}$ ✓

Question 2 (12 Marks)

(a) (i) $P\left(B\tilde{B}\text{ or }\tilde{B}B\right)=0.9\times0.1+0.1\times0.9$

$=0.18$ ✓

(ii) $P\left(\tilde{B}\tilde{B}\right)=0.1\times0.1=0.01$ ✓

(b) (i) $8(4-3x)^7(-3)$ ✓

$=-24(4-3x)^7$ ✓

(ii) $3x\cos4x\times4+\sin4x\times3$ ✓

$=12x\cos4x+3\sin4x$ ✓

(iii) $\dfrac{x.\dfrac{1}{3x}.3-\ln3x.1}{x^2}$ ✓

$=\dfrac{1-\ln3x}{x^2}$ ✓

(c) $\dfrac{dy}{dx}=\dfrac{1}{2}x^{-\frac{1}{2}}$ ✓

At $x=25$, $m=\dfrac{1}{10}$ ✓

(d) $y-10=x^2-8x$

$y-10+16=x^2-8x+16$

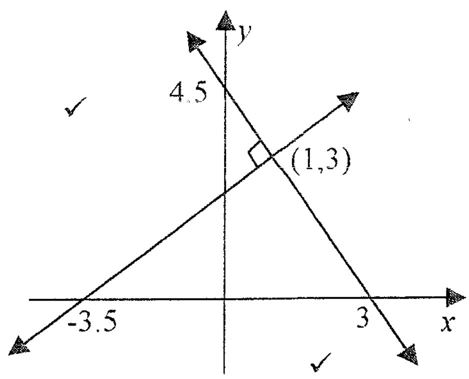
$y+6=(x-4)^2$ ✓

Vertex is $(4,-6)$

Focal length is $\dfrac{1}{4}$ ✓

Question 3 (12 Marks)

(a) (i)



(ii) Gradient of line is $-\dfrac{3}{2}$.

Gradient of normal is $\dfrac{2}{3}$. ✓

Equation of the normal is

$y-3=\dfrac{2}{3}(x-1)$

$\therefore\ 2x-3y+7=0.$ ✓

(iii) Normal cuts x-axis at $-3\frac{1}{2}$ ✓

Refer to diagram. ✓

(iv) $A=\frac{1}{2}bh=\frac{1}{2}\times\frac{15}{2}\times3$ ✓

$=11\frac{1}{4}$ sq. units ✓

(b) $2x-y=12$

$3x+y=13$

$\therefore\ x=5,\ y=-2.$ ✓

Point of intersection is $(5,-2)$. ✓

(c) Perpendicular distance is

$$\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$$

$$=\left|\frac{6-16-1}{\sqrt{9+16}}\right| \checkmark$$

$$=\frac{11}{5}=2\frac{1}{5}. \checkmark$$

Question 4 (12 Marks)

(a) $l=r\theta\Rightarrow9=r.\frac{2\pi}{3}$ ✓

$\therefore\ r=4.3$ cm (to 1 d.p.) ✓

(b) $T_1\times T_2=32$

$a\times ar=32$

$\therefore\ a^2r=32$ .... (i)

$T_3\times T_4=2$

$ar^2\times ar^3=2$

$\therefore\ a^2r^5=2$ .... (ii) ✓

Sub. (i) into (ii)

$32r^4=2$ ✓

$r=\pm\frac{1}{2};\ a=\pm8$ ✓

(c) $y=x^3+6x^2$

$\frac{dy}{dx}=3x^2+12x$

(i) Stationary points occur when $y'=0$

$3x(x+4)=0$

$x=0$ or -4

$y=0$ or 32

$\frac{d^2y}{dx^2}=6x+12$

At $x=0,\frac{d^2y}{dx^2}>0,\ \therefore$ Min at $(0,0)$. ✓

At

$x=-4,\ \frac{d^2y}{dx^2}<0;\ \therefore$ Max at $(-4,32)$. ✓

(ii) Point of inflexion when $\frac{d^2y}{dx^2}=0$

$6x+12=0$

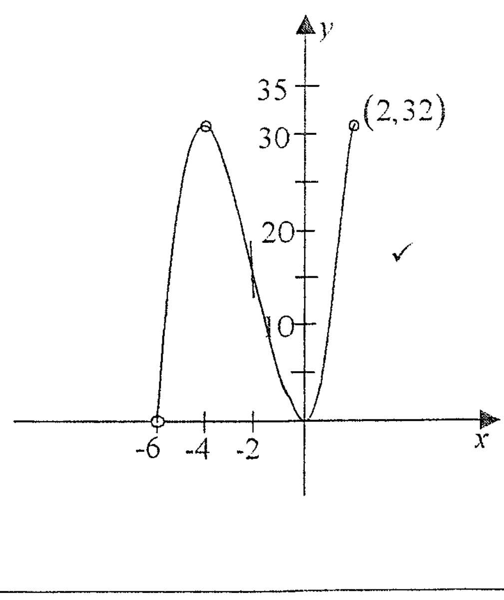
$x=-2\ ,\ y=16$ ✓

Test for concavity.

$f''(-1)>0$ and $f''(-3)<0$ ✓

$\therefore$ point of inflexion at $(-2,16)$.

(iii)



(d) $\left(\alpha^2+1\right)\left(\beta^2+1\right)$

$=(\alpha\beta)^2+(\alpha+\beta)^2-2\alpha\beta+1$ ✓

$=\left(\frac{6}{2}\right)^2+\left(\frac{4}{2}\right)^2-2\times\frac{6}{2}+1=8$ ✓

Question 5 (12 Marks)

(a) (i) Diagram ✓

$\angle BAF=\frac{4\times180^0}{6}=120^0$ ✓

(ii) $\angle GAF=60^0$ (Straight angle) ✓
$\angle AFG=60^0$ (Straight angle with $\angle EFA$) ✓

$\therefore\ \angle AGH=60^0+60^0=120^0$
(Exterior angle of $\triangle AFG$) ✓

(b) (ii) Using Sine rule:

$\frac{\sin\angle H}{213}=\frac{\sin 7^0}{136}$ ✓

$\sin\angle H=\frac{213\times\sin 7^0}{136}$

$\angle H=11^0$ ✓

$\therefore\ \angle R=180^0-\left(7^0+11^0\right)$ (Angle sum of $\triangle$)

$=162^0$ ✓

Using Cosine rule:

$r^2=213^2+136^2-2\times213\times136\times\cos162^0$ ✓

$r^2=118965.4$

$r=345$ m ✓

(c) $7-4x<13$

$-4x<6$ ✓

$x>-\frac{6}{4}$

$x>-1\frac{1}{2}$ ✓

Question 6 (12 Marks)

(a) $h=6\left(5+9t-3t^2\right)$

Greatest height reached when $\frac{dh}{dt}=0$

(i) $\frac{dh}{dt}=54-36t$

$0=9-6t$

$\therefore\ t=1\frac{1}{2}$ sec. ✓

(ii) Greatest height when $t=1\frac{1}{2}$ s

$h=70\frac{1}{2}$ m. ✓

(iii) $\frac{dh}{dt}=v$

$\therefore\ v=54-36t$ ✓

When $t=2\frac{1}{2}$ sec, $v=-36$ m/s. ✓

(iv) $a=\frac{dv}{dt}=-36$ m/s^2 ✓

(b) $x^6=10x^3-16$

$x^6-10x^3+16=0$

Let $u=x^3$

$u^2-10u+16=0$ ✓

$(u-2)(u-8)=0$

$x^3=2$ or 8. ✓

$x=\sqrt[3]{2}$ or 2. ✓

(c) (i) $y=2+\cos^2 x$

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	3	2.5	2	2.5	3
	y_1	y_2	y_3	y_4	y_5 ✓✓

(ii) Using Simpson's rule

$$A \approx \frac{h}{3}\{(y_1 + y_5) + 4(y_2 + y_4) + 2y_3\}$$
$$= \frac{\pi}{3}\{(3+3) + 4(2.5+2.5) + 2(2)\} \checkmark$$
$$= 10\pi \text{ sq. units or } 31.42 \text{ u}^2 \text{ (to 2d.p.)} \checkmark$$

Question 7 (12 Marks)

(a) (i) $f(x) = 5 + 4\cos 3x$

$f'(x) = 0 + 4(-\sin 3x).3$

$= -12\sin 3x$ ✓

(ii) $\int_{\frac{\pi}{3}}^{0} \frac{-12\sin 3x}{5+4\cos 3x}dx = \int_{\frac{\pi}{3}}^{0} \frac{f'(x)}{f(x)}dx$ ✓

$= \left[\ln(5+4\cos 3x)\right]_{\frac{\pi}{3}}^{0}$ ✓

$= \ln(5+4\cos 0) - \ln(5+4\cos\pi)$

$= 2.197$ ✓

(b) (i) $P(E) = \frac{50}{200} \times \frac{49}{100} \times \frac{48}{198} = 0.0149$ ✓✓

(ii) $P(E) = 1 - P(\text{No tickets between 1 \& 50})$

$= 1 - \frac{150}{200} \times \frac{149}{199} \times \frac{148}{198}$ ✓

$= 0.58$ ✓

(c) (i) $\int_0^1 2e^{4x}dx = \left[\frac{2e^{4x}}{4}\right]_0^1$ ✓

$= \frac{1}{2}(e^4 - e^0) = \frac{1}{2}(e^4 - 1)$ ✓

(ii) $\int_0^{\frac{\pi}{2}} 2\sec^2\frac{1}{2}\theta\, d\theta = \left[4\tan\frac{1}{2}\theta\right]_0^{\frac{\pi}{2}}$ ✓

$4\tan\frac{\pi}{4} - 4\tan 0 = 4$ ✓

Question 8 (12 Marks)

(a) $x^2 + y^2 - 6x + 4y + 4 = 0$

$x^2 - 6x + y^2 + 4y = -4$

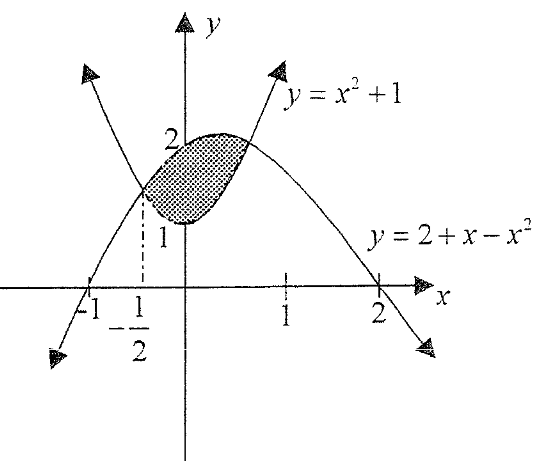
$(x^2 - 6x + 9) + (y^2 + 4y + 4) = -4 + 9 + 4$ ✓

$(x-3)^2 + (y+2)^2 = 9$ ✓

Centre $(3, -2)$ ✓

Radius = 3 units ✓

(b)



$x^2 + 1 = 2 + x - x^2$ ✓

$2x^2 - x - 1 = 0$

$x = -\frac{1}{2}, 1$ ✓

$A_C = \int_{-\frac{1}{2}}^{1} (2 + x - x^2) - (x^2 + 1)\, dx$ ✓

$= \int_{-\frac{1}{2}}^{1} 1 + x - 2x^2\, dx$

$= \left[x + \frac{x^2}{2} - \frac{2x^3}{3}\right]_{-\frac{1}{2}}^{1}$ ✓

$= 1\frac{1}{8}$ sq. units ✓

(c) (i) $T = T_0 e^{-0.06t}$

$T = 8500e^{-0.06 \times 4}$

$T = 6686^0C$ ✓

(ii) $T = T_0 e^{-0.06t}$

$4250 = 8500e^{-0.06t}$

$\frac{1}{2} = e^{-0.06t}$ ✓

$\ln\left(\frac{1}{2}\right) = -0.06t$

$t = 11.55$ million years. ✓

Question 9 (12 Marks)

(a) $T_7 = 14$, $T_{13} = 32$ in A.P.

(i) $a + 6d = 14$ ✓

$a + 12d = 32$ ✓

$-6d = -18$

$d = 3$ ✓

$a = -4$ ✓

(ii) $S_{70} = \frac{n}{2}\left[2a + (n-1)d\right]$

$= \frac{70}{2}[-8 + 69 \times 3]$ ✓

$= 6965$ ✓

(b) $V = \pi\int_3^6 \left(\sqrt{y-1}\right)^2 dy$ ✓

$= \pi\int_3^6 y - 1\, dy$

$= \pi\left[\frac{y^2}{2} - y\right]_3^6$ ✓

$= \pi\left[\left(\frac{36}{2} - 6\right) - \left(\frac{9}{2} - 3\right)\right]$

$= \frac{21\pi}{2}$ cu. Units. ✓

(c) $A_n = P\left(1 + \frac{r}{100}\right)^n$

$A_n = 15000\left(1 + \frac{3.5}{100}\right)^{10}$ ✓

$A_n = 15000 \times 1.035^{10}$ ✓

$A_n = \$21\,159$ ✓

Question 10 (12 Marks)

(a) $T_2 = ar$, Limiting sum $= \frac{a}{1-r}$

$\frac{ar}{\frac{a}{1-r}} = \frac{2}{9}$ ✓

$9ar = \frac{2a}{1-r}$ ✓

$9ar(1-r) = 2a$

$9r - 9r^2 = 2$ ✓

$9r^2 - 9r + 2 = 0$

$(3r-1)(3r-2) = 0$ ✓

$r = \frac{1}{3}$ or $\frac{2}{3}$

(b) (i) $x + y + x = 200$

$2x + y = 200$

$y = 200 - 2x$ ✓

$A = x(200 - 2x) = 200x - 2x^2$

$\frac{dA}{dx} = 200 - 4x$

$200 - 4x = 0$

$x = 50$ ✓

When $x = 50$, $\frac{d^2A}{dx^2} < 0$ ✓

Max. area $= 100 \times 50 = 5\,000\text{ m}^2$ ✓

(ii) Circumference of semicircle = 200

$$\frac{\pi d}{2} = 200$$

$$\pi d = 400$$

$$d = 127.32 \checkmark$$

Area of semi-circle

$$A = \frac{\pi r^2}{2} = 6366.2 \text{ m}^2 . \checkmark$$

(c) $3\log_4 2 = \log_4 2x - \log_4 5$

$$\log_4 2^3 = \log_4\left(\frac{2x}{5}\right) \checkmark$$

$$8 = \frac{2x}{5}$$

$$x = 20 \checkmark$$

End of examination